Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
November 2022

Mathematics

StemUp Model Solution

Unit M3
(With calculator)
Higher Tier



[GMC31]

GMC31

TUESDAY 29 NOVEMBER, 9.15am–11.15am

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page. **You are provided with Higher Tier Additional Support Materials for use with this paper.**
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all twenty-five** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

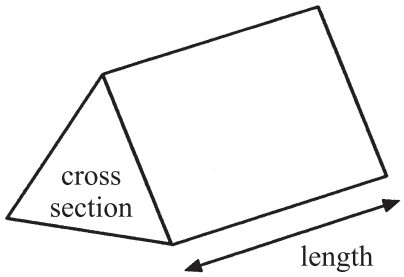
INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

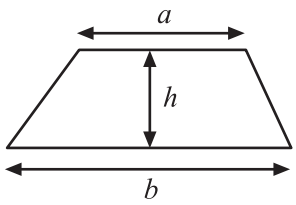
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Formula Sheet

Volume of prism = area of cross section × length

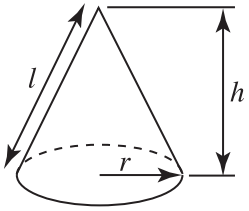


Area of trapezium = $\frac{1}{2}(a+b)h$



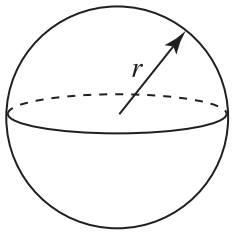
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

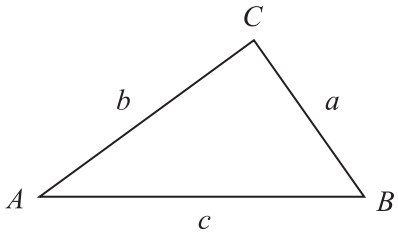


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

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1

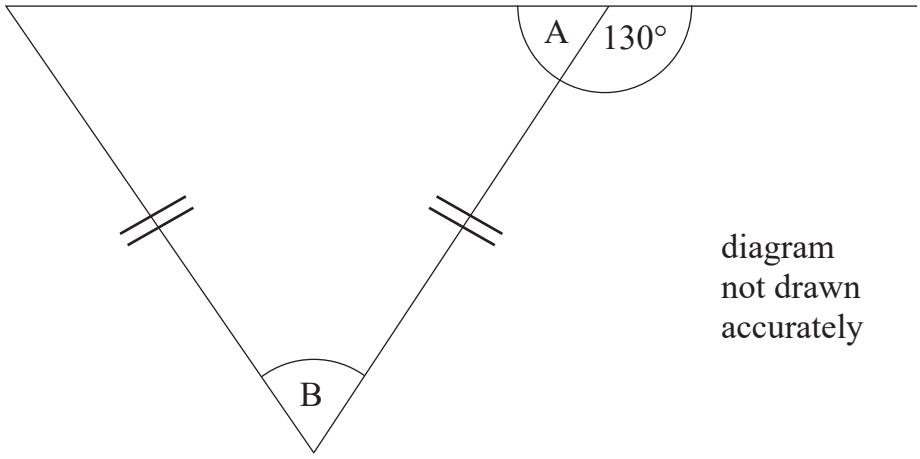


diagram
not drawn
accurately

Work out the sizes of the angles marked A and B in the diagram.

A = 180 − 130 = 50° (angles on a straight line)

Isosceles triangle, so the base angles are equal: 50° and 50°

B = 180 − (2 × 50) = 180 − 100 = 80° (angles in a triangle)

Answers A = 50 °

B = 80 °

[3]

