Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
November 2022

# Mathematics

StemUp Model Solution

Unit M8 Paper 1
**(Non-Calculator)**
Higher Tier



**[GMC81]**

*GMC81*

**THURSDAY 1 DECEMBER, 9.15am–10.30am**

**TIME**

1 hour 15 minutes.

**INSTRUCTIONS TO CANDIDATES**

Write your Centre Number and Candidate Number in the spaces provided at the top of this page. **You are provided with Higher Tier Additional Support Materials for use with this paper.**
**You must answer the questions in the spaces provided.**
**Do not write outside the boxed area on each page, on blank pages or tracing paper.**
Complete in black ink only. **Do not write with a gel pen.**
Answer **all eleven** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **must not** use a calculator for this paper.

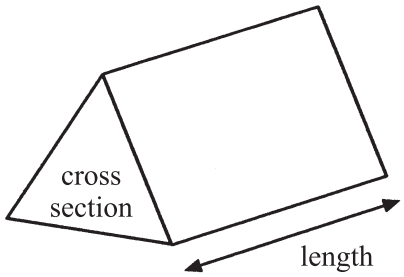
**INFORMATION FOR CANDIDATES**

The total mark for this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a ruler, compasses and a protractor.
The Formula Sheet is on page 2.

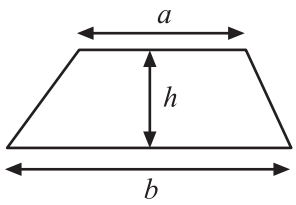
13335

# Formula Sheet

**Volume of prism** = area of cross section × length

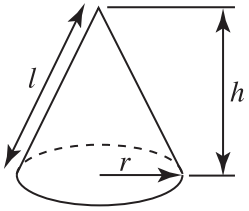



**Area of trapezium** = $\frac{1}{2}(a+b)h$



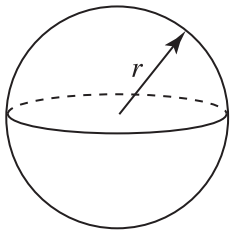

**Volume of cone** = $\frac{1}{3}\pi r^2 h$

**Curved surface area of cone** = $\pi r l$

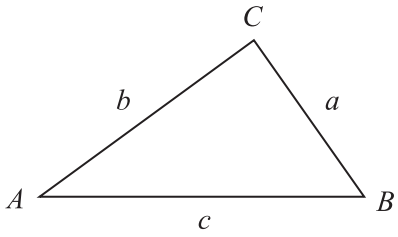



**Volume of sphere** = $\frac{4}{3}\pi r^3$

**Surface area of sphere** = $4\pi r^2$




**In any triangle *ABC***




**Quadratic Equation**

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Sine Rule:** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine Rule:** $a^2 = b^2 + c^2 - 2bc\cos A$

**Area of triangle** = $\frac{1}{2}ab\sin C$

13335

**1** A shape has a perimeter of 16 cm and an area of 10 $cm^2$

It is enlarged using a scale factor of 2

**(a)** What is the perimeter of the enlarged shape?

*16 × 2 = 32*

Answer *32* cm [1]

**(b)** How many times bigger is the area of the enlarged shape than the area of the original shape?

*area scale factor = $2^2$ = 4*

Answer *4 times* [1]

**2** Each interior angle of a regular polygon is 140°.

How many sides has the polygon?

*exterior angle = 180° − 140° = 40°*

*number of sides = 360° ÷ 40° = 9*

Answer *9* [2]

