Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
November 2021

Mathematics

StemUp Model Solution

Unit M4
(With calculator)
Higher Tier



[GMC41]

GMC41

MONDAY 29 NOVEMBER, 9.15am–11.15am

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You are provided with Higher Tier Additional Support Materials for use with this paper.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all twenty-four** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

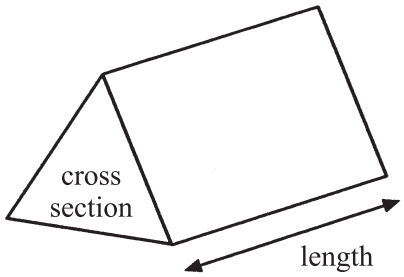
INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

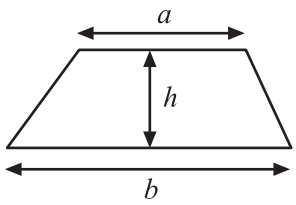
12920

Formula Sheet

Volume of prism = area of cross section × length

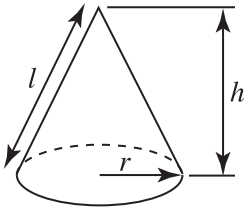


Area of trapezium = $\frac{1}{2}(a+b)h$



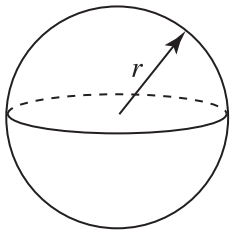
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

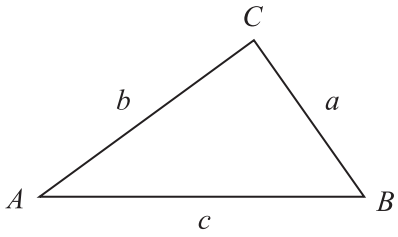


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

12920

1 A trapezium is drawn below.

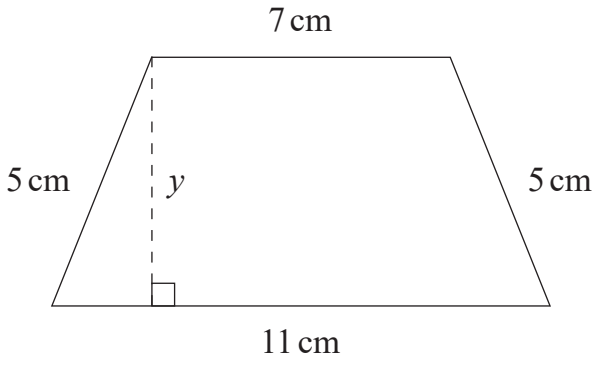


diagram not drawn accurately

Calculate its height y.

Base of each right-angled triangle = (11 − 7) ÷ 2 = 2 cm

$2^2 + y^2 = 5^2$

$4 + y^2 = 25$

$y^2 = 21$

$y = \sqrt{21} = 4.582575695...$

Answer 4.58 cm [4]

[Turn over

2 A number, expressed as a product of its prime factors, is $2^2 \times 3 \times 5^2$

(a) This number is multiplied by 9

Write the new number as a product of its prime factors.

$2^2 \times 3 \times 5^2 \times 3^2 = 2^2 \times 3^3 \times 5^2$

Answer $2^2 \times 3^3 \times 5^2$ [1]

(b) Is this new number a square number?

You must explain your answer.

Answer No because the power of 3 is odd (3^3), so not all the prime factors are squared. $2^2 \times 3^3 \times 5^2 = 2700$, which is not a square number. [1]

3 Jane completes a 5 km race in 24 minutes.

Calculate her average speed in km/hr.

$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{5}{(24 \div 60)} = 5 \div 0.4$

$= 12.5$ km/hr

Answer 12.5 km/hr [2]

4 Last year a company made a profit of £152 650

This year the company made a profit of £104 760

Work out the percentage decrease in the company's profit.

Give your answer to 1 decimal place.