[Turn over

2 A local drama group sold the following tickets.

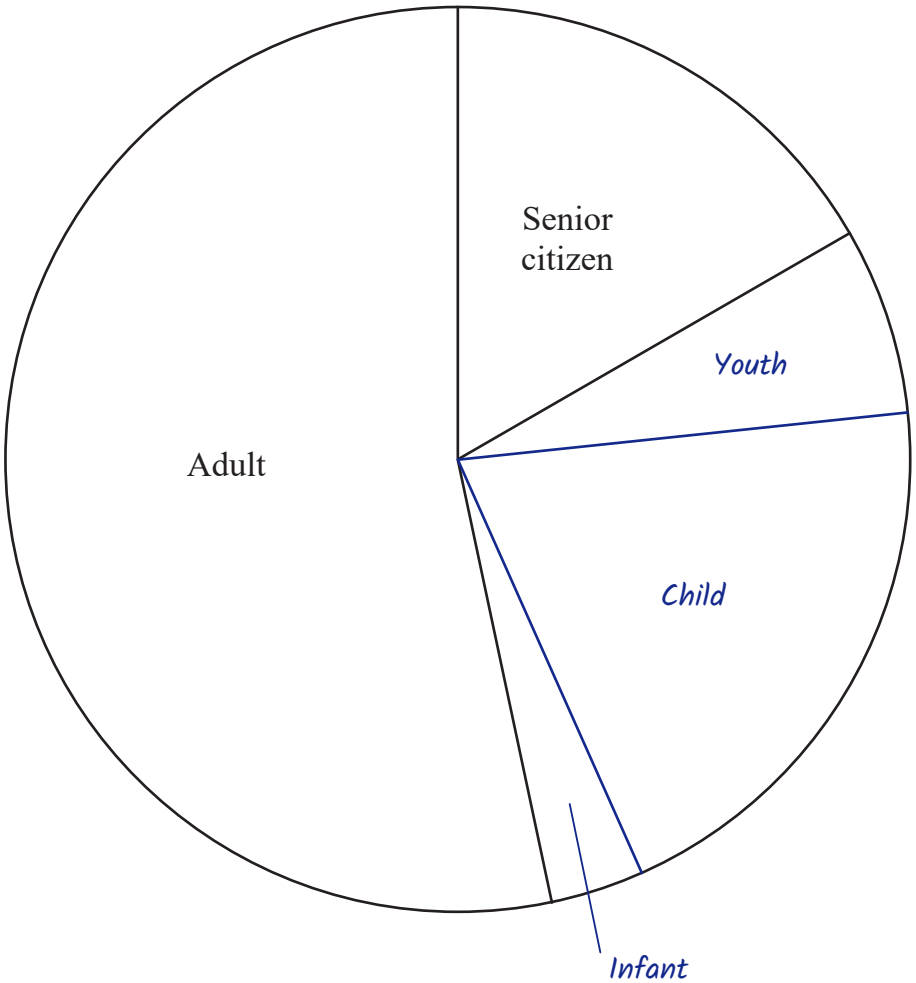
Type of ticket	Number of tickets sold
Senior citizen	15
Adult	48
Youth	6
Child	18
Infant	3

Total = 90 tickets
360 ÷ 90 = 4° each
Youth: 6 × 4 = 24°
Child: 18 × 4 = 72°
Infant: 3 × 4 = 12°

(a) The drama group leader started to draw a pie chart representing the number of tickets sold in each age category.

The first two sectors have been drawn.

Complete the pie chart.



[3]

(b) The table below includes the cost of the tickets.

Calculate the total income from ticket sales.

Type of ticket	Cost of ticket	No. of tickets sold
Senior citizen	£6	15
Adult	£10	48
Youth	£5	6
Child	£3	18
Infant	Free	3

$(6 \times 15) + (10 \times 48) + (5 \times 6) + (3 \times 18) + (0 \times 3)$

$= 90 + 480 + 30 + 54 + 0$

$= 654$

Answer £ 654 [2]

(c) At the interval, hot and cold drinks were sold as shown in the table below.

	Small	Medium	Large
Hot	4	24	7
Cold	3	12	15

The drama group leader said:

"Half of the medium drinks sold were cold."

Is this statement correct? Explain your answer.

Answer No because half of 36 medium drinks is 18, but only 12 were cold, which is one third of the medium drinks [2]

[Turn over

3 Calculate $5.9^2 + \sqrt{80}$

Give your answer correct to 1 decimal place.

$5.9^2 + \sqrt{80} = 34.81 + 8.94427191...$

$= 43.75427191$

Answer 43.8 [2]

4 Mr Jenkins is a bus driver.

His standard rate of pay is £12 per hour.

At weekends he gets "time and a half".

If he works a split shift (two separate blocks of time) on any day he gets an additional £18 for that day.

Mr Jenkins' timesheet for the first week in January is shown below.

Monday	0800–1400
Tuesday	0730–0930 and 1500–2200
Wednesday	Off
Thursday	1500–2230
Friday	Off
Saturday	0800–1500
Sunday	1230–1830

Calculate Mr Jenkins' total pay for this week.

Weekdays: Monday 6 h, Tuesday 2 h + 7 h = 9 h, Thursday 7.5 h

6 + 9 + 7.5 = 22.5 hours, 22.5 × 12 = £270

Weekend: Saturday 7 h, Sunday 6 h = 13 hours

time and a half = 12 × 1.5 = £18 per hour, 13 × 18 = £234

Split shift on Tuesday = £18

Total = 270 + 234 + 18 = £522

Answer £ 522 [4]

[Turn over

5 **(a)** A farm worker can pick 90 strawberries every 5 minutes.

How many strawberries can the farm worker pick in an hour?

60 ÷ 5 = 12, 90 × 12 = 1080

Answer 1080 [1]

(b) The average picking rate for each worker is 1000 strawberries per hour.

There are 8 workers on the farm.

They need to pick 58 000 strawberries for a special order.

If they pick at the average rate, how long should it take them?

Give your answer in hours and minutes.

8 workers: 8 × 1000 = 8000 strawberries per hour

58 000 ÷ 8000 = 7.25 hours

0.25 × 60 = 15 minutes

Answer 7 hours and 15 minutes [2]

6 The formula $P = \dfrac{ab}{a+b}$

is used in a Science experiment to calculate P for values of a and b.

