Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2024

Mathematics

StemUp Model Solution

Unit M4
(With calculator)
Higher Tier

[GMC41]



GMC41

THURSDAY 16 MAY, 9.15 am–11.15 am

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

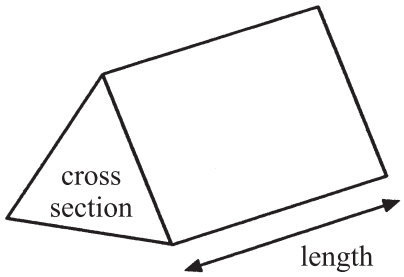
Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all twenty-three** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

INFORMATION FOR CANDIDATES

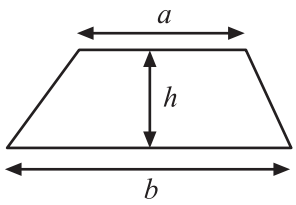
The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

Formula Sheet

Volume of prism = area of cross section × length

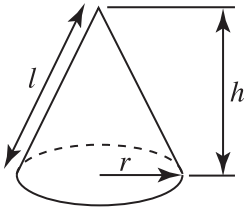


Area of trapezium = $\frac{1}{2}(a+b)h$



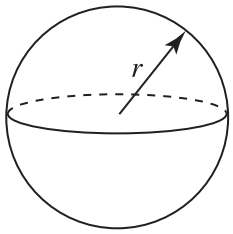
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

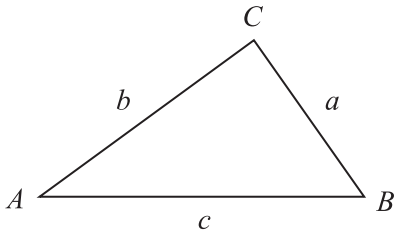


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

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1 ABC is a right-angled triangle.

AC = 15 cm.

BC = 7.5 cm.

Work out the length of AB.

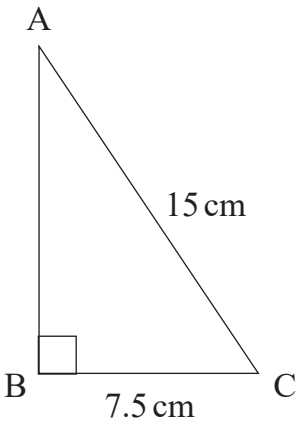


diagram not drawn accurately

$AC^2 = AB^2 + BC^2$

$15^2 = AB^2 + 7.5^2$

$AB^2 = 225 - 56.25 = 168.75$

$AB = \sqrt{168.75}$

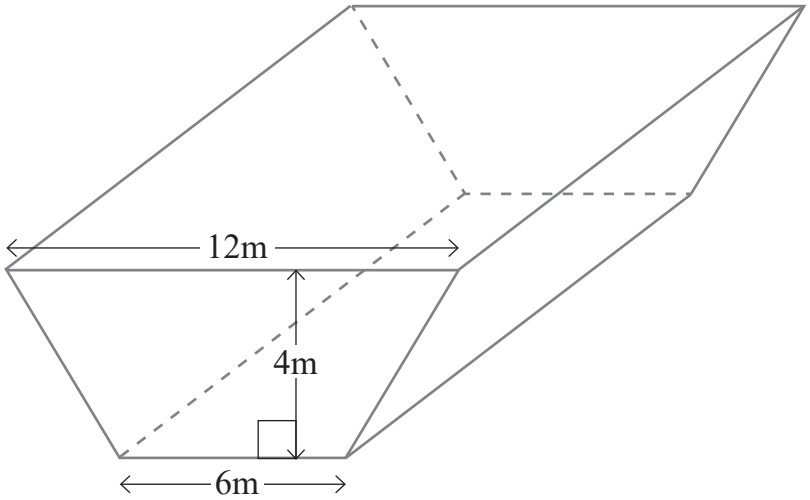
$= 12.99038106...$

$= 12.99$ cm

Answer 12.99 cm [3]

[Turn over

2 A water tank has the dimensions as shown.



The cross-section is a trapezium.

The volume of the tank is $522\,m^3$

Calculate the length of the tank.