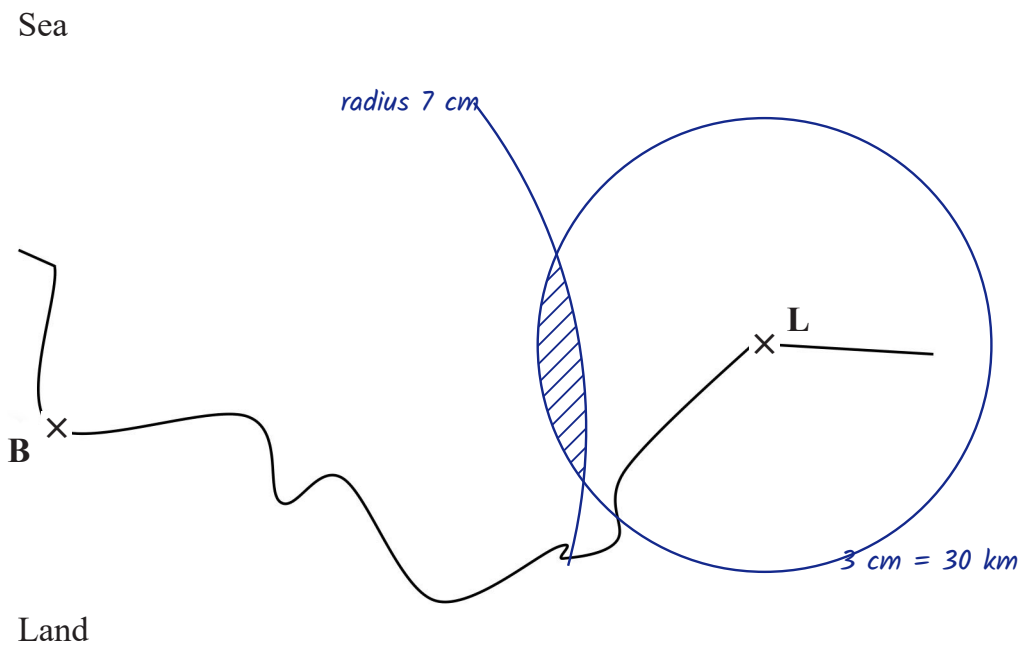
**[Turn over**

**3** The diagram shows a section of coastline with a lifeboat station marked at B and a lighthouse marked at L.

A sinking ship sends a distress signal.

The ship is less than 70 km from B and less than 30 km from L.

Using a scale of 1 cm = 10 km, **shade the region** in which the ship could be.




[3]

**4** Tom has some 20p coins and some 50p coins.

The total value of Tom's coins is £8.60

Seana has twice as many 20p coins as Tom, but only half as many 50p coins as Tom.

The total value of Seana's coins is £8.20

Find how many of each type of coin Tom has.

A solution by trial and improvement will not be accepted.

Let Tom have x 20p coins and y 50p coins.

Tom: $20x + 50y = 860$ (1)

Seana: $20(2x) + 50(\frac{1}{2}y) = 820$

$40x + 25y = 820$ (2)

(2) × 2: $80x + 50y = 1640$ (3)

(3) − (1): $60x = 780$

$x = 13$

Sub into (1): $260 + 50y = 860$

$50y = 600$

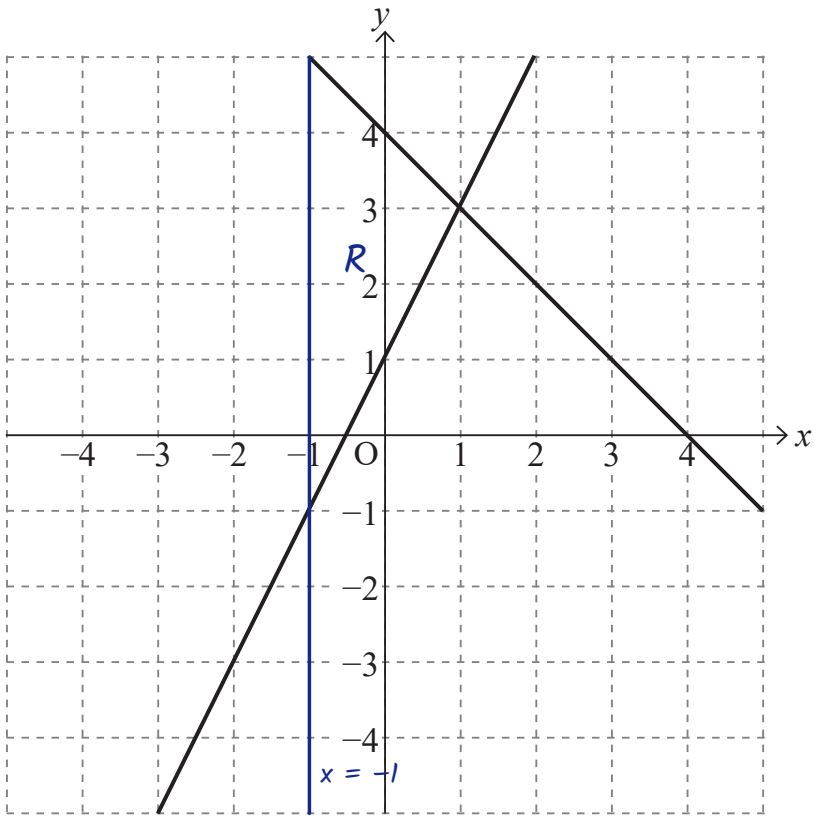
$y = 12$

Check Seana: 26 × 20p + 6 × 50p = £5.20 + £3.00 = £8.20

Answer 13 20p coins, 12 50p coins [5]