Decrease = 152 650 − 104 760 = £47 890

$$\% \text{ decrease} = \frac{47\,890}{152\,650} \times 100 = 31.37235506\ldots$$

= 31.4%

Answer 31.4 % [3]

5 James can throw a javelin 49 metres.

His target is to throw it 4% further each year.

If he stays on target, how many years will it be before he can throw the javelin 55 metres?

You must show working to justify your answer.

Increase by 4% each year: multiply by 1.04

After 1 year: $49 \times 1.04 = 50.96$ m

After 2 years: $49 \times 1.04^2 = 52.9984$ m

After 3 years: $49 \times 1.04^3 = 55.118336$ m

After 2 years 52.9984 < 55, after 3 years 55.118... > 55

so it will be 3 years.

Answer *3* years [4]

6 The number of hours of daily sunshine is recorded at a resort during four months.

Hours of daily sunshine	Frequency	Midpoint x	$f \times x$
$0 < h \leq 3$	18	1.5	27
$3 < h \leq 6$	45	4.5	202.5
$6 < h \leq 9$	37	7.5	277.5
$9 < h \leq 12$	19	10.5	199.5
$12 < h \leq 15$	4	13.5	54

Calculate an estimate of the mean number of hours of daily sunshine at the resort during the four months.

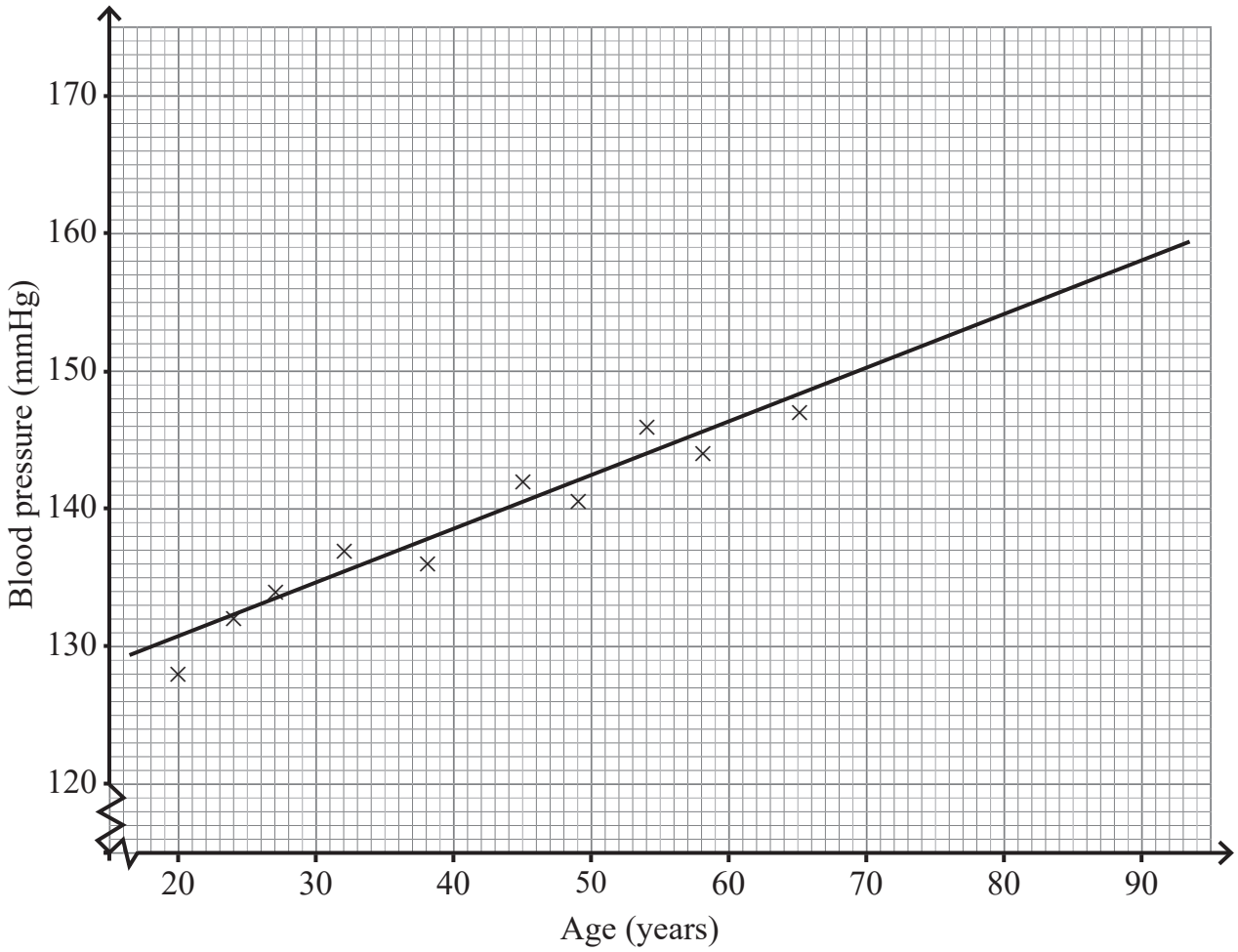
$\Sigma f = 18 + 45 + 37 + 19 + 4 = 123$ $\Sigma fx = 27 + 202.5 + 277.5 + 199.5 + 54 = 760.5$

