Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
November 2022

Mathematics

StemUp Model Solution

Unit M4
(With calculator)
Higher Tier



[GMC41]

GMC41

TUESDAY 29 NOVEMBER, 9.15am–11.15am

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page. **You are provided with Higher Tier Additional Support Materials for use with this paper.**
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all twenty-three** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

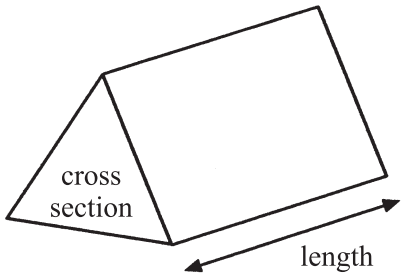
INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

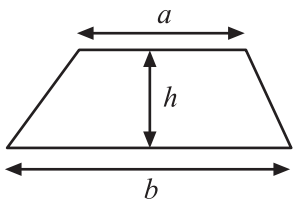
13328

Formula Sheet

Volume of prism = area of cross section × length

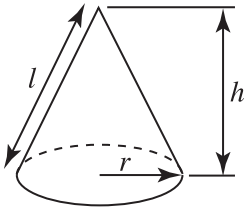


Area of trapezium = $\frac{1}{2}(a+b)h$



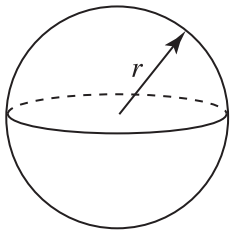
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

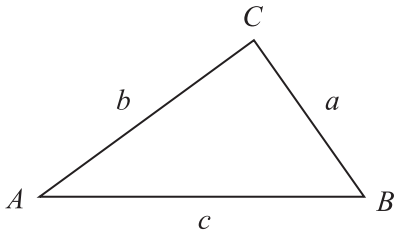


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

13328

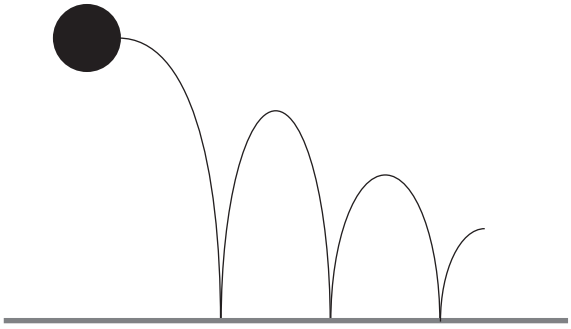
1 A ball is dropped from a height of 2.5 m above the ground.

Each time it hits the ground it bounces to $\frac{3}{5}$ of its previous height.

Kevin says that after it hits the ground for the third time it will not bounce above half a metre.

Is Kevin correct?

You must show all your working.



$\frac{3}{5} \times 2.5 = 1.5$ m (first bounce)

$\frac{3}{5} \times 1.5 = 0.9$ m (second bounce)

$\frac{3}{5} \times 0.9 = 0.54$ m (third bounce)

0.54 m > 0.5 m, so after the third bounce the ball goes above half a metre. Kevin is not correct.

Answer No [3]

[Turn over

2 Two points P(4, 8) and Q(x, y) are joined to form a straight line.

The midpoint of the line PQ has coordinates (1, 7)

Find the coordinates of Q.

$\frac{4 + x}{2} = 1$ $\quad$ $\frac{8 + y}{2} = 7$

$4 + x = 2$ $\quad$ $8 + y = 14$

$x = -2$ $\quad$ $y = 6$

Answer Q (-2 , 6) [3]

3 **(a)** Write down any two numbers whose highest common factor (HCF) is 8

Answer 8 and 16 [2]

(b) Given that $600 = 2^a \times 3 \times 5^b$ find the values of a and b

2 | 600
2 | 300
2 | 150
3 | 75
5 | 25
5 | 5
1

$600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5$

$600 = 2^3 \times 3 \times 5^2$

Answer a = 3 b = 2 [2]

4 Mary and Anne both go to a shop, each with the same amount of money.

Mary buys 7 bars of chocolate at y pence each and receives 19p change.

Anne buys 5 bars of chocolate at y pence each and receives 65p change.