Calculate the value of P when $a = 8.4$ and $b = -1.2$

$P = \dfrac{8.4 \times (-1.2)}{8.4 + (-1.2)} = \dfrac{-10.08}{7.2} = -1.4$

Answer -1.4 [3]

7 **(a)** Simplify $5a + 2b - 3a - 8b$

Answer $2a - 6b$ [2]

(b) Multiply out $4(3x - 5)$

Answer $12x - 20$ [2]

(c) Solve $8x - 10 = 6x + 2$

$8x - 6x = 2 + 10$

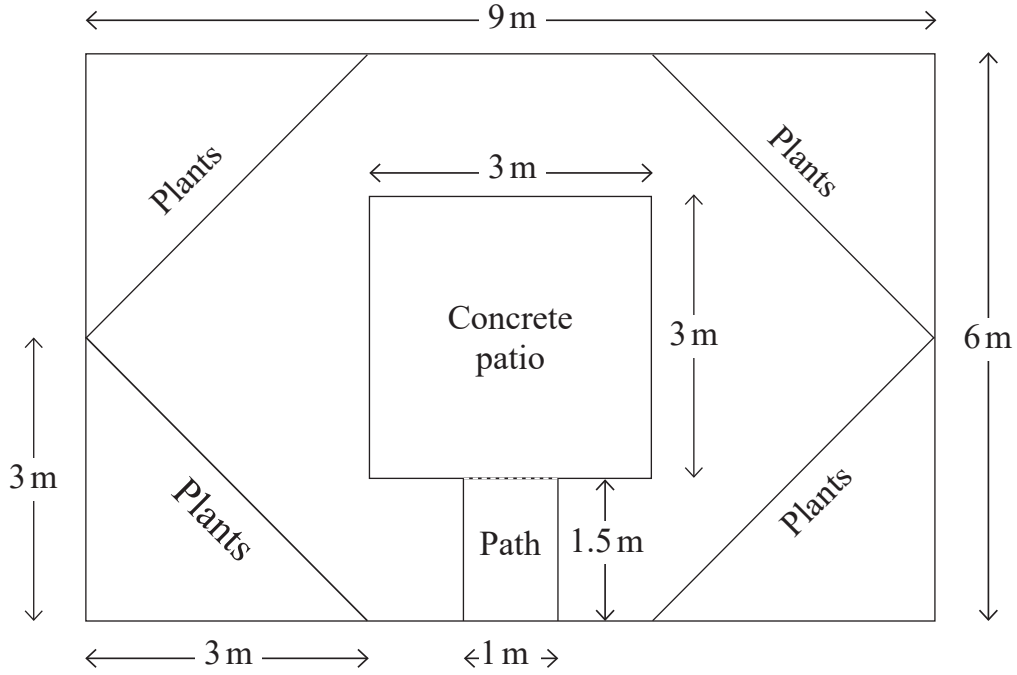
$2x = 12$

$x = 6$

Answer $x =$ 6 [3]

[Turn over

8 The diagram shows the layout of a rectangular garden.



(a) There are four identical triangular sections of the garden covered with plants.

Calculate the area of one of these triangular sections.

You must include units with your answer.

Area = ½ × 3 × 3 = 9 ÷ 2 = 4.5

Answer 4.5 m^2 [3]

(b) The garden also has a rectangular path leading to a square concrete patio.

The remainder of the garden is covered by grass.

Calculate the total area of grass in the garden.