Area of trapezium = $\frac{1}{2} \times (12 + 6) \times 4$

$= 36\ m^2$

Volume = area × length

$522 = 36 \times \text{length}$

$\text{length} = 522 \div 36$

$= 14.5\ m$

Answer 14.5 m [3]

3 The heights, in metres, of a number of trees is recorded for a science experiment.

The results are shown in the table below.

Height h (m)	Frequency
$100 \leqslant h < 110$	6
$110 \leqslant h < 120$	13
$120 \leqslant h < 130$	12
$130 \leqslant h < 140$	15
$140 \leqslant h < 150$	3
$150 \leqslant h < 160$	1

(a) Calculate an estimate for the mean height.

Midpoints: 105, 115, 125, 135, 145, 155

$\Sigma fx = 6\times105 + 13\times115 + 12\times125 + 15\times135 + 3\times145 + 1\times155$

$= 630 + 1495 + 1500 + 2025 + 435 + 155$

$= 6240$

$\Sigma f = 6 + 13 + 12 + 15 + 3 + 1 = 50$

$\text{Mean} = \frac{6240}{50} = 124.8$ m

Answer 124.8 m [4]

(b) In which class interval does the median lie?

50 trees: median is the 25.5th value. 6 + 13 = 19, 19 + 12 = 31

Answer $120 \leqslant h < 130$ [1]

[Turn over

4 Alan wants to invest £2400 for 2 years.

He sees the following offers for compound interest.

Bank A	Bank B
3% for the first year 1% for each extra year	4% for the first year 0.5% for each extra year

Calculate, for each bank, the value of the investment after 2 years.

Bank A:

3% of £2400 = £72 Year 1: 2400 + 72 = £2472

1% of £2472 = £24.72 Year 2: 2472 + 24.72 = £2496.72

(check: 2400 × 1.03 × 1.01 = 2496.72)

Bank B:

4% of £2400 = £96 Year 1: 2400 + 96 = £2496

0.5% of £2496 = £12.48 Year 2: 2496 + 12.48 = £2508.48

(check: 2400 × 1.04 × 1.005 = 2508.48)

Answer **Bank A**: £ 2496.72

Bank B: £ 2508.48 [4]

5 **(a)** Write 126 as a product of its prime factors.

$126 = 2 \times 63$

$= 2 \times 3 \times 21$

$= 2 \times 3 \times 3 \times 7$

Answer $2 \times 3^2 \times 7$ [2]

(b) Find the HCF of 126 and 90

$126 = 2 \times 3 \times 3 \times 7$

$90 = 2 \times 3 \times 3 \times 5$

$HCF = 2 \times 3 \times 3 = 18$

Answer 18 [2]

[Turn over

6 A rectangular basketball court is x metres wide.

The length of the court is 13 metres longer than its width.

The perimeter of the court is 86 metres.

Form and solve an equation to find the value of x.

width = x, length = x + 13

x + x + 13 + x + x + 13 = 86

4x + 26 = 86

4x = 60

x = 15

Answer x = 15 m [3]

7 (2, 5) is the midpoint of the line joining (a, b) and $(3a, 3)$.

Find the values of a and b.

$$\frac{a + 3a}{2} = 2$$

$$4a = 4$$

$$a = 1$$

$$\frac{b + 3}{2} = 5$$

$$b + 3 = 10$$

$$b = 7$$

Answer $a =$ 1 , $b =$ 7 [3]

[Turn over

8 A stepladder with sides of 2.3 m is opened so that the feet are 1.4 m apart.



Source: © Getty Images

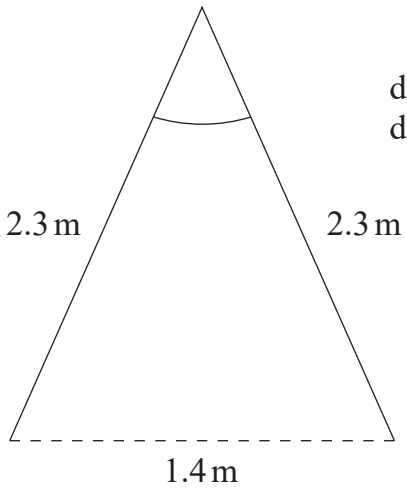


diagram not drawn accurately

Calculate the angle to which the stepladder has been opened.