$\text{Mean} = \frac{760.5}{123} = 6.182926829... = 6.18$

Answer 6.18 hours [4]

[Turn over

7 Janet sees a scatter graph which displays the age and blood pressure of 10 adults.



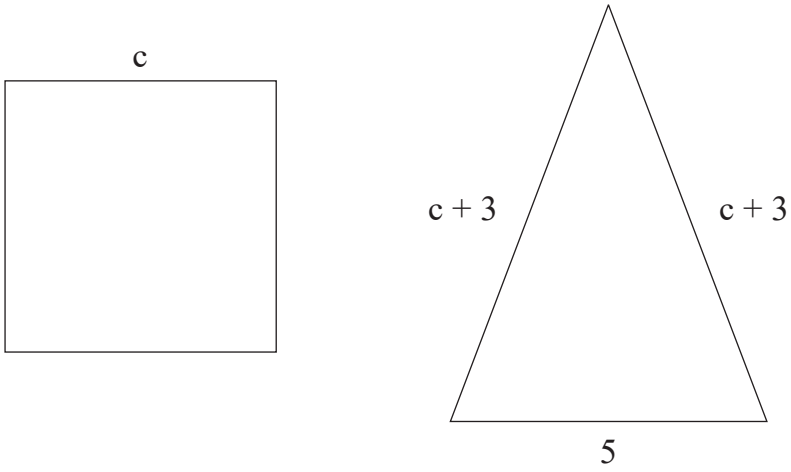
Janet is aged 41 and her father is 84

She comments that a good estimate for her blood pressure would be 139 whilst a good estimate for her father's would be 156

Do you think her estimates are reliable? Explain your reasoning clearly.

Her own estimate (139) is reliable because age 41 lies within the range of ages plotted (about 20 to 65), so it is interpolation. Her father's estimate (156) is not reliable as 84 is outside the range of the data (extrapolation). [2]

8 The diagrams below show a square and an isosceles triangle.



diagrams not drawn accurately

They have the same perimeter.

By forming and solving an equation, work out the perimeter.

Square perimeter = 4c

Triangle perimeter = c + 3 + c + 3 + 5 = 2c + 11

4c = 2c + 11

2c = 11

c = 5.5

Perimeter = 4 × 5.5 = 22

Answer *22* [4]

[Turn over

9 Calculate the surface area of a sphere with diameter 12 cm.

radius = 12 ÷ 2 = 6 cm

Surface area = $4\pi r^2$ = 4 × π × 6^2

= 452.3893421...