Whole garden: $9 \times 6 = 54\ m^2$

Plants: $4 \times 4.5 = 18\ m^2$

Patio: $3 \times 3 = 9\ m^2$

Path: $1.5 \times 1 = 1.5\ m^2$

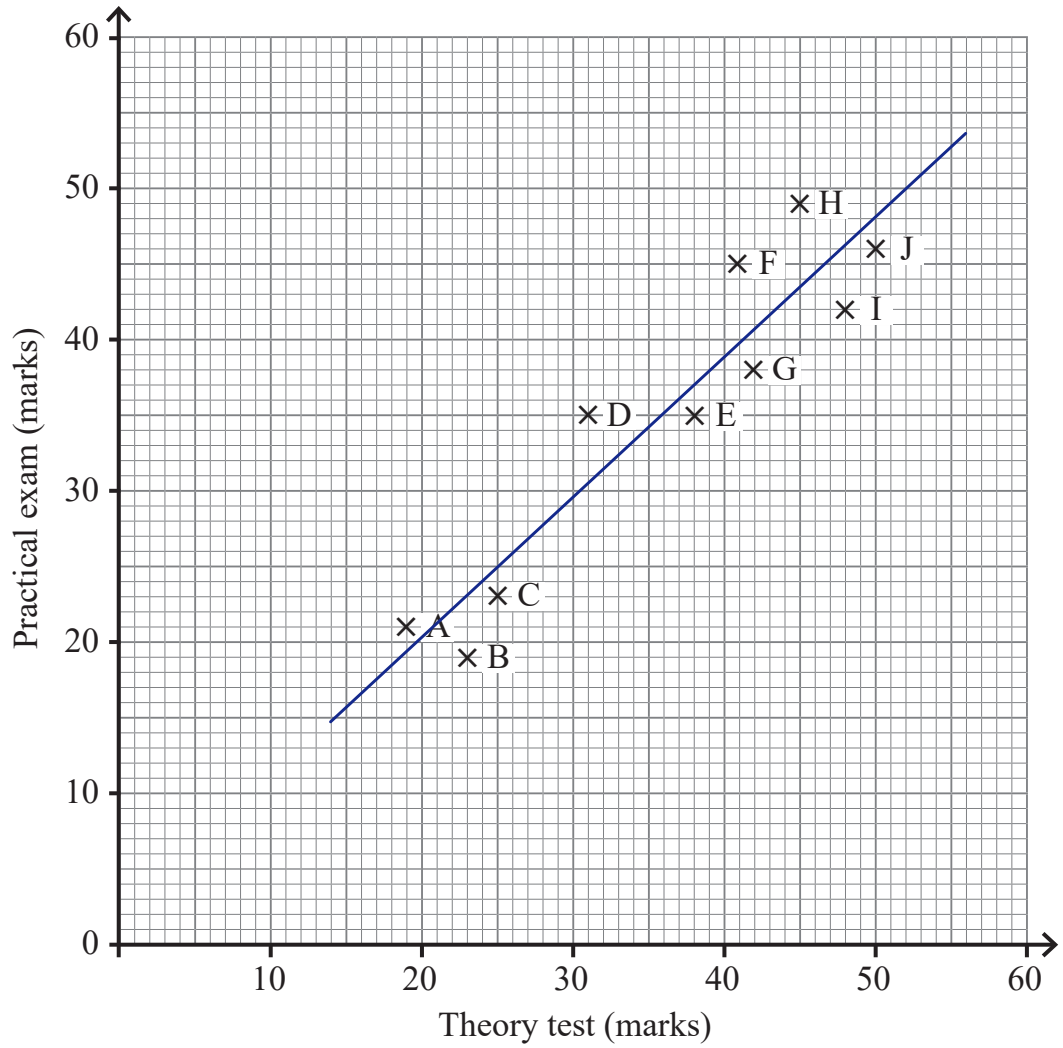
Grass $= 54 - 18 - 9 - 1.5 = 25.5\ m^2$

Answer 25.5 m^2 [3]

[Turn over

9 Ten job applicants completed a theory test and a practical exam.

The marks are shown in the scatter graph below.



(a) What was the highest mark obtained in the practical exam?

Answer 49 [1]

(b) One applicant scored a total of 80 marks.

Which applicant was this?

G: 42 + 38 = 80

Answer G [1]

13327

(c) Draw a line of best fit on the graph. [1]

[Turn over

10 The stem and leaf diagram shows the heights of 19 members of a sports team.

16	3	4	9						
17	1	2	4	5	6	7	8	9	9
18	0	2	3	5	5	8			
19	0								

Key 17 | 5 = 175 cm

Four new players attend trials to become the 20th member of the team.

Their heights are

Player A	185 cm
Player B	179 cm
Player C	177 cm
Player D	191 cm

If added to the team, which player would make

(a) the modal height greater than 180 cm?

two 185s and a third would make 185 the mode

Answer A [1]

(b) the range of the heights 28 cm?

191 – 163 = 28 cm

Answer D [1]

(c) the median height 177.5 cm?

10th and 11th values: 177 and 178, median = 177.5

Answer C [1]

11 A coat has an original price of £87

Online Shop A offers a 33% discount.

Online Shop B offers a third off.

Which shop offers the greatest saving and how much greater is the saving in pence?

You must show all your working.

Shop A: 33% of £87 = 0.33 × 87 = £28.71

Shop B: one third of £87 = 87 ÷ 3 = £29

Shop B saves more: £29 − £28.71 = £0.29 = 29p

Answer Shop ___B___ offers greater saving by ___29___ p [4]

[Turn over

12 **(a)** Factorise

(i) $8p + 12t$

Answer $4(2p + 3t)$ [1]

(ii) $r - r^2$

Answer $r(1 - r)$ [1]

(iii) $81 - t^2$

Answer $(9 - t)(9 + t)$ [1]