Split the isosceles triangle into two right-angled triangles:

half of 1.4 = 0.7 m

$\sin\theta = \frac{0.7}{2.3}$

$\theta = \sin^{-1}(0.7 \div 2.3) = 17.71893187...°$

Angle = 2 × 17.71893187...

= 35.43786375...°

= 35.44°

Answer 35.44 ° [4]

9 A cylindrical flask has diameter 12 cm and height 25 cm.

The flask is full of coffee.

A coffee cup is a hemisphere of diameter 9 cm.

Each coffee cup is filled to 90% capacity.

How many cups of coffee can be poured from the flask?

Volume of flask = $\pi \times 6^2 \times 25$

$= 2827.433388... \text{ cm}^3$

Volume of cup = $\frac{2}{3} \times \pi \times 4.5^3$

$= 190.8517537... \text{ cm}^3$

90% of cup = $0.9 \times 190.8517537...$

$= 171.7665783... \text{ cm}^3$

Number of cups = $2827.433388... \div 171.7665783...$

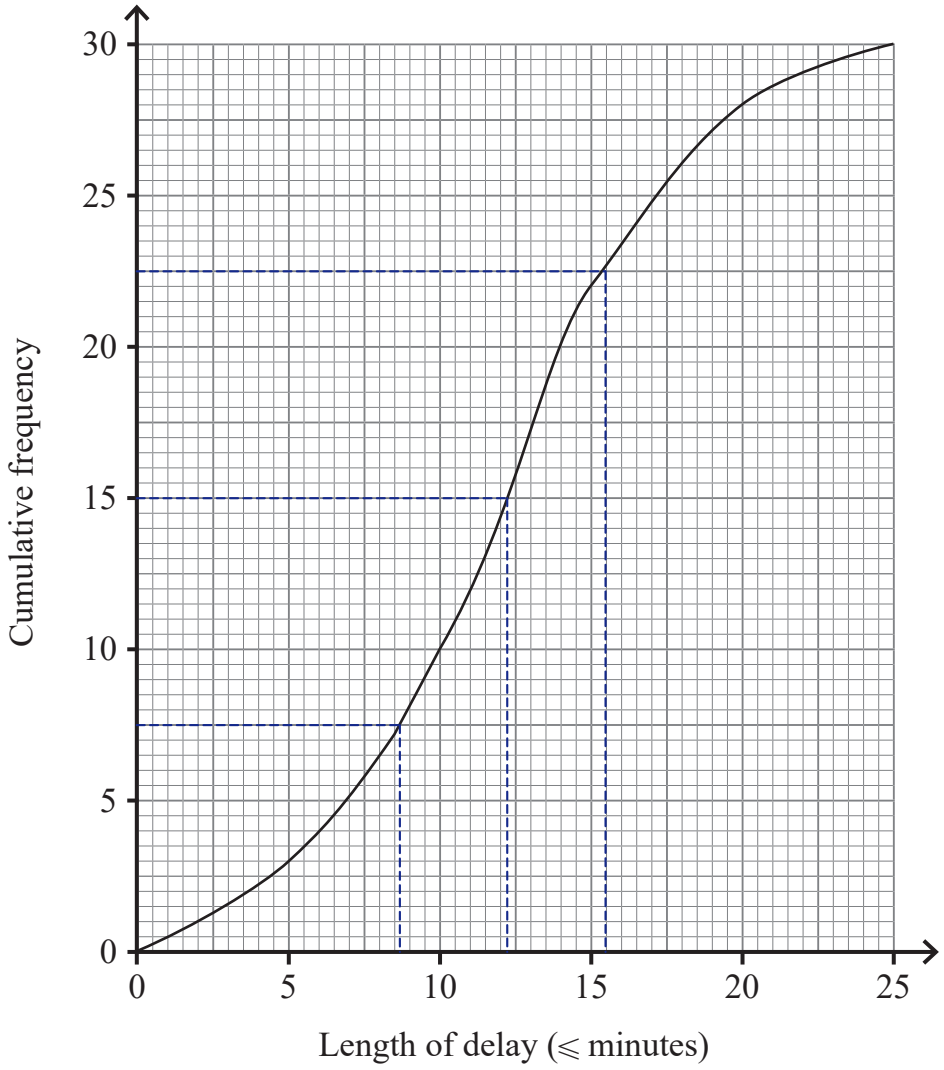
$= 16.46090535...$

So 16 full cups can be poured.