**[Turn over**

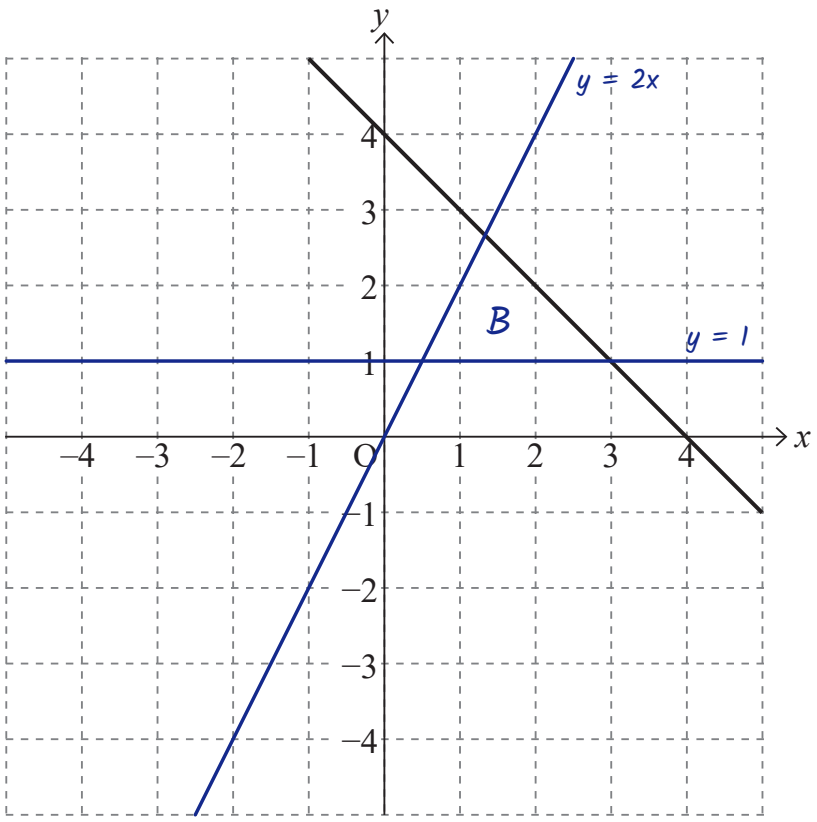
**5 (a)**




The lines $y = 2x + 1$ and $x + y = 4$ have been drawn on the grid.

By drawing another line on the grid above, indicate clearly by the letter R the region satisfying $y \geq 2x + 1$ and $x + y \leq 4$ and $x \geq -1$ [2]

**(b)**




By drawing more lines on the grid above, indicate clearly by the letter B the region satisfying $y \leq 2x$ and $x + y \leq 4$ and $y \geq 1$ [2]