By forming and solving an equation, work out how much money each girl had going to the shop.

You must show all your working.

A solution by trial and improvement will not be acceptable.

Mary: $7y + 19$ Anne: $5y + 65$ (both in pence)

$7y + 19 = 5y + 65$

$7y - 5y = 65 - 19$

$2y = 46$

$y = 23$

Money $= 7 \times 23 + 19 = 161 + 19 = 180p$

Check: $5 \times 23 + 65 = 115 + 65 = 180p$

$180p = £1.80$

Equation $7y + 19 = 5y + 65$ [1]

Answer: Each girl had £1.80 going to the shop. [3]

[Turn over

5 A cuboid has length 90 cm, width 45 cm and height 30 cm.

It has a mass of 24 300 g.

Calculate the density of the cuboid.

Volume = 90 × 45 × 30 = 121 500 cm^3

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{24\ 300}{121\ 500}$$

= 0.2 g/cm^3

Answer 0.2 g/cm^3 [3]

6 Leah walks 4.7 m in a straight line.

She then turns 90° clockwise and walks 2.5 m in another straight line.

How far is Leah from where she started?

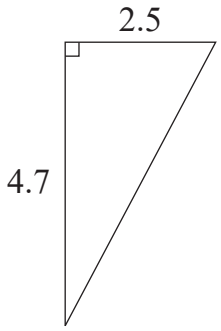


diagram not drawn accurately

$d^2 = 4.7^2 + 2.5^2$

$d^2 = 22.09 + 6.25 = 28.34$

$d = \sqrt{28.34}$

$d = 5.323532662...$

$d = 5.32$ m

Answer 5.32 m [3]

[Turn over

7 The front door of Martin's house is wooden.

The top of the door is a semicircle.

It has a window in the shape of a rhombus as shown in the diagram.

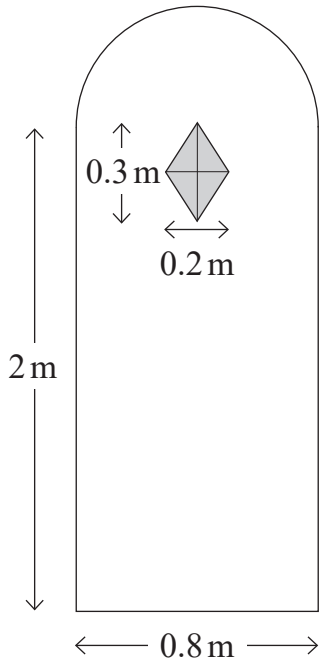


diagram not drawn accurately

What is the area of the wooden part of the door?

Rectangle = $2 \times 0.8 = 1.6$

Semicircle = $\frac{1}{2} \times \pi \times 0.4^2 = 0.2513274123...$

Rhombus window = $\frac{1}{2} \times 0.2 \times 0.3 = 0.03$

Wood = $1.6 + 0.2513274123... - 0.03$

$= 1.821327412... = 1.82\ m^2$

Answer 1.82 m^2 [5]

8 One third of the pupils in a school are girls.

15% of the girls are in the Senior School.

There are 306 girls in the Junior School.

How many pupils are in the school?

Junior School girls = 100% − 15% = 85% of the girls

85% = 306

1% = 306 ÷ 85 = 3.6

100% = 3.6 × 100 = 360 girls

Girls are one third, so pupils = 360 × 3 = 1080

Answer *1080* [4]

[Turn over



9 Given that $(n-1)^2 + n + (n-a) \equiv n^2 - 2$

find the value of a

$(n-1)^2 + n + (n-a) \equiv n^2 - 2$

$n^2 - 2n + 1 + n + n - a \equiv n^2 - 2$

$n^2 + 1 - a \equiv n^2 - 2$

Compare constants: $1 - a = -2$

$a = 3$

Answer $a =$ 3 [3]



10 A line L passes through the points with coordinates (0, 2) and (2, 8).

Find the equation of any line parallel to line L.

gradient $= \frac{8-2}{2-0} = \frac{6}{2} = 3$

Parallel lines have the same gradient, 3

$y = 3x + c$ with $c \neq 2$, e.g. $c = 5$

Answer $y = 3x + 5$ [4]

11 When the square of the number n is added to double the number n, the total is 24

By forming and solving an equation, find the value of n where n is a negative number.