Answer 16 cups [5]

[Turn over

10 **(a)** The cumulative frequency diagram illustrates the lengths of delays to trains at Northside Station.



(i) Find the median.

Answer 12.25 minutes [1]

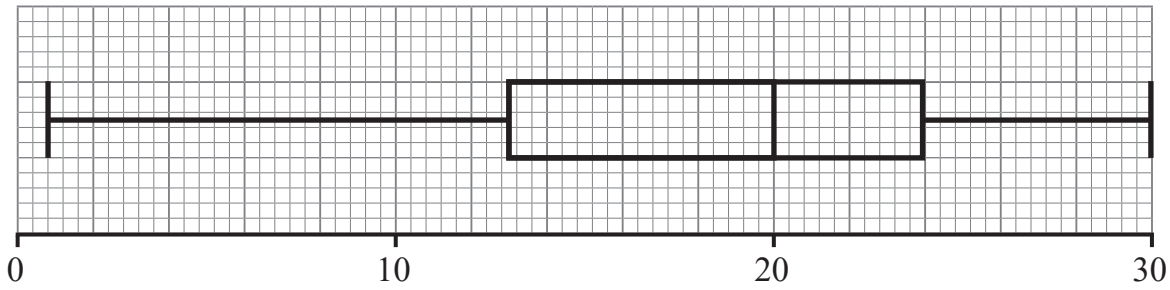
n = 30: median at 15th, LQ at 7.5th, UQ at 22.5th

(ii) Find the interquartile range.

UQ − LQ = 15.5 − 8.7

Answer 6.8 minutes [2]

(b) The box plot illustrates the lengths of delays to trains at Southend Station.



Make two comparisons between the lengths of delays at Northside Station to those at Southend Station.

1st comparison *The median delay at Southend (20 min) is greater than at Northside (12.25 min), so delays are longer on average.*

2nd comparison *Delays at Southend are more varied: IQR about 24 – 13 = 11 min compared with 6.8 min at Northside.* [2]

11 The price for 900 litres of oil increased by 28%

The new price for 900 litres of oil is £736

How much did 900 litres of oil cost before the increase?

128% = £736

1% = 736 ÷ 128 = £5.75

100% = 5.75 × 100 = £575

(check: 736 ÷ 1.28 = 575)

Answer £ *575* [3]

[Turn over

12 **(a)** The diagonal of a rectangle is 26 cm.

The width of the rectangle is x cm.

The length of the rectangle is $(34 - x)$ cm.

Show that $x^2 - 34x + 240 = 0$

$x^2 + (34 - x)^2 = 26^2$

$x^2 + 1156 - 68x + x^2 = 676$

$2x^2 - 68x + 1156 - 676 = 0$

$2x^2 - 68x + 480 = 0$

$\div 2$: $x^2 - 34x + 240 = 0$

[4]

(b) Solve the equation to find the possible values of x.

$x^2 - 34x + 240 = 0$

$(x - 10)(x - 24) = 0$

$x = 10$ or $x = 24$

Answer $x =$ 10 or 24 [2]



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13 A rectangle has length $(4x - 5)$ and width $(3x + 4)$.

What is the quadratic expression for its area?

Area = $(4x - 5)(3x + 4)$

$= 12x^2 + 16x - 15x - 20$

$= 12x^2 + x - 20$

Answer $12x^2 + x - 20$ [2]

[Turn over

14 In the triangle below, angle ABC = 90°

AB = 50 cm. BD = 22 cm.

Calculate the length of AC.