= 452.39 cm^2

Answer 452.39 cm^2 [2]

10 Calculate the size of the largest angle in the rhombus.

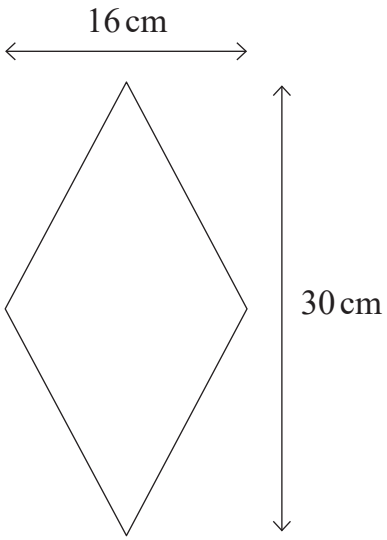


diagram
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You must show your working.

Diagonals of a rhombus bisect each other at right angles,
giving right-angled triangles with sides 8 cm and 15 cm.

$\tan\theta = \frac{15}{8}$ (θ = half of the largest angle)

$\theta = \tan^{-1}(15 \div 8) = 61.92751306...°$

Largest angle = $2 \times 61.92751306...$

$= 123.8550261...°$

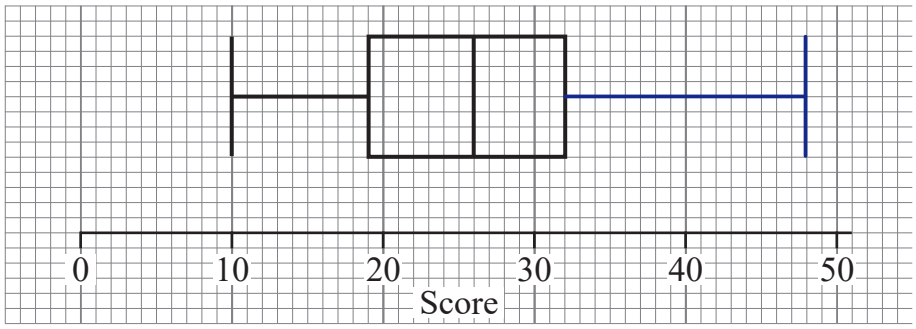
$= 123.86°$

Answer 123.86 ° [4]

[Turn over

11 Mr Davison's class did a test.

Their scores are shown on the box plot, but the box plot is incomplete.



(a) The range of scores is 25 more than the interquartile range.

Use this information to complete the box plot.

LQ = 19, UQ = 32, so IQR = 32 − 19 = 13

Range = 13 + 25 = 38

Highest score = 10 + 38 = 48

[2]

(b) Explain why the interquartile range may be a better measure of spread for this distribution than the range.

The range uses only the extreme values, which may be affected by an outlier;

the IQR uses the middle 50% so is not affected by extreme values. [1]

(c) Kevin scored 32 marks in the test.

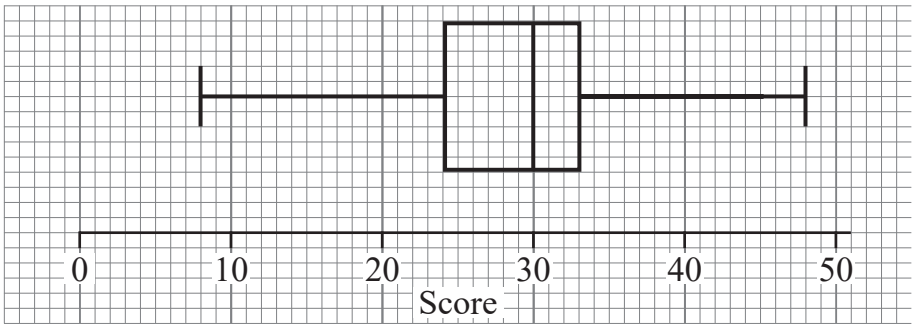
What percentage of the class scored lower than Kevin?

32 is the upper quartile

Answer *75* % [1]

(d) Mrs Clarke's class did the same test.

Their scores are shown on the box plot below.



Write two comparisons between the results of the two classes.

On average Mrs Clarke's class got higher scores (median 30 compared with 26).

Mrs Clarke's class scores were more consistent / less spread out

(IQR about 9 compared with 13). [2]

[Turn over

12 A survey was carried out to estimate how many people own a smartphone.

The results of the sample are shown below.