(b) Solve the equation $y^2 + y - 6 = 0$

$y^2 + y - 6 = 0$

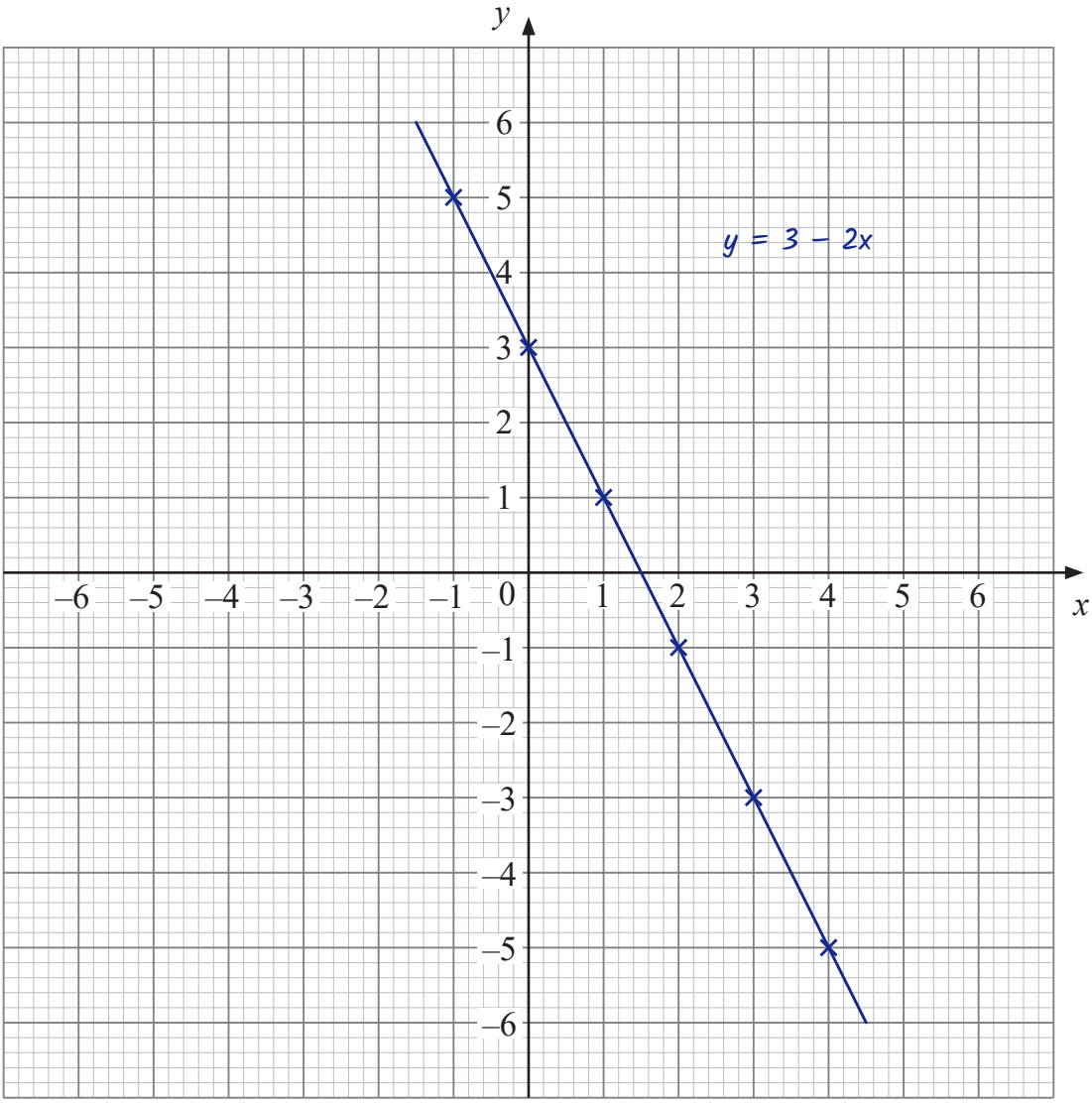
$(y + 3)(y - 2) = 0$

$y + 3 = 0$ or $y - 2 = 0$

Answer $y =$ -3 or 2 [3]

13 **(a)** On the grid below draw the graph of $y = 3 - 2x$

x:	−1	0	1	2	3	4
y:	5	3	1	−1	−3	−5



[3]

(b) Write down the equation of any line parallel to $y = 3 - 2x$

Answer y = 7 − 2x [1]

[Turn over

14 Two points P(4, 8) and Q(x, y) are joined to form a straight line.

The midpoint of the line PQ has coordinates (1, 7)

Find the coordinates of Q.

$\frac{4 + x}{2} = 1$ $\frac{8 + y}{2} = 7$

$4 + x = 2, \quad x = -2$

$8 + y = 14, \quad y = 6$

Answer Q (-2 , 6) [3]

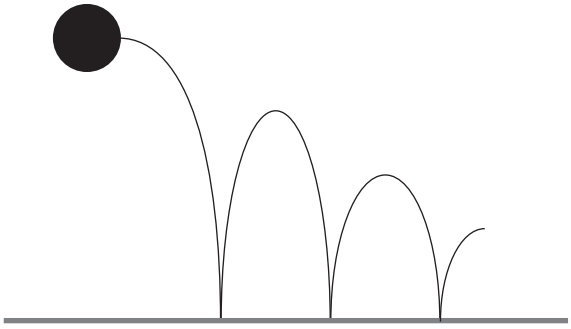
15 A ball is dropped from a height of 2.5 m above the ground.

Each time it hits the ground it bounces to $\frac{3}{5}$ of its previous height.

Kevin says that after it hits the ground for the third time it will not bounce above half a metre.

Is Kevin correct?

You must show all your working.