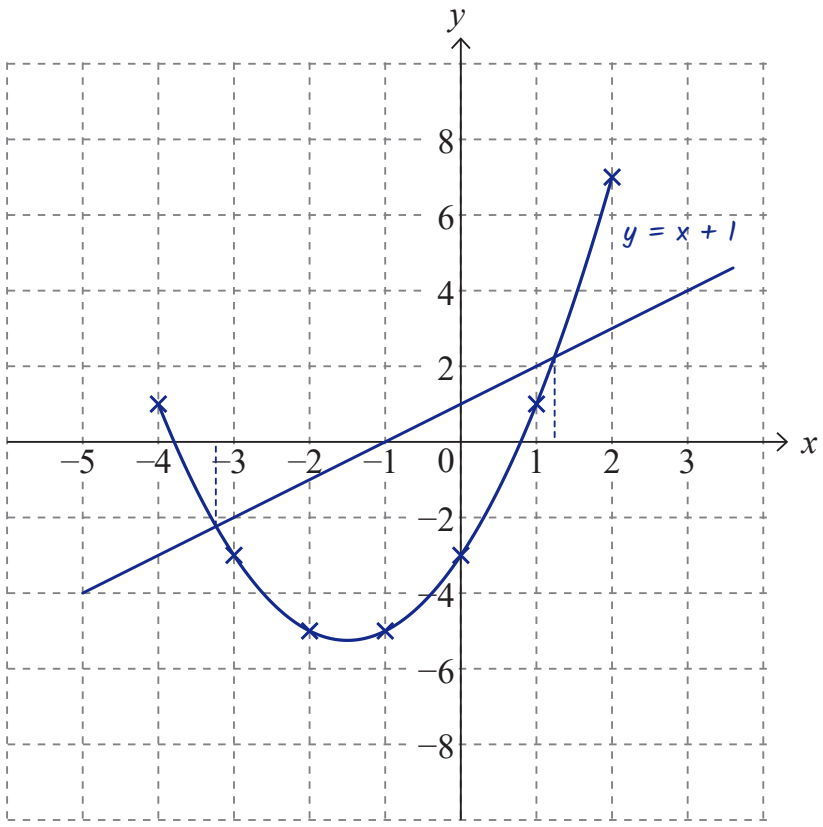
**[Turn over**

**6** **(a)** Complete the table for $y = x^2 + 3x - 3$

| $x$ | −4 | −3 | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| $y$ | 1 | −3 | −5 | −5 | −3 | 1 | 7 |

[2]

**(b)** Draw the graph of $y = x^2 + 3x - 3$ from $x = -4$ to $x = 2$ [2]




**(c)** Use your graph to estimate the solutions to $x^2 + 3x - 3 = 0$

curve crosses the x axis (y = 0)

Answer x ≈ −3.8 and x ≈ 0.8 [2]




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**(d) (i)** By drawing a line on the grid, solve $x^2 + 3x - 3 = x + 1$

Draw $y = x + 1$ on the grid and read off where it crosses the curve.

Answer $x \approx -3.2$ and $x \approx 1.2$ [2]

**(ii)** Write down in simplest form the quadratic equation that has been solved in **(i)**.

$x^2 + 3x - 3 - x - 1 = 0$

Answer $x^2 + 2x - 4 = 0$ [1]

**(e)** What line would be drawn on the grid to solve $x^2 + 4x - 6 = 0$?

$x^2 + 4x - 6 = 0$

$x^2 + 3x - 3 + x - 3 = 0$

$x^2 + 3x - 3 = -x + 3$

Answer $y = -x + 3$ [2]

**[Turn over**

**7** **(a)** Write the decimal number 15 as a binary number.

8 4 2 1
1 1 1 1 8 + 4 + 2 + 1 = 15

Answer 1111 [1]

**(b)** Write the binary number 1000000 as a decimal number.

64 32 16 8 4 2 1
1 0 0 0 0 0 0

Answer 64 [1]

**8** Change 0.08181818181… into a fraction.

$x = 0.0818181...$

$100x = 8.1818181...$

$100x - x = 8.1818... - 0.0818...$

$99x = 8.1$

$x = \frac{8.1}{99} = \frac{81}{990} = \frac{9}{110}$

Answer $\frac{9}{110}$ [2]

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**(Questions continue overleaf)**

**[Turn over**

**9** A (8, −6) and B (−8, 6) are ends of the diameter AB of a circle.

**(a)** What is the centre of the circle?

midpoint of AB

Answer (0, 0) [1]

**(b)** What is the equation of the circle?

$r^2 = 8^2 + (-6)^2 = 64 + 36 = 100$

$r = 10$

Answer $x^2 + y^2 = 100$ [1]

**(c)** What is the gradient of the diameter AB?

$\text{gradient} = \frac{6 - (-6)}{-8 - 8} = \frac{12}{-16} = -\frac{3}{4}$

Answer $-\frac{3}{4}$ [1]

**(d)** Find the equation of the tangent to the circle at A.

gradient of radius OA $= -\frac{3}{4}$

tangent $\perp$ radius, so gradient $= \frac{4}{3}$

$y = \frac{4}{3}x + c$ through (8, −6)

$-6 = \frac{4}{3} \times 8 + c = \frac{32}{3} + c$

$c = -6 - \frac{32}{3} = -\frac{50}{3}$

Answer $y = \frac{4}{3}x - \frac{50}{3}$ [3]