A solution by trial and improvement will not be accepted.

$n^2 + 2n = 24$

$n^2 + 2n - 24 = 0$

$(n + 6)(n - 4) = 0$

$n = -6$ or $n = 4$

n is negative so $n = -6$

Equation $n^2 + 2n = 24$ [1]

Answer $n =$ -6 [3]

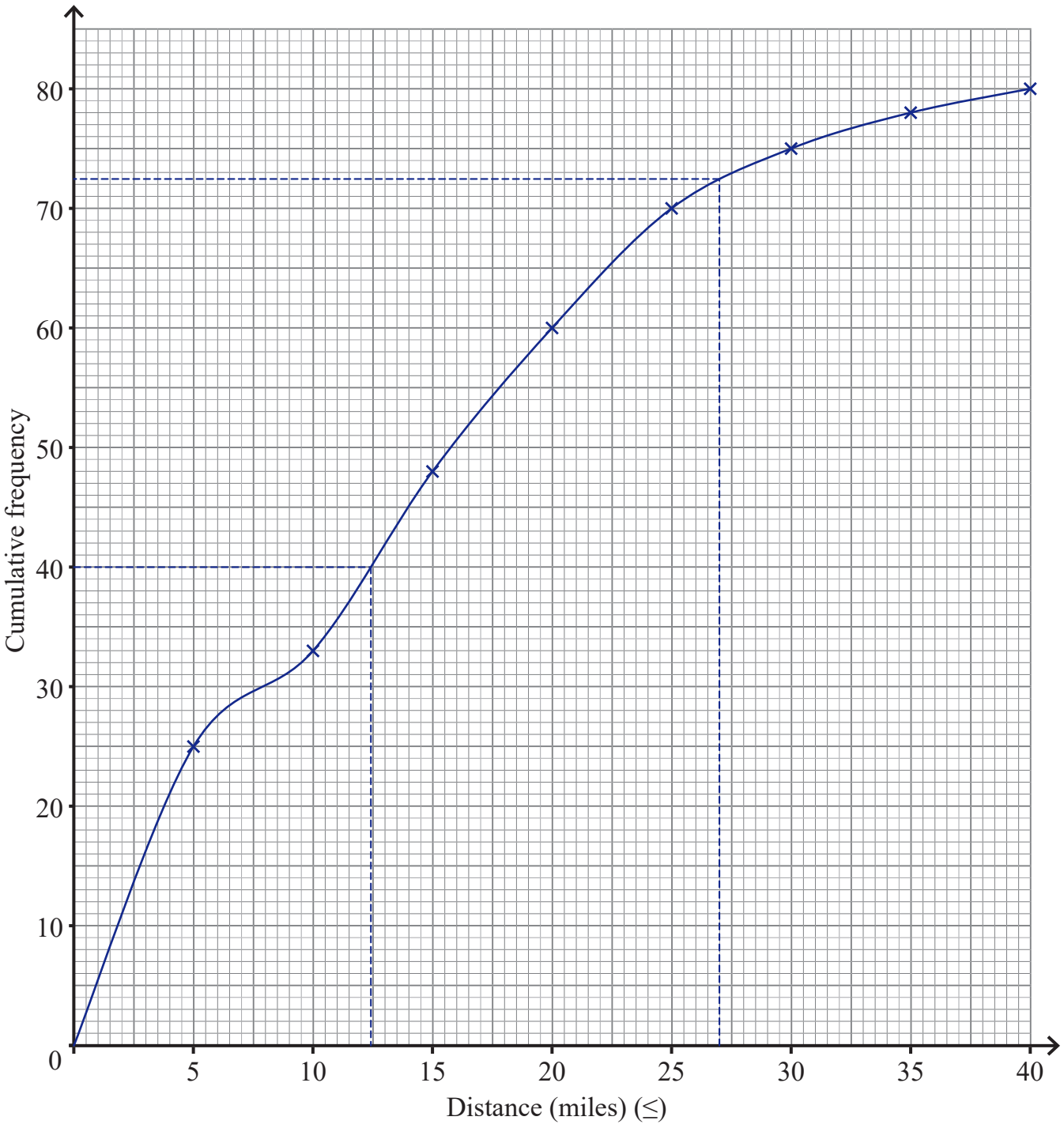
[Turn over

12 The frequency table shows the distance travelled to school by 80 teachers.

Distance (miles)	Number of teachers	Cumulative frequency
$0 < d \leq 5$	25	25
$5 < d \leq 10$	8	33
$10 < d \leq 15$	15	48
$15 < d \leq 20$	12	60
$20 < d \leq 25$	10	70
$25 < d \leq 30$	5	75
$30 < d \leq 35$	3	78
$35 < d \leq 40$	2	80