$\frac{3}{5} \times 2.5 = 1.5$ m after the first bounce

$\frac{3}{5} \times 1.5 = 0.9$ m after the second bounce

$\frac{3}{5} \times 0.9 = 0.54$ m after the third bounce

0.54 m > 0.5 m, so it does bounce above half a metre.

Kevin is not correct.

Answer No [3]

[Turn over

16 **(a)** Write down any two numbers whose highest common factor (HCF) is 8

Answer 16 and 24 [2]

(b) Given that $600 = 2^a \times 3 \times 5^b$ find the values of a and b

$600 = 2 \times 300$

$= 2 \times 2 \times 150$

$= 2 \times 2 \times 2 \times 75$

$= 2 \times 2 \times 2 \times 3 \times 25$

$= 2 \times 2 \times 2 \times 3 \times 5 \times 5$

$= 2^3 \times 3 \times 5^2$

Answer $a =$ 3 $b =$ 2 [2]

17 Mary and Anne both go to a shop, each with the same amount of money.

Mary buys 7 bars of chocolate at y pence each and receives 19p change.

Anne buys 5 bars of chocolate at y pence each and receives 65p change.

By forming and solving an equation, work out how much money each girl had going to the shop.

You must show all your working.

A solution by trial and improvement will not be accepted.

Money = $7y + 19 = 5y + 65$

$7y - 5y = 65 - 19$

$2y = 46$

$y = 23$

$7 \times 23 + 19 = 161 + 19 = 180p$

check: $5 \times 23 + 65 = 115 + 65 = 180p$

Equation $7y + 19 = 5y + 65$ [1]

Answer: Each girl had £1.80 going to the shop. [3]

[Turn over

18 A cuboid has length 90 cm, width 45 cm and height 30 cm.

It has a mass of 24 300 g.

Calculate the density of the cuboid.

Volume = 90 × 45 × 30 = 121 500 cm^3

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{24\ 300}{121\ 500} = 0.2$$

Answer 0.2 g/cm^3 [3]

19 Leah walks 4.7 m in a straight line.

She then turns 90° clockwise and walks 2.5 m in another straight line.

How far is Leah from where she started?

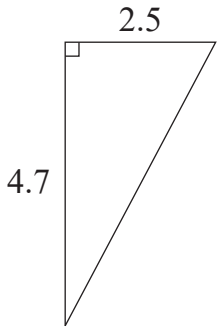


diagram not drawn accurately

$x^2 = 4.7^2 + 2.5^2$

$x^2 = 22.09 + 6.25 = 28.34$

$x = \sqrt{28.34} = 5.323532662$

Answer 5.32 m [3]

[Turn over

20 The front door of Martin's house is wooden.

The top of the door is a semicircle.

It has a window in the shape of a rhombus as shown in the diagram.

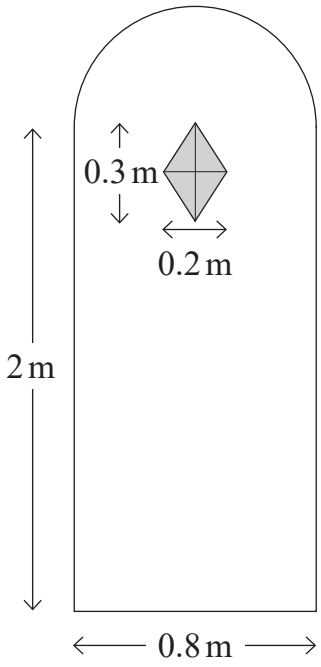


diagram not drawn accurately

What is the area of the wooden part of the door?

Rectangle: $2 \times 0.8 = 1.6\ m^2$

Semicircle: radius $= 0.8 \div 2 = 0.4$ m

$\frac{1}{2} \times \pi \times 0.4^2 = 0.2513274123\ m^2$

Rhombus window: $\frac{1}{2} \times 0.2 \times 0.3 = 0.03\ m^2$

Wood $= 1.6 + 0.2513274123 - 0.03$

$= 1.8213274... = 1.82\ m^2$

Answer 1.82 m^2 [5]

21 Given that $(n-1)^2 + n + (n-a) \equiv n^2 - 2$

find the value of a

$(n-1)^2 = n^2 - 2n + 1$

$n^2 - 2n + 1 + n + n - a \equiv n^2 - 2$

$n^2 + 1 - a \equiv n^2 - 2$

$1 - a = -2$

$a = 3$

Answer $a =$ 3 [3]

[Turn over

22 One third of the pupils in a school are girls.

15% of the girls are in the Senior School.

There are 306 girls in the Junior School.

How many pupils are in the school?

Junior School girls = 100% − 15% = 85% of the girls

85% = 306

1% = 306 ÷ 85 = 3.6

100% = 360 girls

Girls are one third, so pupils = 360 × 3 = 1080

Answer *1080* [4]

23 The length of the side of a square is 8.3cm, correct to 1 decimal place.

Work out the lower bound for the area of the square.