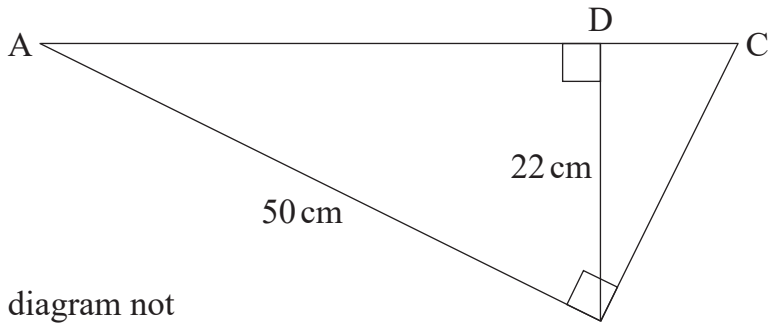


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drawn accurately

In triangle ABD (right angle at D):

$\sin A = \frac{22}{50}$

$A = \sin^{-1}(22 \div 50) = 26.10388...°$

In triangle ABC (right angle at B):

$\cos 26.10388... = \frac{50}{AC}$

$AC = \frac{50}{\cos 26.10388...}$

$= 55.67764...$

$= 55.68$ cm

Answer 55.68 cm [5]

15 A sample of 16 squirrels was taken from a forest.

Each squirrel was marked with a dye and released back into the forest.

A year later a sample of 20 squirrels was taken from the same forest.

3 of the squirrels in this sample were found to be marked with the dye.

(a) Calculate an estimate of the population of squirrels in the forest.

$\frac{3}{20} = \frac{16}{N}$

$N = 16 \times 20 \div 3$

$= 106.666...$

so about 107 squirrels

Answer 107 [2]

(b) State one fault with the sampling method used and suggest how it could be improved.

Fault The sample sizes (16 and 20) are too small.

Improvement Use much larger samples. [2]

[Turn over

16 The formula for h is

$$h = \frac{u^2}{2a}$$

Given that

$u = 21.34$ correct to 4 significant figures,

$a = 9.8$ correct to 1 decimal place,

calculate the upper bound of h.

u: upper bound = 21.345

a: lower bound = 9.75

Upper bound of h uses max u and min a:

$h = \frac{21.345^2}{2 \times 9.75} = \frac{455.609025}{19.5}$

$= 23.36456538...$

Answer 23.36456538 [3]

17 Find the equation of the straight line that passes through the point (–2, 6) and is perpendicular to the straight line $3x + 2y = 7$

$3x + 2y = 7$

$2y = -3x + 7$

$y = -\frac{3}{2}x + \frac{7}{2}$ gradient $= -\frac{3}{2}$

Perpendicular gradient $= \frac{2}{3}$ $(-\frac{3}{2} \times \frac{2}{3} = -1)$

$y - 6 = \frac{2}{3}(x + 2)$

$y = \frac{2}{3}x + \frac{4}{3} + 6$

$y = \frac{2}{3}x + \frac{22}{3}$

Answer $y = \frac{2}{3}x + \frac{22}{3}$ [4]

[Turn over

18 **(a)** Factorise $50a^2x^2 - 18y^2$

$50a^2x^2 - 18y^2 = 2(25a^2x^2 - 9y^2)$

$= 2(5ax + 3y)(5ax - 3y)$

Answer $2(5ax + 3y)(5ax - 3y)$ [3]

(b) Write as a single fraction and simplify

$$2 + \frac{4}{2x-1} - \frac{3}{x+2}$$

$$= \frac{2(2x-1)(x+2) + 4(x+2) - 3(2x-1)}{(2x-1)(x+2)}$$

$$= \frac{2(2x^2 + 3x - 2) + 4x + 8 - 6x + 3}{(2x-1)(x+2)}$$

$$= \frac{4x^2 + 6x - 4 + 4x + 8 - 6x + 3}{(2x-1)(x+2)}$$

$$= \frac{4x^2 + 4x + 7}{(2x-1)(x+2)}$$

Answer $\dfrac{4x^2 + 4x + 7}{(2x-1)(x+2)}$ [4]

19 **(a)** O is the centre of a circle.

A, B, C and D are points on the circumference of the circle.

