Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2024

Mathematics

StemUp Model Solution

Unit M8 Paper 2
(With calculator)
Higher Tier



[GMC82]

GMC82

MONDAY 3 JUNE, 10.45 am–12 NOON

TIME

1 hour 15 minutes.

INSTRUCTIONS TO CANDIDATES

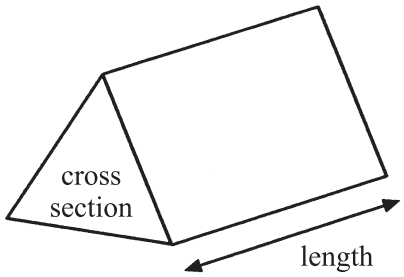
Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all twelve** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

INFORMATION FOR CANDIDATES

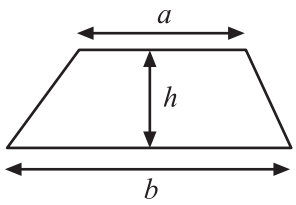
The total mark for this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

Formula Sheet

Volume of prism = area of cross section × length

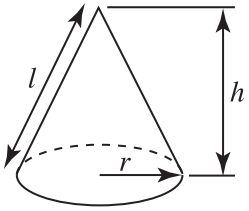


Area of trapezium = $\frac{1}{2}(a+b)h$



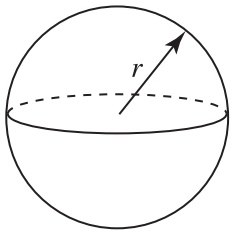
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl

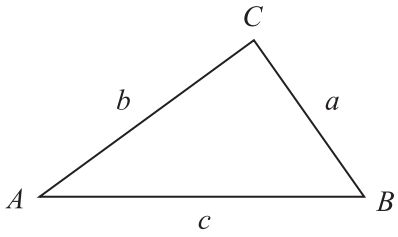


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

14392

1 Sarah rolls a fair dice and tosses a fair coin.

What is the probability that she gets a number less than 5 and a Head?

Show your working out.

$P(\text{less than 5}) = \frac{4}{6}$

$P(\text{Head}) = \frac{1}{2}$

$P(\text{less than 5 and Head}) = \frac{4}{6} \times \frac{1}{2} = \frac{4}{12} = \frac{1}{3}$

Answer $\frac{1}{3}$ [2]

2 $x^a \times x^4 = x^{12}$

Find the value of a.

$x^{a+4} = x^{12}$

$a + 4 = 12$

$a = 12 - 4 = 8$

Answer $a =$ 8 [1]

[Turn over

3 The equation $x^3 - 12x - 2 = 0$ has a solution between 2 and 5

Use a trial and improvement method to find this solution, correct to one decimal place.

You must show all your working.

x	$x^3 - 12x - 2$	
3	−11	too small
4	14	too big
3.5	−1.125	too small
3.6	1.456	too big
3.55	0.138875	too big

The solution lies between 3.5 and 3.55

so x = 3.5 correct to one decimal place

Answer $x =$ 3.5 [4]

4 The table shows the probability of getting each number on a biased dice.

Number on dice	1	2	3	4	5	6
Probability	0.18	0.2	0.3	0.1	0.1	0.12

The dice is rolled twice and the numbers are added together.

What is the probability of getting a total of 5?

Total of 5: (1, 4), (4, 1), (2, 3), (3, 2)

P(1, 4) = 0.18 × 0.1 = 0.018

P(4, 1) = 0.1 × 0.18 = 0.018

P(2, 3) = 0.2 × 0.3 = 0.06

P(3, 2) = 0.3 × 0.2 = 0.06

P(total 5) = 0.018 + 0.018 + 0.06 + 0.06

= 0.156

Answer 0.156 [4]