Do own a smartphone	236
Do not own a smartphone	64

(a) Based on this sample, estimate the number of people in a town with a population of 15 000 who might own a smartphone.

$\frac{236}{300} \times 15\,000 = 11\,800$

Answer 11 800 [1]

(b) The sample data was obtained from a group of 17 year olds.

Do you think your result in **(a)** is an overestimate or an underestimate for the number of people in the town who own a smartphone?

Explain your reasoning.

Answer Overestimate because 17 year olds are more likely to own a smartphone than other age groups (e.g. older people), so the proportion in the sample is likely to be higher than in the town population as a whole. [1]

(c) Outline a better way of obtaining the sample data.

Select 300 people at random from the whole town population (e.g. from the electoral register) so that all age groups are included and it is not biased. [1]

13 Solve $\frac{2x-1}{5}+\frac{4x+5}{3}=\frac{20}{3}$

A solution by trial and improvement will not be accepted.

Multiply every term by 15:

$3(2x - 1) + 5(4x + 5) = 100$

$6x - 3 + 20x + 25 = 100$

$26x + 22 = 100$

$26x = 78$

$x = 3$

Answer 3 [4]

[Turn over

14 The value of John's house has risen by 3.5%

It is now worth £150 075

What was the original value of John's house?

103.5% = £150 075

1% = 150 075 ÷ 103.5 = £1450

100% = 1450 × 100 = £145 000

Answer £ 145 000 [3]

stemup.co.uk

15 **(a)** Expand and simplify $(2x - 5)(3x + 2)$

$(2x - 5)(3x + 2) = 6x^2 + 4x - 15x - 10$

$= 6x^2 - 11x - 10$

Answer $6x^2 - 11x - 10$ [3]

(b) Simplify $\dfrac{x^2 - 49}{2x - 14}$

$\dfrac{x^2 - 49}{2x - 14} = \dfrac{(x + 7)(x - 7)}{2(x - 7)} = \dfrac{x + 7}{2}$

Answer $(x + 7)/2$ [2]

(c) Factorise $8x^2 - 10x - 3$

$8 \times (-3) = -24$: factors -12 and 2

$8x^2 - 12x + 2x - 3$

$= 4x(2x - 3) + 1(2x - 3)$

$= (4x + 1)(2x - 3)$

Answer $(4x + 1)(2x - 3)$ [3]

[Turn over

16 The area of rectangle A is four times the area of square B.

By setting up and solving a quadratic equation, find the value of x.

A solution by trial and improvement will receive no marks.

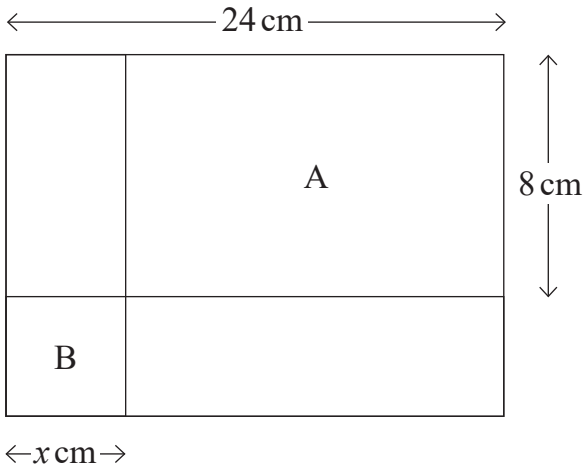


diagram not drawn accurately

Area of A = $8(24 - x)$ Area of B = x^2

$8(24 - x) = 4x^2$

$192 - 8x = 4x^2$

$4x^2 + 8x - 192 = 0$

$x^2 + 2x - 48 = 0$

$(x + 8)(x - 6) = 0$

$x = -8$ or $x = 6$; x is a length so $x > 0$, $x = 6$

Answer $x =$ 6 [6]

17

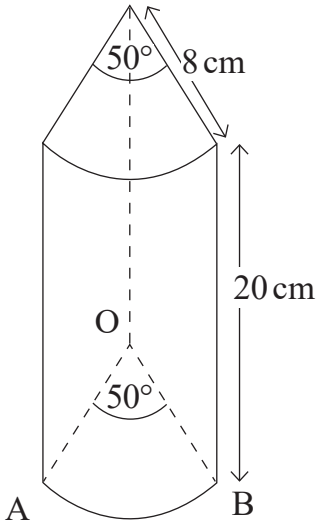


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A solid prism has base OAB, the sector of a circle, and height 20 cm.