Angle BOD = 114°

On the diagram below mark clearly an angle that is 57° [1]

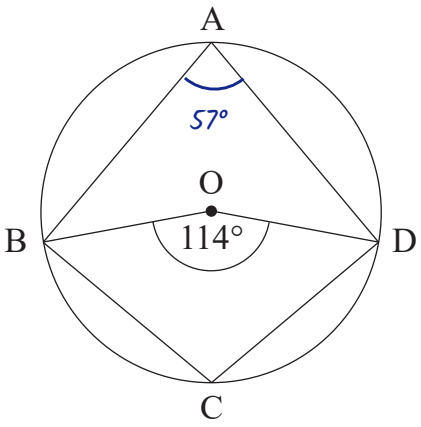


diagram not drawn accurately

(b) O is the centre of a circle.

E, F and G are points on the circumference of the circle.

HFJ is a tangent to the circle at the point F.

Angle HFE = 49°

On the diagram below mark clearly another angle that is 49° [1]

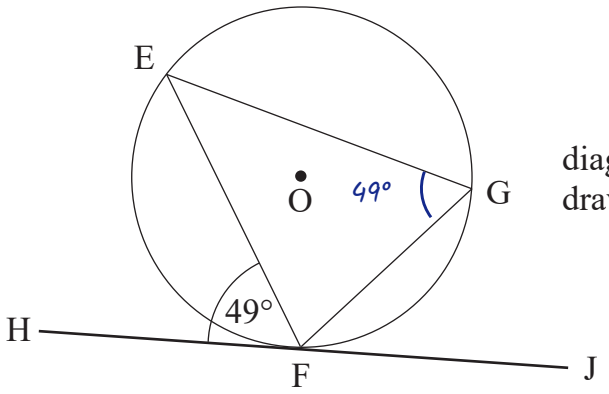


diagram not drawn accurately

[Turn over

(c) L, M and N are three points on the circumference of a circle.

LP and NP are tangents to the circle.

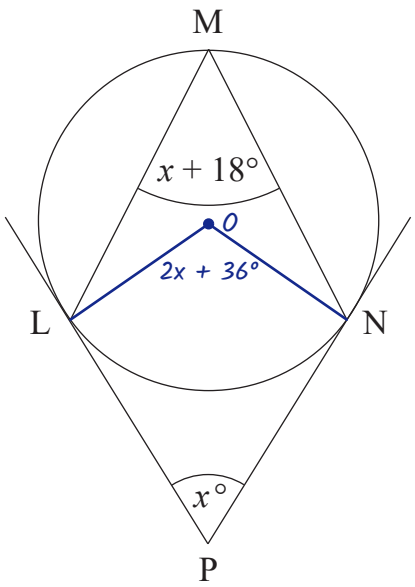


diagram not drawn accurately

Work out the value of x.

Show each step of your working clearly.

Join O (centre) to L and N.

Angle LON = 2 × (x + 18) = 2x + 36 (angle at centre = 2 × angle at circumference)

Angle OLP = angle ONP = 90° (tangent is perpendicular to radius)

OLPN is a quadrilateral, so angles add to 360°:

2x + 36 + 90 + 90 + x = 360

3x + 216 = 360

3x = 144

x = 48

Answer $x =$ 48 [4]