**(e)** Write down the equations of the two tangents to this circle which are parallel to the $y$ axis.

centre (0, 0), radius 10

Answer $x = 10$ and $x = -10$ [2]

**(f)** Find the points of intersection of these tangents with the tangent at A.

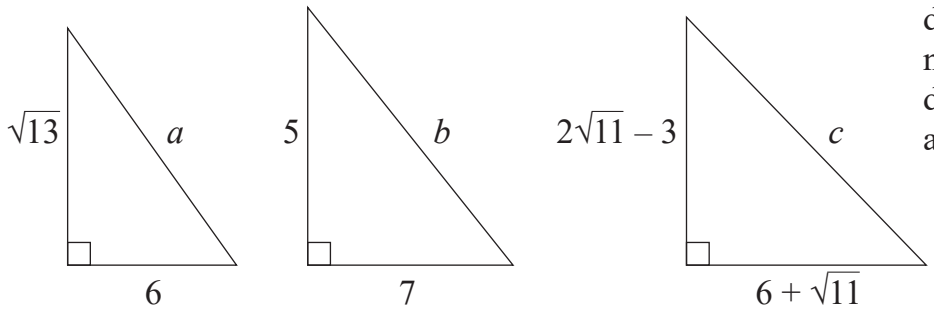
$x = 10$: $y = \frac{4}{3} \times 10 - \frac{50}{3} = \frac{40 - 50}{3} = -\frac{10}{3}$

$x = -10$: $y = \frac{4}{3} \times (-10) - \frac{50}{3} = \frac{-40 - 50}{3} = -30$

Answer $(10, -\frac{10}{3})$ and $(-10, -30)$ [2]

**[Turn over**

**10**




diagrams not drawn accurately

For each of the right-angled triangles above, state whether the length of the hypotenuse is rational or irrational, giving reasons for your answers.

$a^2 = (\sqrt{13})^2 + 6^2 = 13 + 36 = 49, \quad a = \sqrt{49} = 7$

$b^2 = 5^2 + 7^2 = 25 + 49 = 74, \quad b = \sqrt{74}$

$c^2 = (2\sqrt{11} - 3)^2 + (6 + \sqrt{11})^2$

$= (44 - 12\sqrt{11} + 9) + (36 + 12\sqrt{11} + 11)$

$= 53 - 12\sqrt{11} + 47 + 12\sqrt{11}$

$= 100, \quad c = \sqrt{100} = 10$

Answer $a$ is *rational* because *a = 7, which is an integer*

$b$ is *irrational* because *b = √74 and 74 is not a square number*

$c$ is *rational* because *c = √100 = 10, which is an integer*

[6]

**11** $4^{-\frac{1}{2}}$ $\left(\frac{1}{9}\right)^{-\frac{1}{2}}$ $64^{\frac{2}{3}}$ $3^{\frac{3}{2}}$ $1^{\frac{5}{7}}$

Prove that one of the above is a prime number and one is a surd.

$4^{-1/2} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

$(\frac{1}{9})^{-1/2} = 9^{1/2} = \sqrt{9} = 3$, which is a prime number

$64^{2/3} = (\sqrt[3]{64})^2 = 4^2 = 16$

$3^{3/2} = \sqrt{(3^3)} = \sqrt{27} = 3\sqrt{3}$, which is a surd (irrational)

$1^{5/7} = 1$

So $(\frac{1}{9})^{-1/2} = 3$ is prime and $3^{3/2} = 3\sqrt{3}$ is a surd.

[3]

**THIS IS THE END OF THE QUESTION PAPER**

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| For Examiner's use only | |
|---|---|
| **Question Number** | **Marks** |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
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| 6 | |
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| 9 | |
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| 11 | |
| **Total Marks** | |

**Examiner Number**

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General Certificate of Secondary Education
November 2022

# GCSE Mathematics

# HIGHER TIER
# ADDITIONAL SUPPORT MATERIALS
# (For use in November 2022)



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## HIGHER TIER ADDITIONAL SUPPORT MATERIALS (November 2022)

### Numbers

Highest Common Factor (HCF): The highest common factor is the largest number that divides evenly into two or more numbers, e.g. 4 is the HCF of 8 and 20

### Trial and Improvement

This is a method of trying different values in an equation until you get a suitable solution (e.g. to 1 decimal place).