Radius OA = 8 cm and angle AOB = 50°.

Calculate the total surface area of the prism.

Curved surface = $\frac{50}{360} \times 2 \times \pi \times 8 \times 20 = 139.6263402...$

Two rectangles (sides) = $2 \times 8 \times 20 = 320$

Area of sector AOB = $\frac{50}{360} \times \pi \times 8^2 = 27.92526803...$

Total surface area = $2 \times 27.92526803... + 139.6263402... + 320$

$= 515.4768763...$

$= 515.48\ cm^2$

Answer 515.48 cm^2 [7]

[Turn over

18 Find the equation of the straight line which is perpendicular to the line $y = 3x + 2$ and passes through the point (9,1).

Give your answer in the form $y = mx + c$

Gradient of $y = 3x + 2$ is 3, so perpendicular gradient $m = -\frac{1}{3}$

$y = -\frac{1}{3}x + c$

At (9, 1): $1 = -\frac{1}{3} \times 9 + c$

$1 = -3 + c$

$c = 4$

Answer $y = -\frac{1}{3}x + 4$ [3]

19

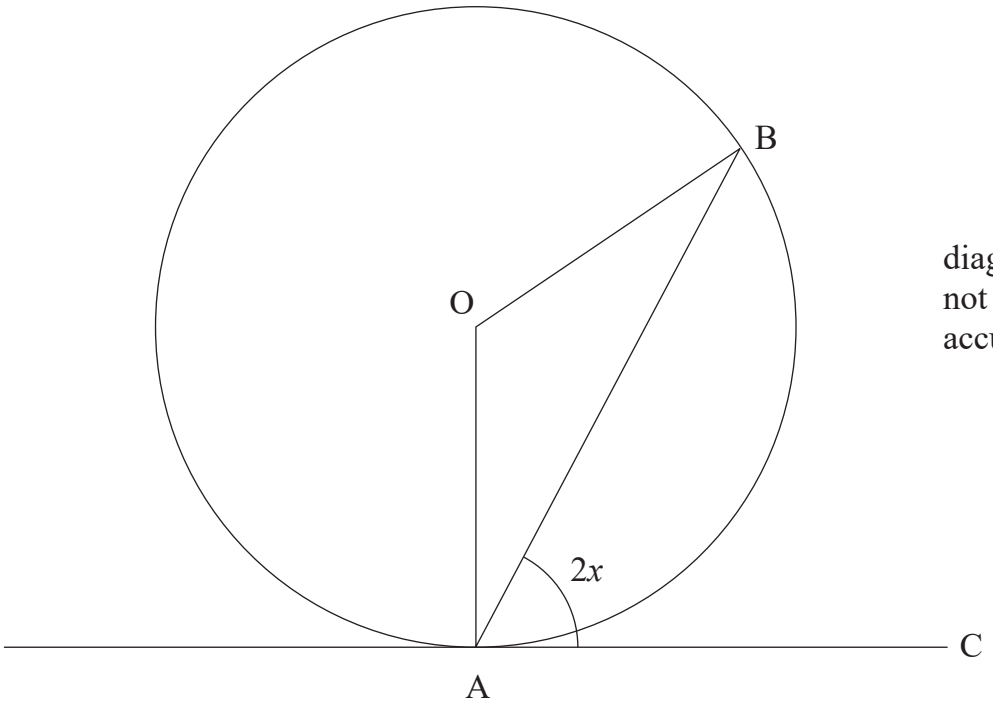


diagram not drawn accurately

A and B are points on the circumference of a circle, centre O.