20 Prove that a sphere of radius r and a cube of side r can never have the same surface area.

Surface area of sphere = $4\pi r^2$

Surface area of cube = $6r^2$

If they were equal: $4\pi r^2 = 6r^2$

$r \neq 0$, so divide by r^2: $4\pi = 6$

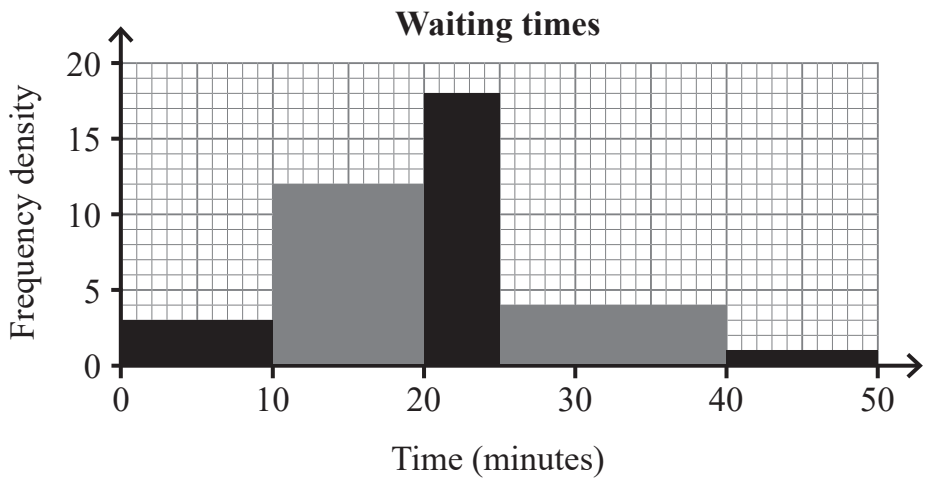
$\pi = 6 \div 4 = 1.5$

But $\pi = 3.14159...$, so $\pi \neq 1.5$ $(4\pi = 12.566..., \text{not } 6)$.

This is impossible, so the surface areas can never be the same.

[2]

21 **(a)** The histogram shows patient waiting times in a hospital clinic during one week.



The hospital target is for 50% of patients to have to wait less than 20 minutes.

Has the hospital target been met?

Explain your answer clearly.

f = fd × width: 3×10 + 12×10 + 18×5 + 4×15 + 1×10 = 30 + 120 + 90 + 60 + 10 = 310

50% of 310 = 155. Less than 20 min: 30 + 120 = 150

Answer *No* because *only 150 of the 310 patients waited less than 20 minutes, which is less than 155 (50%).* [3]

(b) 38 patients waited longer than T minutes.

Work out an estimate for the value of T.

40 to 50 min: 1 × 10 = 10 patients, so need 38 − 10 = 28 more from 25 to 40

28 ÷ 4 (frequency density) = 7 minutes

T = 40 − 7 = 33

Answer *33* minutes [3]

22 The diagram shows a square of side $(3x - 1)$ cm and a trapezium with base length $(5x - 3)$ cm and perpendicular height 4 cm.

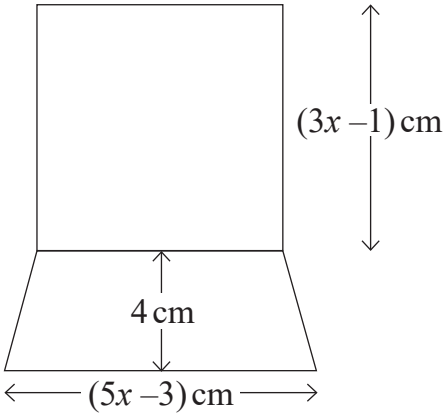


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The area of the square is one and a half times the area of the trapezium.

By setting up and solving a quadratic equation, find the value of x, to 2 decimal places.

Area of square = $(3x - 1)^2 = 9x^2 - 6x + 1$

Area of trapezium = $\frac{1}{2}(5x - 3 + 3x - 1) \times 4 = 2(8x - 4) = 16x - 8$

$9x^2 - 6x + 1 = 1.5(16x - 8)$

$9x^2 - 6x + 1 = 24x - 12$

$9x^2 - 30x + 13 = 0$

$x = \frac{30 \pm \sqrt{((-30)^2 - 4 \times 9 \times 13)}}{2 \times 9} = \frac{30 \pm \sqrt{432}}{18}$

$x = 2.821367...$ or $x = 0.512076...$

$x = 0.51$ is not possible as $5x - 3$ would be negative

so $x = 2.82$

Answer $x =$ 2.82 [6]

[Turn over

23 Solve the equation

$$\frac{1}{2} - \frac{3}{3x+1} = \frac{5}{5x-9}$$

$\frac{1}{2} - \frac{3}{3x+1} = \frac{3x+1-6}{2(3x+1)} = \frac{3x-5}{6x+2}$

$\frac{3x-5}{6x+2} = \frac{5}{5x-9}$

$(3x-5)(5x-9) = 5(6x+2)$

$15x^2 - 27x - 25x + 45 = 30x + 10$

$15x^2 - 52x + 45 = 30x + 10$

$15x^2 - 82x + 35 = 0$

$(x-5)(15x-7) = 0$

$x = 5$ or $x = \frac{7}{15}$

Answer $x =$ 5 or $\frac{7}{15}$ [7]

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