(a) Complete the cumulative frequency column in the table. [1]

(b) On the graph below draw a cumulative frequency graph for this information. [3]



[Turn over

stemup.co.uk

(c) Use your graph to estimate

(i) the median distance

½ × 80 = 40th

Answer 12.4 miles [1]

(ii) the number of teachers who travel more than 27 miles.

Reading at 27 miles = 72 teachers

80 − 72 = 8

Answer 8 [2]

13 An aeroplane is flying at a height of 9000 m.

The angle of depression from the aeroplane to a ship in the sea is 32°

Find the distance from the aeroplane to the ship.

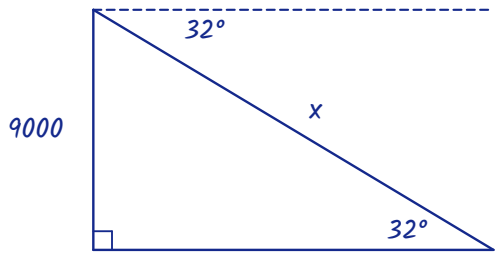
Angle of elevation from ship = 32° (alternate)

$\sin 32° = \frac{9000}{x}$

$x = \frac{9000}{\sin 32°}$

x = 16983.71923...

x = 16 983.7 m



Answer 16 983.7 m [3]

14 An equation used in physics is $W = Fd$

where W = work done in Joules (J)

F = force in Newtons (N)

d = distance in metres (m)

The work done in moving a car is given as 12 500 J correct to 3 significant figures.

The car moves a distance of 215 m correct to the nearest metre.

Calculate the maximum and minimum force applied to the car.

W: 12 450 ≤ W < 12 550 d: 214.5 ≤ d < 215.5

F = W ÷ d

$$\text{Maximum } F = \frac{\text{max } W}{\text{min } d} = \frac{12\,550}{214.5} = 58.50815851...$$

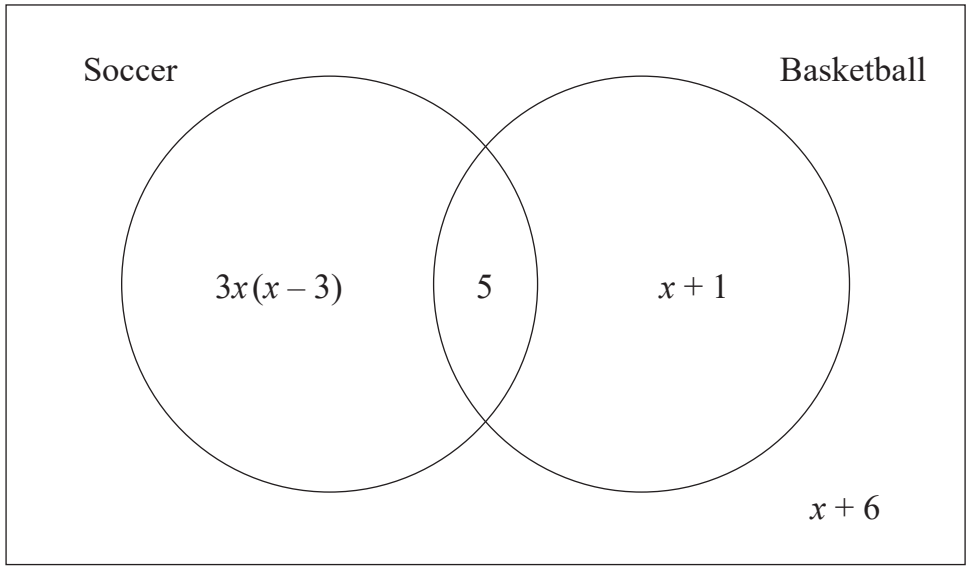
$$\text{Minimum } F = \frac{\text{min } W}{\text{max } d} = \frac{12\,450}{215.5} = 57.77262181...$$

Answer maximum force = 58.50815851 N

minimum force = 57.77262181 N [4]

[Turn over

15 The Venn diagram shows information about 32 pupils in a class.



By setting up and solving a quadratic equation, calculate how many pupils in the class play soccer only.