AC is a tangent to the circle.

Angle BAC = $2x$

Find the size of the angle AOB, in terms of x, giving a reason for each stage of your working.

Angle OAC = 90° (angle between a tangent and a radius is 90°)

Angle OAB = 90° − 2x

OA = OB (radii), so triangle OAB is isosceles:

Angle OBA = 90° − 2x (base angles of an isosceles triangle are equal)

Angle AOB = 180° − (90° − 2x) − (90° − 2x) = 4x

(angles in a triangle add up to 180°)

Answer 4x ° [3]

[Turn over

20 Factorise $2x^2 - 5xy - 3y^2$

$2 \times (-3) = -6$: factors -6 and 1

$2x^2 - 6xy + xy - 3y^2$

$= 2x(x - 3y) + y(x - 3y)$

$= (2x + y)(x - 3y)$

Answer $(2x + y)(x - 3y)$ [3]

21 Jack says the distance from Larne to Enniskillen is 110 miles to the nearest 10 miles.

He drove this distance at an average speed of 45 mph to the nearest 5 mph.

Calculate the least amount of time the journey could have taken.

Give your answer in hours and minutes, to the nearest minute.

Lower bound of distance = 105 miles

Upper bound of speed = 47.5 mph

Least time = $\frac{105}{47.5}$ = 2.210526316... hours

0.210526316... × 60 = 12.63... minutes

= 2 hr 13 min (to the nearest minute)

Answer 2 hr 13 min [4]

[Turn over

22 Simplify

$$\frac{ax + 2x - ay - 2y}{x^2 - y^2}$$

$$\frac{x(a+2) - y(a+2)}{(x-y)(x+y)} = \frac{(a+2)(x-y)}{(x-y)(x+y)} = \frac{a+2}{x+y}$$

Answer $(a+2)/(x+y)$ [3]

23 Solve the following equation giving your answers to 2 decimal places.

$$\frac{x-3}{x+4} - \frac{2x+1}{x+2} = 3$$

$$\frac{(x-3)(x+2) - (2x+1)(x+4)}{(x+4)(x+2)} = 3$$

$$\frac{x^2 - x - 6 - 2x^2 - 9x - 4}{x^2 + 6x + 8} = 3$$

$$-x^2 - 10x - 10 = 3(x^2 + 6x + 8)$$

$$-x^2 - 10x - 10 = 3x^2 + 18x + 24$$

$$4x^2 + 28x + 34 = 0$$

$$x = \frac{-28 \pm \sqrt{(28^2 - 4 \times 4 \times 34)}}{2 \times 4} = \frac{-28 \pm \sqrt{240}}{8}$$