[Turn over

5 A varies directly as the square of x.

When $x = 5$, $A = 20$

Calculate the value of x when $A = 45$

$A = kx^2$

$20 = k \times 5^2$

$k = \frac{20}{25} = 0.8$

$45 = 0.8x^2$

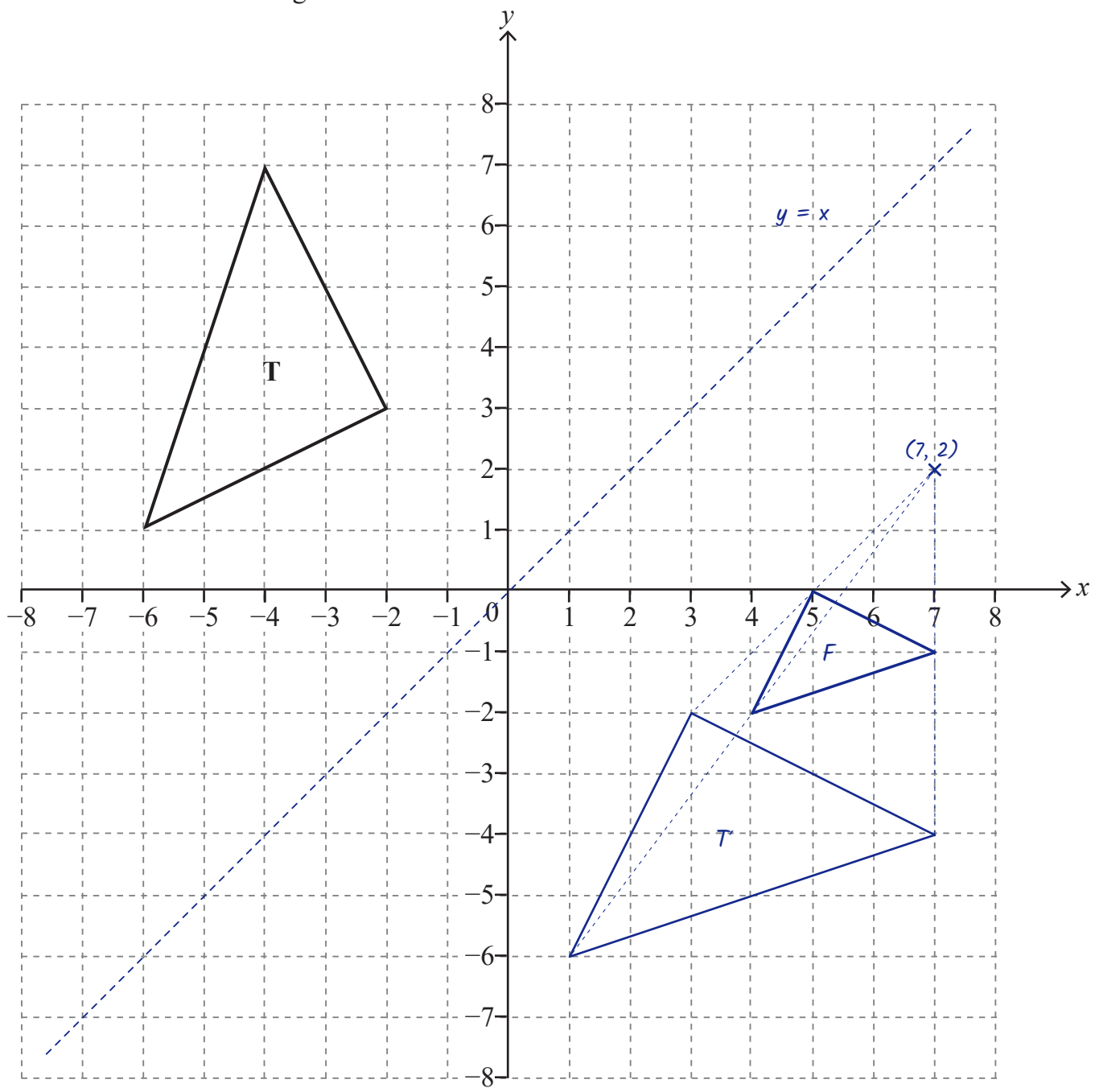
$x^2 = 45 \div 0.8 = 56.25$

$x = \sqrt{56.25} = 7.5$

Answer $x =$ 7.5 [4]

6 Triangle T is reflected in the line $y = x$ and then enlarged by a scale factor of $\frac{1}{2}$ using the centre of enlargement (7, 2).

Draw the final image and label it F.



[4]

[Turn over

7 There are 200 students in Year 8 and 160 students in Year 9

There are 120 girls in Year 8 and 100 girls in Year 9

The probability that a girl plays a musical instrument is 0.3

The probability that a boy plays a musical instrument is 0.25

A student is taken at random from the students who play a musical instrument.

What is the probability that it is a boy?