$3x(x-3) + 5 + (x+1) + (x+6) = 32$

$3x^2 - 9x + 5 + x + 1 + x + 6 = 32$

$3x^2 - 7x + 12 = 32$

$3x^2 - 7x - 20 = 0$

$(3x+5)(x-4) = 0$

$x = 4$ or $x = -5/3$ (not possible, gives negative numbers)

Soccer only $= 3x(x-3) = 3 \times 4 \times (4-3) = 12$

Answer 12 pupils [5]

16 Factorise fully

$$63x^2 - 28$$

$63x^2 - 28 = 7(9x^2 - 4)$

$= 7((3x)^2 - 2^2)$

$= 7(3x + 2)(3x - 2)$

Answer $7(3x + 2)(3x - 2)$ [3]

17 Expand and simplify

$$(4x - y)(2y - 7x)$$

$(4x - y)(2y - 7x)$

$= 8xy - 28x^2 - 2y^2 + 7xy$

$= 15xy - 28x^2 - 2y^2$

Answer $15xy - 28x^2 - 2y^2$ [3]

[Turn over

18 The straight line L_1 has equation

$$2x - 5y + 6 = 0$$

The straight line L_2 is perpendicular to L_1 and passes through the point (3,1)

Find the equation of the straight line L_2

$2x - 5y + 6 = 0$

$5y = 2x + 6$

$y = \frac{2}{5}x + \frac{6}{5}$ gradient of $L_1 = \frac{2}{5}$

Perpendicular gradient $= -\frac{5}{2}$ (since $\frac{2}{5} \times -\frac{5}{2} = -1$)

$y = -\frac{5}{2}x + c$ through (3, 1):

$1 = -\frac{5}{2} \times 3 + c$ so $1 = -\frac{15}{2} + c$

$c = 1 + \frac{15}{2} = \frac{17}{2}$

L_2: $y = -\frac{5}{2}x + \frac{17}{2}$ (or $5x + 2y = 17$)

Answer $y = -\frac{5}{2}x + \frac{17}{2}$ [4]

19 In the diagram shown

QR = RS

PQ is parallel to SR

TPV is a tangent to the circle at the point P

Angle SPV = 52°

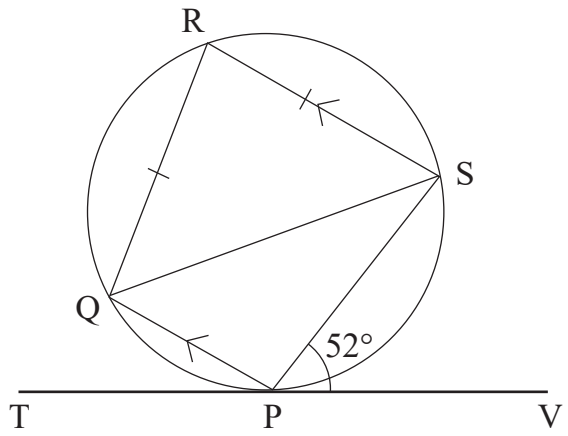


diagram not drawn accurately

Find the size of angle SPQ explaining clearly each step of your solution.

Angle PQS = 52° because of the alternate segment theorem (angle between tangent and chord = angle in alternate segment)

Angle QSR = 52° because alternate angles are equal (PQ parallel to SR)

Angle RQS = 52° because base angles of an isosceles triangle are equal (QR = RS), so angle SRQ = 180 − 52 − 52 = 76° (angles in a triangle)

Angle SPQ = 180 − 76 = 104° because opposite angles in a cyclic quadrilateral add up to 180°

Answer Angle SPQ = 104 ° [6]

[Turn over

20 Solve the equation

$$\frac{4}{3x+1} - \frac{5}{7-2x} = \frac{3}{4}$$

giving your answers correct to 2 decimal places.