Lower bound of side = 8.25 cm

8.25 × 8.25 = 68.0625

Answer *68.0625* cm^2 [2]

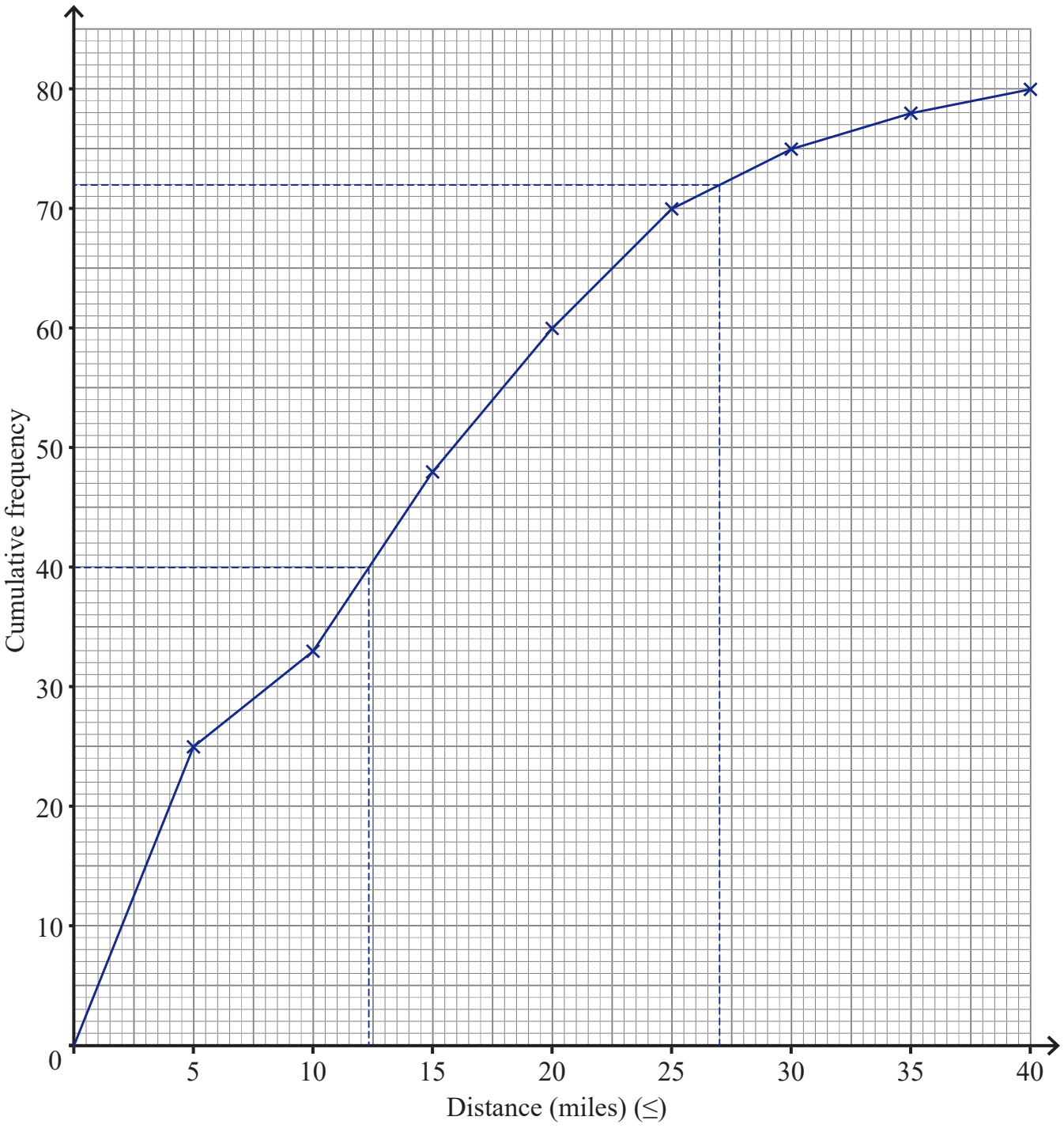
[Turn over

24 The frequency table shows the distance travelled to school by 80 teachers.

Distance (miles)	Number of teachers	Cumulative frequency
$0 < d \leq 5$	25	25
$5 < d \leq 10$	8	33
$10 < d \leq 15$	15	48
$15 < d \leq 20$	12	60
$20 < d \leq 25$	10	70
$25 < d \leq 30$	5	75
$30 < d \leq 35$	3	78
$35 < d \leq 40$	2	80

(a) Complete the cumulative frequency column in the table. [1]

(b) On the graph below draw a cumulative frequency graph for this information. [3]



[Turn over

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(c) Use your graph to estimate

(i) the median distance

CF = 40 → 12.3

Answer 12.3 miles [1]

(ii) the number of teachers who travel more than 27 miles.

27 miles → CF = 72

80 − 72 = 8

Answer 8 [2]

25 A line L passes through the points with coordinates (0, 2) and (2, 8).

Find the equation of any line parallel to line L.