Boys: Year 8 $200 - 120 = 80$, Year 9 $160 - 100 = 60$

Girls $= 120 + 100 = 220$, Boys $= 80 + 60 = 140$

Girls who play $= 0.3 \times 220 = 66$

Boys who play $= 0.25 \times 140 = 35$

Total who play $= 66 + 35 = 101$

$P(\text{boy}) = \frac{35}{101}$

Answer $\frac{35}{101}$ [4]

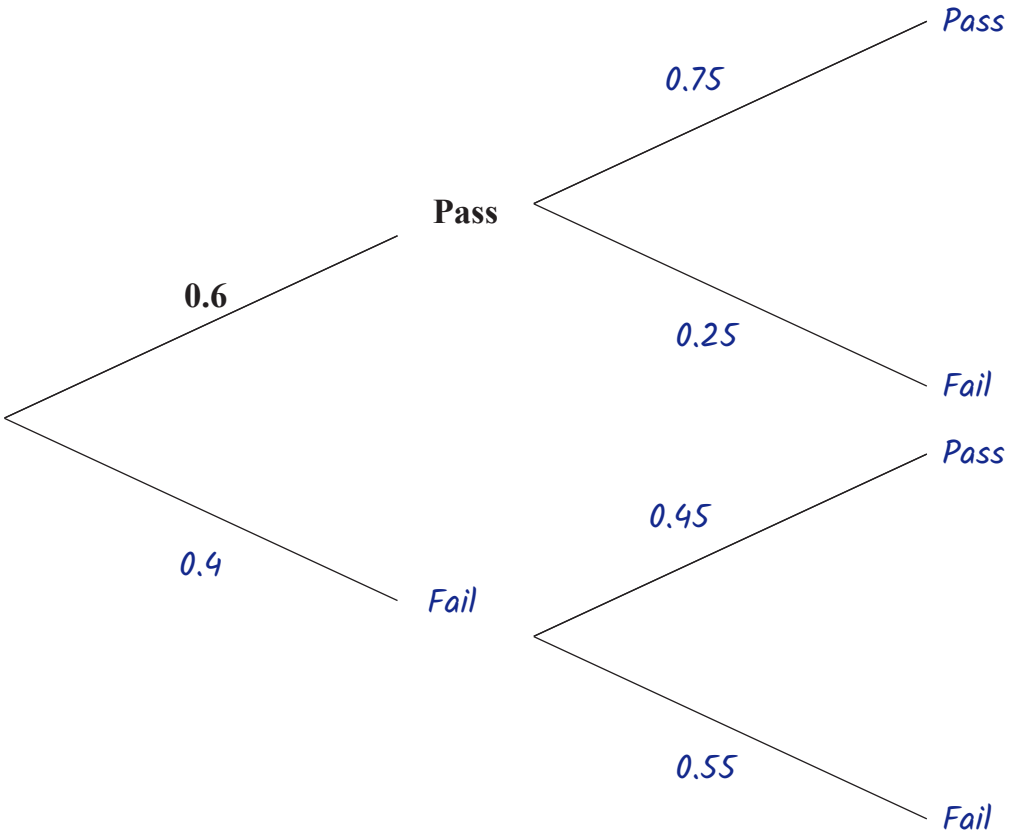
8 **(a)** Athletes at a fitness course complete two fitness tests.

The probability of passing the first test is 0.6

If the athlete passes the first test, the probability of passing the second test is 0.75

If the athlete fails the first test, the probability of passing the second test is 0.45

Complete the tree diagram.



[1]

(b) Work out the probability of an athlete passing only one of the tests.