$\frac{4(7-2x) - 5(3x+1)}{(3x+1)(7-2x)} = \frac{3}{4}$

$\frac{28 - 8x - 15x - 5}{21x - 6x^2 + 7 - 2x} = \frac{3}{4}$

$\frac{23 - 23x}{7 + 19x - 6x^2} = \frac{3}{4}$

$4(23 - 23x) = 3(7 + 19x - 6x^2)$

$92 - 92x = 21 + 57x - 18x^2$

$18x^2 - 149x + 71 = 0$

$x = \frac{149 \pm \sqrt{((-149)^2 - 4(18)(71))}}{2 \times 18} = \frac{149 \pm \sqrt{17089}}{36}$

$x = (149 + 130.7249...) \div 36 = 7.770136...$

$x = (149 - 130.7249...) \div 36 = 0.507641...$

$x = 7.77$ or $x = 0.51$

Answer $x =$ 7.77 or 0.51 [7]

21 There are 3200 passengers on a cruise ship.

A stratified sample is taken to get feedback on the entertainment on board.

The table shows some information about the people on board the ship.

	Men	Women	Children
Total on ship	1152	1408	640
Number in sample	27	33	15

Complete the table showing all working out.

Men: $\frac{1152}{3200} \times n = 27$ so $n = 27 \times \frac{3200}{1152} = 75$ (sample size)

Women = $\frac{1408}{3200} \times 75 = 33$

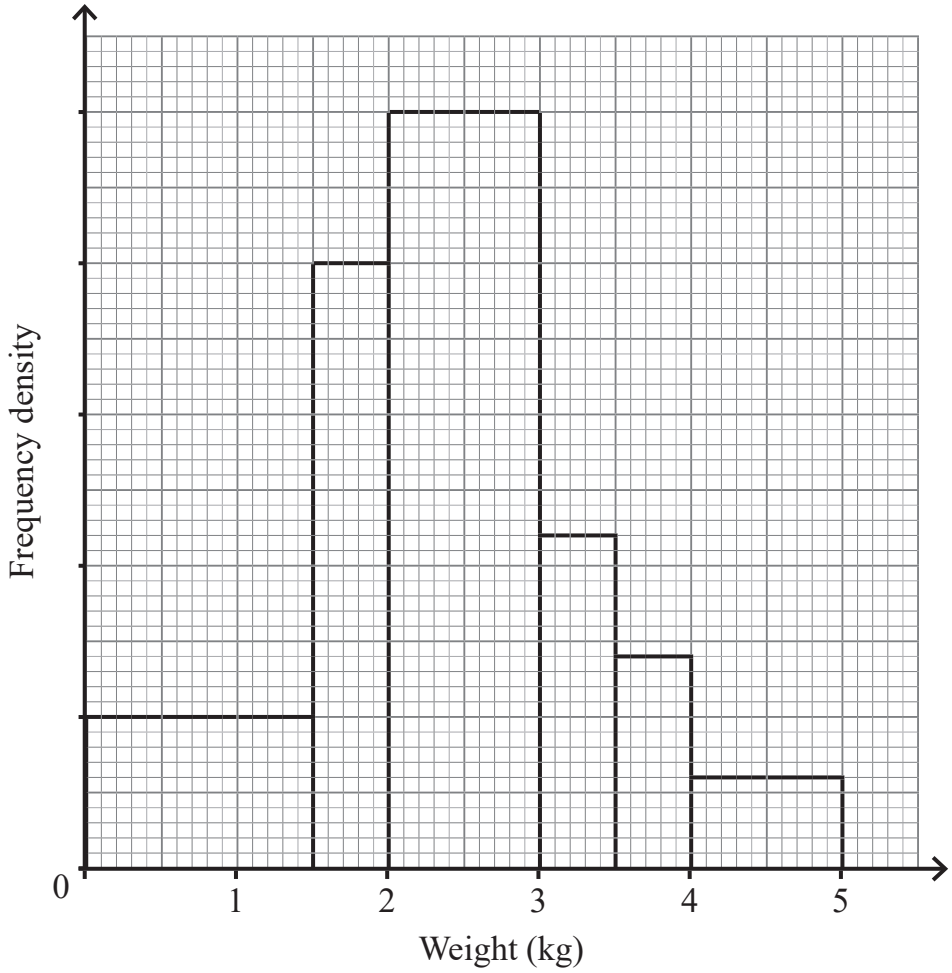
Children = $\frac{640}{3200} \times 75 = 15$

Check: $27 + 33 + 15 = 75$

[3]

[Turn over

22 The histogram shows information about the weights, in kg, of some newborn babies.



There are 35 newborn babies with weight less than 2 kg.