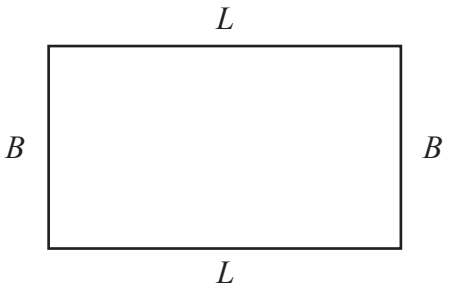
### Range

The range of a set of data is the difference between the largest value and the smallest value in the data set.

### Mean

The mean of a set of data is the sum of all the data values divided by the number of data values.

### Area and Volume




The area of a rectangle is found by multiplying the length of the rectangle by the breadth.

$A = L \times B$ where $A$ is area, $L$ is length and $B$ is breadth.

The area of a triangle is found by multiplying half the length of the base by the perpendicular height of the triangle.

$A = \frac{1}{2}bh$ where $b$ is the base and $h$ is the perpendicular height of the triangle.

The area of a circle is $A = \pi r^2$ where $r$ is the radius of the circle.

The volume of a cuboid is found by multiplying the length by the breadth by the height of the cuboid.

$V = L \times B \times H$ where $V$ is volume, $L$ is length, $B$ is breadth and $H$ is height.

**Angles**

There are 180° on a straight line.
There are 180° inside a triangle.
An isosceles triangle is a triangle with 2 equal sides and 2 equal angles.
The sum of all the angles inside a polygon is given by $180(n - 2)$ where $n$ is the number of sides in the polygon.

**Pie Chart**

In a pie chart, the total angle that corresponds to the entire data set is 360°

**Probability**

The sum of the probabilities of all outcomes equals 1

**Compound Measures**

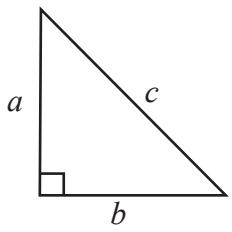
$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

**Pythagoras' Theorem**

If $a$, $b$ and $c$ are the sides of a right angled triangle shown below, then

$$a^2 + b^2 = c^2$$






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**Midpoint of a line**

If $(x_1, y_1)$ and $(x_2, y_2)$ are the end points of a line, then the coordinates of the midpoint $M$ of the line are

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Gradient of a line**

If $(x_1, y_1)$ and $(x_2, y_2)$ are two points on a line, then the gradient $m$ of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**Lines**

Parallel lines have the same gradient.

If a straight line has gradient $m$, then a line which is perpendicular to this line has a gradient $-\frac{1}{m}$

**Equation of a circle**

The equation of a circle of radius $r$ and centre, the origin (0, 0), is given by $x^2 + y^2 = r^2$ where $(x, y)$ are the coordinates of any point on the circumference of the circle.

**Tangent/Radius property**

The tangent to a circle is perpendicular to the radius at the point of contact with the circle.

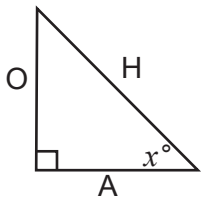
**Cyclic Quadrilateral property**

The opposite angles of a cyclic quadrilateral add up to 180°

**Alternate Segment Theorem**

In a circle, the angle between a chord and a tangent through one of the end points of the chord is equal to the angle in the alternate segment.

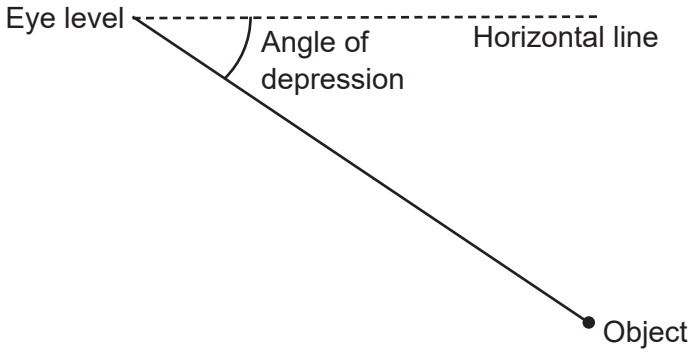
**Trigonometric ratios in right angled triangles**




$\sin x° = \frac{O}{H}$ $\cos x° = \frac{A}{H}$ $\tan x° = \frac{O}{A}$

**Angle of depression**

If a person stands and looks down at an object, the **angle of depression** is the angle between the horizontal line of sight and the object.




**Frequency density in histograms**

$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$