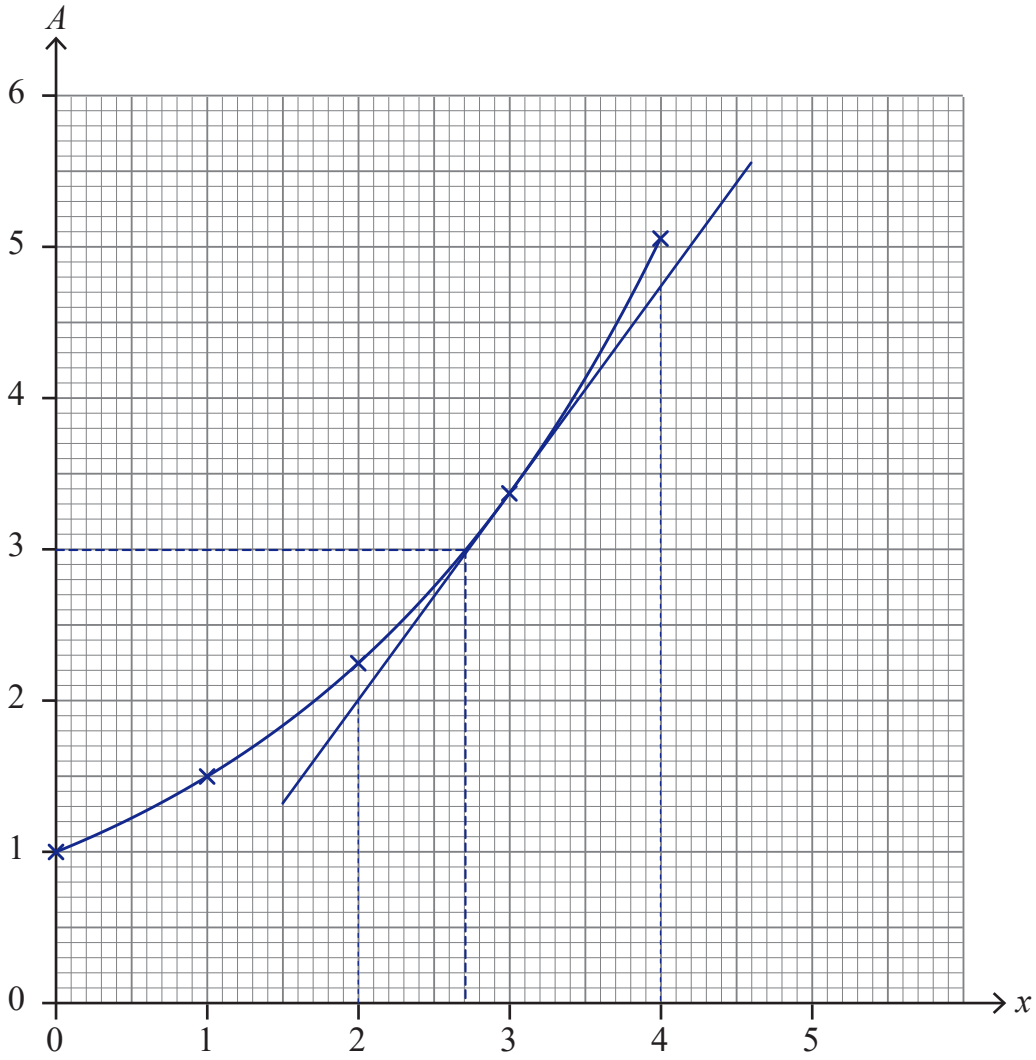
P(pass, fail) + P(fail, pass)
= 0.6 × 0.25 + 0.4 × 0.45
= 0.15 + 0.18 = 0.33

Answer 0.33 [2]

[Turn over

9 **(a)** Draw the graph of $A = (1.5)^x$ for values of x between 0 and 4

x	0	1	2	3	4
A	1	1.5	2.25	3.375	5.0625



[3]

(b) Use the graph to find when the initial value of A has trebled in value.

Initial value: $x = 0$, $A = 1.5^0 = 1$

Trebled: $A = 3 \times 1 = 3$

From the graph, $A = 3$ when $x \approx 2.7$

Answer $x =$ 2.7 [1]

(c) By drawing a suitable line on the graph, estimate the instantaneous rate of increase in the value of A when $x = 3$

Tangent drawn at (3, 3.375)

Points on tangent: (2, 2.0) and (4, 4.75)

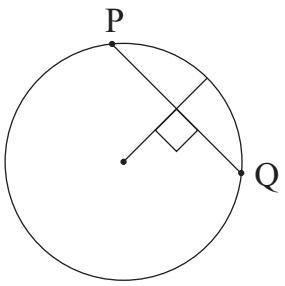
gradient = (4.75 − 2.0) ÷ (4 − 2) = 2.75 ÷ 2 = 1.375

Answer 1.4 [2]

[Turn over

10 PQ is a chord of the circle $x^2 + y^2 = 160$

(4, 8) is the midpoint of PQ.



(a) What is the gradient of the radius of the circle which passes through (4, 8)?

gradient of radius $= \frac{8-0}{4-0} = \frac{8}{4} = 2$

Answer 2 [1]

(b) The radius intersects the chord at right angles.