(a) Calculate an estimate for the number of newborn babies with weights between 2.5 kg and 4.5 kg.

Heights (small squares): 10, 40, 50, 22, 14, 6

Under 2 kg: let FD of 0–1.5 bar be x, so 1.5–2 bar is 4x

(1.5 × x) + (0.5 × 4x) = 35 → 3.5x = 35 → x = 10

so 1 small square = 1 unit of frequency density

(0.5 × 50) + (0.5 × 22) + (0.5 × 14) + (0.5 × 6)

= 25 + 11 + 7 + 3 = 46

Answer 46 [4]

(b) Calculate an estimate for the median weight of a newborn baby.

Frequencies: 15, 20, 50, 11, 7, 6 total = 109

Median = $\frac{109 + 1}{2}$ = 55th value, in 2–3 kg (35 before it)

= $2 + \frac{20}{50} \times 1 = 2.4$

Answer 2.4 kg [2]

(c) Calculate an estimate for the interquartile range.

LQ: 109 ÷ 4 = 27.25 → 28th value, in 1.5–2 kg: $1.5 + \frac{13}{20} \times 0.5 = 1.825$

UQ: 3 × 27.25 = 81.75 → 82nd value, in 2–3 kg: $2 + \frac{47}{50} \times 1 = 2.94$

IQR = 2.94 − 1.825 = 1.115

Answer 1.115 kg [3]

[Turn over

23 Simplify

$$\left(\frac{1}{x+4}+\frac{1}{2x-3}\right)\div\frac{3x^2-11x-4}{2x^2-32}$$

$\frac{1}{x+4}+\frac{1}{2x-3}=\frac{(2x-3)+(x+4)}{(x+4)(2x-3)}=\frac{3x+1}{(x+4)(2x-3)}$

$3x^2-11x-4=(3x+1)(x-4)$

$2x^2-32=2(x^2-16)=2(x-4)(x+4)$

$\frac{3x+1}{(x+4)(2x-3)}\div\frac{3x^2-11x-4}{2x^2-32}$

$=\frac{3x+1}{(x+4)(2x-3)}\times\frac{2(x-4)(x+4)}{(3x+1)(x-4)}$

$=\frac{2}{2x-3}$

Answer $\frac{2}{2x-3}$ [6]

THIS IS THE END OF THE QUESTION PAPER

BLANK PAGE

DO NOT WRITE ON THIS PAGE

BLANK PAGE

DO NOT WRITE ON THIS PAGE

stemup.co.uk

BLANK PAGE

DO NOT WRITE ON THIS PAGE

Sources: All images © CCEA unless stated

DO NOT WRITE ON THIS PAGE

For Examiner's use only	
Question Number	**Marks**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
Total Marks	

Examiner Number

Permission to reproduce all copyright material has been applied for.
In some cases, efforts to contact copyright holders may have been unsuccessful and CCEA will be happy to rectify any omissions of acknowledgement in future if notified.

Rewarding Learning

General Certificate of Secondary Education
November 2022

GCSE Mathematics

HIGHER TIER
ADDITIONAL SUPPORT MATERIALS
(For use in November 2022)

stemup.co.uk

HIGHER TIER ADDITIONAL SUPPORT MATERIALS (November 2022)

Numbers

Highest Common Factor (HCF): The highest common factor is the largest number that divides evenly into two or more numbers, e.g. 4 is the HCF of 8 and 20

Trial and Improvement

This is a method of trying different values in an equation until you get a suitable solution (e.g. to 1 decimal place).