$$x = (-28 + 15.49193338) \div 8 = -1.563508327...$$

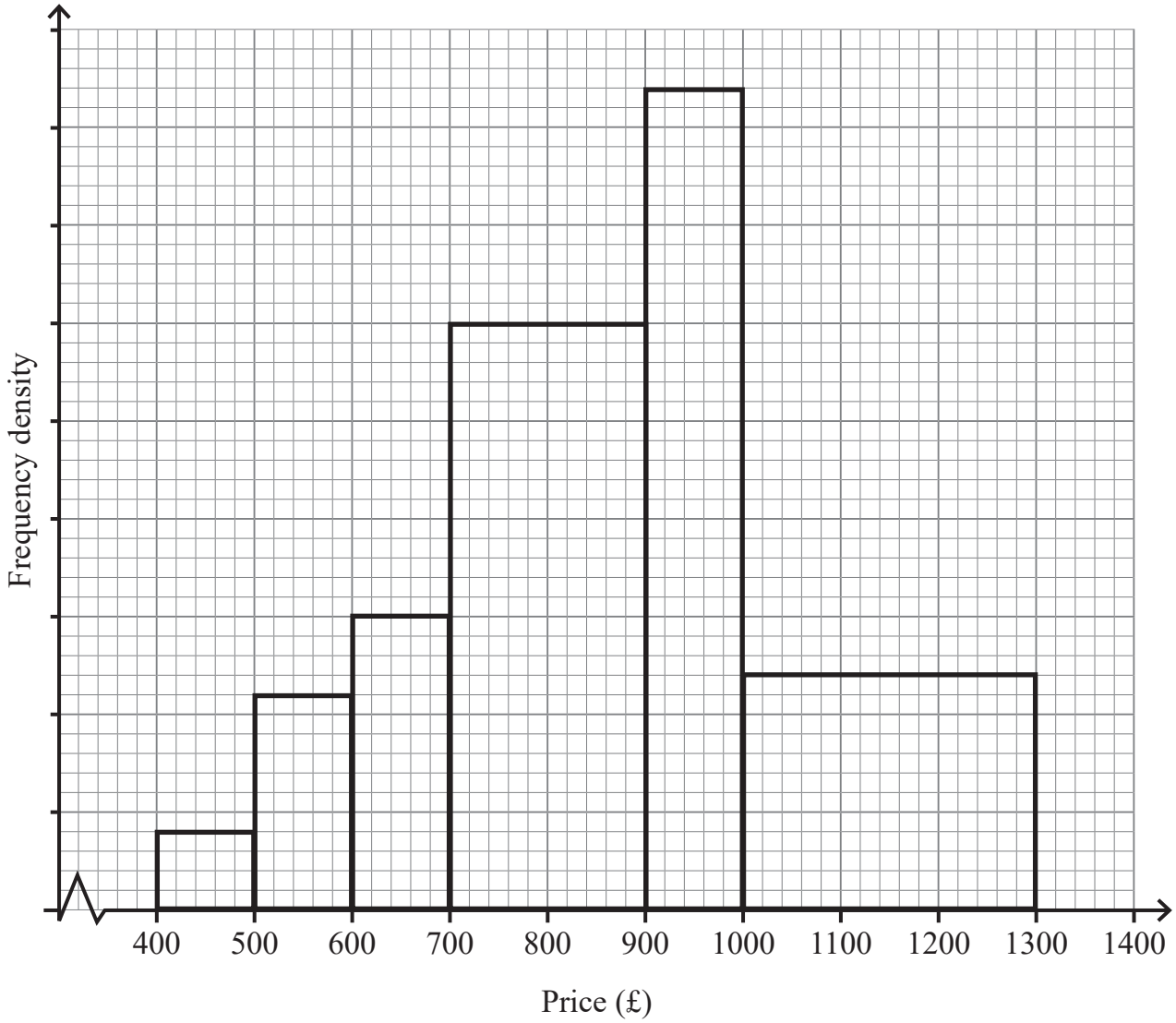
$$x = (-28 - 15.49193338) \div 8 = -5.436491673...$$

$x = -1.56$ or $x = -5.44$

Answer $-1.56, -5.44$ [8]

[Turn over

24 The histogram below shows the prices of televisions sold online one month.



150 televisions cost less than £600

stemup.co.uk

(a) Calculate the number of televisions costing between £900 and £1200

Bar heights in small squares: £400–£500 is 4, £500–£600 is 11

Area for £400–£600 = 100 × 4 + 100 × 11 = 1500 ↔ 150 televisions

so 1 small square = frequency density 0.1

Frequency densities: £900–£1000 = 4.2, £1000–£1300 = 1.2

Number = 100 × 4.2 + 200 × 1.2 = 420 + 240 = 660

Answer 660 [4]

(b) A stratified sample is taken.

Given that 11 televisions are sampled from the £500–£600 group, find the number of televisions in the sample.

£500–£600: 100 × 1.1 = 110 televisions

Total = 150 + 100 × 1.5 + 200 × 3 + 100 × 4.2 + 300 × 1.2

= 150 + 150 + 600 + 420 + 360 = 1680

$\frac{11}{110} = \frac{1}{10}$ of each group is sampled

$\frac{1}{10} \times 1680 = 168$

Answer 168 [4]

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Question Number	**Marks**
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Examiner Number

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