Show that the equation of the chord PQ is $2y + x = 20$

gradient of chord $= \frac{-1}{2}$ $\left(2 \times \frac{-1}{2} = -1\right)$

$y = mx + c$ through (4, 8):

$8 = \frac{-1}{2} \times 4 + c$

$8 = -2 + c$ so $c = 10$

$y = \frac{-1}{2}x + 10$

$2y = -x + 20$ so $2y + x = 20$

[3]

(c) Find the coordinates of P and Q.

$x = 20 - 2y$

$(20 - 2y)^2 + y^2 = 160$

$400 - 80y + 4y^2 + y^2 = 160$

$5y^2 - 80y + 240 = 0$

$y^2 - 16y + 48 = 0$

$(y - 12)(y - 4) = 0$

$y = 12$ or $y = 4$

$y = 12$: $x = 20 - 24 = -4$

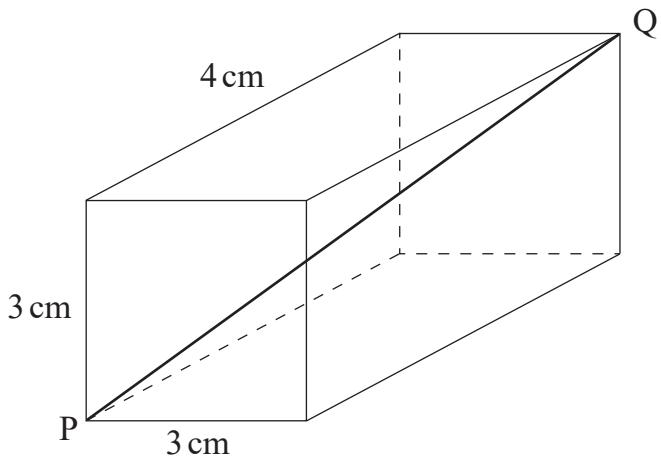
$y = 4$: $x = 20 - 8 = 12$

Answer $(-4, 12)$ and $(12, 4)$ [5]

[Turn over



11 Calculate the angle between the space diagonal PQ and the base of the cuboid.



Base diagonal = $\sqrt{(3^2 + 4^2)} = \sqrt{25} = 5$ cm

$\tan\theta = \frac{3}{5} = 0.6$

$\theta = \tan^{-1}(0.6) = 30.96375...°$

Answer 30.96 ° [3]

12 Work out the area of ABCD.

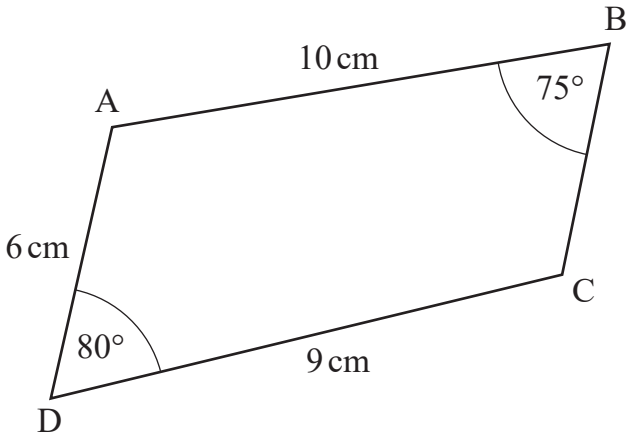


diagram not drawn accurately

Area ACD = 0.5 × 6 × 9 × sin 80° = 26.5898... cm^2

$AC^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 80°$

$AC^2 = 98.24599...$

AC = 9.911911... cm

sin ACB ÷ 10 = sin 75° ÷ 9.911911...

sin ACB = 10 × sin 75° ÷ 9.911911... = 0.974510...

angle ACB = 77.03571...°

angle BAC = 180° − 75° − 77.03571...° = 27.96428...°

Area ABC = 0.5 × 10 × 9.911911... × sin 27.96428...°

= 23.23952... cm^2

Area ABCD = 26.5898... + 23.23952...

= 49.829330...

Answer 49.83 cm^2 [6]

Sources: All images ©CCEA unless stated

THIS IS THE END OF THE QUESTION PAPER

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Question Number	**Marks**
1	
2	
3	
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12	
Total Marks	

Examiner Number

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