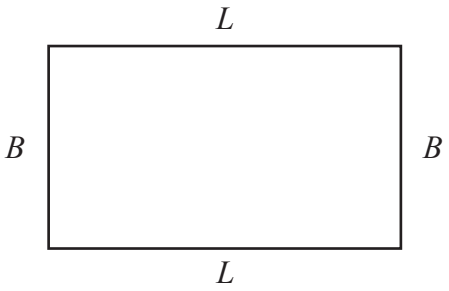
Range

The range of a set of data is the difference between the largest value and the smallest value in the data set.

Mean

The mean of a set of data is the sum of all the data values divided by the number of data values.

Area and Volume



The area of a rectangle is found by multiplying the length of the rectangle by the breadth.

$A = L \times B$ where A is area, L is length and B is breadth.

The area of a triangle is found by multiplying half the length of the base by the perpendicular height of the triangle.

$A = \frac{1}{2}bh$ where b is the base and h is the perpendicular height of the triangle.

The area of a circle is $A = \pi r^2$ where r is the radius of the circle.

The volume of a cuboid is found by multiplying the length by the breadth by the height of the cuboid.

$V = L \times B \times H$ where V is volume, L is length, B is breadth and H is height.

Angles

There are 180° on a straight line.
There are 180° inside a triangle.
An isosceles triangle is a triangle with 2 equal sides and 2 equal angles.
The sum of all the angles inside a polygon is given by $180(n - 2)$ where n is the number of sides in the polygon.

Pie Chart

In a pie chart, the total angle that corresponds to the entire data set is 360°

Probability

The sum of the probabilities of all outcomes equals 1

Compound Measures

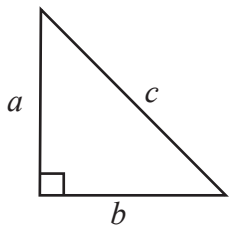
$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Pythagoras' Theorem

If a, b and c are the sides of a right angled triangle shown below, then

$$a^2 + b^2 = c^2$$



stemup.co.uk

Midpoint of a line

If (x_1, y_1) and (x_2, y_2) are the end points of a line, then the coordinates of the midpoint M of the line are

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Gradient of a line

If (x_1, y_1) and (x_2, y_2) are two points on a line, then the gradient m of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Lines

Parallel lines have the same gradient.

If a straight line has gradient m, then a line which is perpendicular to this line has a gradient $-\frac{1}{m}$

Equation of a circle

The equation of a circle of radius r and centre, the origin (0, 0), is given by $x^2 + y^2 = r^2$ where (x, y) are the coordinates of any point on the circumference of the circle.

Tangent/Radius property

The tangent to a circle is perpendicular to the radius at the point of contact with the circle.

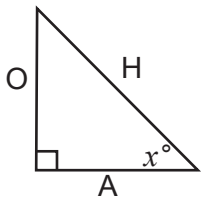
Cyclic Quadrilateral property

The opposite angles of a cyclic quadrilateral add up to 180°

Alternate Segment Theorem

In a circle, the angle between a chord and a tangent through one of the end points of the chord is equal to the angle in the alternate segment.

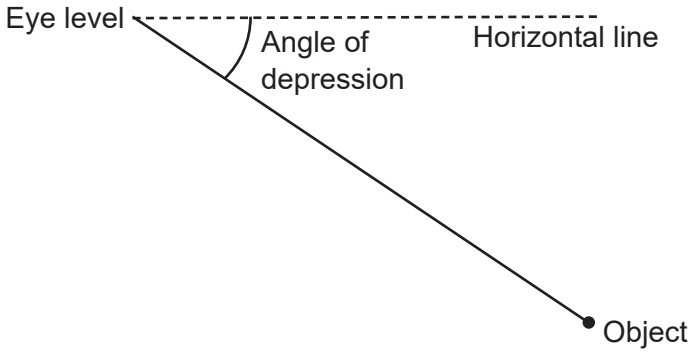
Trigonometric ratios in right angled triangles



$\sin x° = \frac{O}{H}$ $\cos x° = \frac{A}{H}$ $\tan x° = \frac{O}{A}$

Angle of depression

If a person stands and looks down at an object, the **angle of depression** is the angle between the horizontal line of sight and the object.



Frequency density in histograms

$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$