$\text{gradient} = \frac{8-2}{2-0} = \frac{6}{2} = 3$

Parallel lines have the same gradient, so $m = 3$

$y = 3x + c$ with $c \neq 2$, e.g. $c = 5$

Answer $y = 3x + 5$ [4]

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Question Number	Marks
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Rewarding Learning

General Certificate of Secondary Education
November 2022

GCSE Mathematics

HIGHER TIER
ADDITIONAL SUPPORT MATERIALS
(For use in November 2022)

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HIGHER TIER ADDITIONAL SUPPORT MATERIALS (November 2022)

Numbers

Highest Common Factor (HCF): The highest common factor is the largest number that divides evenly into two or more numbers, e.g. 4 is the HCF of 8 and 20

Trial and Improvement

This is a method of trying different values in an equation until you get a suitable solution (e.g. to 1 decimal place).

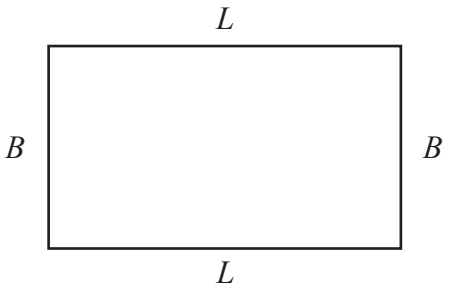
Range

The range of a set of data is the difference between the largest value and the smallest value in the data set.

Mean

The mean of a set of data is the sum of all the data values divided by the number of data values.

Area and Volume



The area of a rectangle is found by multiplying the length of the rectangle by the breadth.

$A = L \times B$ where A is area, L is length and B is breadth.

The area of a triangle is found by multiplying half the length of the base by the perpendicular height of the triangle.

$A = \frac{1}{2}bh$ where b is the base and h is the perpendicular height of the triangle.

The area of a circle is $A = \pi r^2$ where r is the radius of the circle.

The volume of a cuboid is found by multiplying the length by the breadth by the height of the cuboid.

$V = L \times B \times H$ where V is volume, L is length, B is breadth and H is height.

Angles

There are 180° on a straight line.
There are 180° inside a triangle.
An isosceles triangle is a triangle with 2 equal sides and 2 equal angles.
The sum of all the angles inside a polygon is given by $180(n - 2)$ where n is the number of sides in the polygon.

Pie Chart

In a pie chart, the total angle that corresponds to the entire data set is 360°

Probability

The sum of the probabilities of all outcomes equals 1

Compound Measures

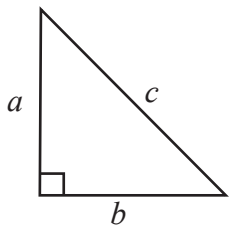
$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Pythagoras' Theorem

If a, b and c are the sides of a right angled triangle shown below, then

$$a^2 + b^2 = c^2$$



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Midpoint of a line

If (x_1, y_1) and (x_2, y_2) are the end points of a line, then the coordinates of the midpoint M of the line are

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Gradient of a line

If (x_1, y_1) and (x_2, y_2) are two points on a line, then the gradient m of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Lines

Parallel lines have the same gradient.

If a straight line has gradient m, then a line which is perpendicular to this line has a gradient $-\frac{1}{m}$

Equation of a circle

The equation of a circle of radius r and centre, the origin (0, 0), is given by $x^2 + y^2 = r^2$ where (x, y) are the coordinates of any point on the circumference of the circle.

Tangent/Radius property

The tangent to a circle is perpendicular to the radius at the point of contact with the circle.

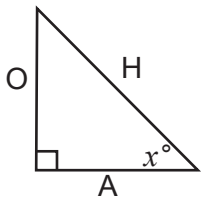
Cyclic Quadrilateral property

The opposite angles of a cyclic quadrilateral add up to 180°

Alternate Segment Theorem

In a circle, the angle between a chord and a tangent through one of the end points of the chord is equal to the angle in the alternate segment.

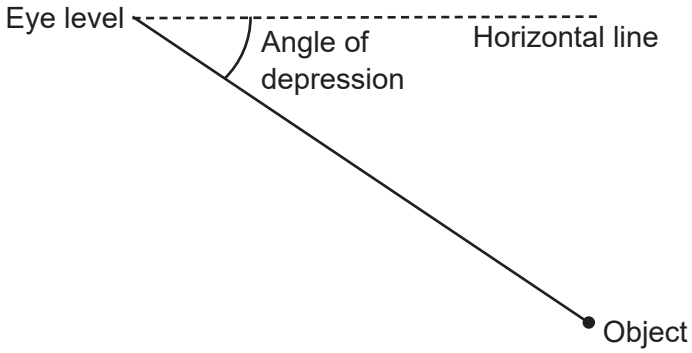
Trigonometric ratios in right angled triangles



$\sin x° = \frac{O}{H}$ $\cos x° = \frac{A}{H}$ $\tan x° = \frac{O}{A}$

Angle of depression

If a person stands and looks down at an object, the **angle of depression** is the angle between the horizontal line of sight and the object.



Frequency density in histograms

$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$