Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2023

# Mathematics

StemUp Model Solution

Unit M8 Paper 2
**(With calculator)**
Higher Tier



**[GMC82]**

*GMC82*

**WEDNESDAY 7 JUNE, 10.45 am–12 NOON**

**TIME**

1 hour 15 minutes.

**INSTRUCTIONS TO CANDIDATES**

Write your Centre Number and Candidate Number in the spaces provided at the top of this page. **You are provided with Higher Tier Additional Support Materials for use with this paper.**
**You must answer the questions in the spaces provided.**
**Do not write outside the boxed area on each page, on blank pages or tracing paper.**
Complete in black ink only. **Do not write with a gel pen.**
Answer **all thirteen** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

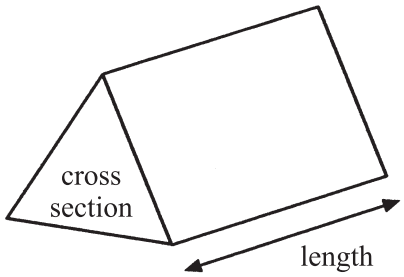
**INFORMATION FOR CANDIDATES**

The total mark for this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

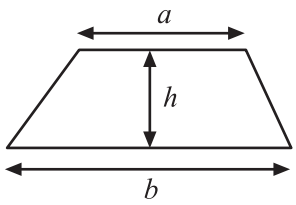
13349.07 **R**

# Formula Sheet

**Volume of prism** = area of cross section × length

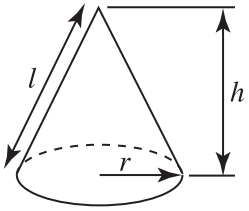



**Area of trapezium** = $\frac{1}{2}(a+b)h$



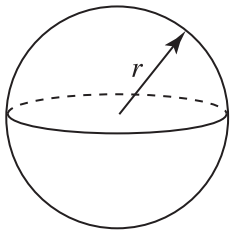

**Volume of cone** = $\frac{1}{3}\pi r^2 h$

**Curved surface area of cone** = $\pi r l$

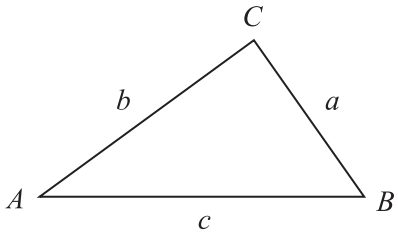



**Volume of sphere** = $\frac{4}{3}\pi r^3$

**Surface area of sphere** = $4\pi r^2$




**In any triangle *ABC***




**Quadratic Equation**

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Sine Rule:** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine Rule:** $a^2 = b^2 + c^2 - 2bc\cos A$

**Area of triangle** = $\frac{1}{2}ab\sin C$

**1** **(a)** Simplify $y^8 \div y^2$

$y^8 \div y^2 = y^{8-2}$

Answer $y^6$ [1]

**(b)** Solve the inequality $7x > 3x - 12$

$7x - 3x > -12$
$4x > -12$
$x > -12 \div 4$

Answer $x > -3$ [2]

**2** **(a)** Write the decimal number 19 as a binary number.

$19 = 16 + 2 + 1$

Answer 10011 [1]

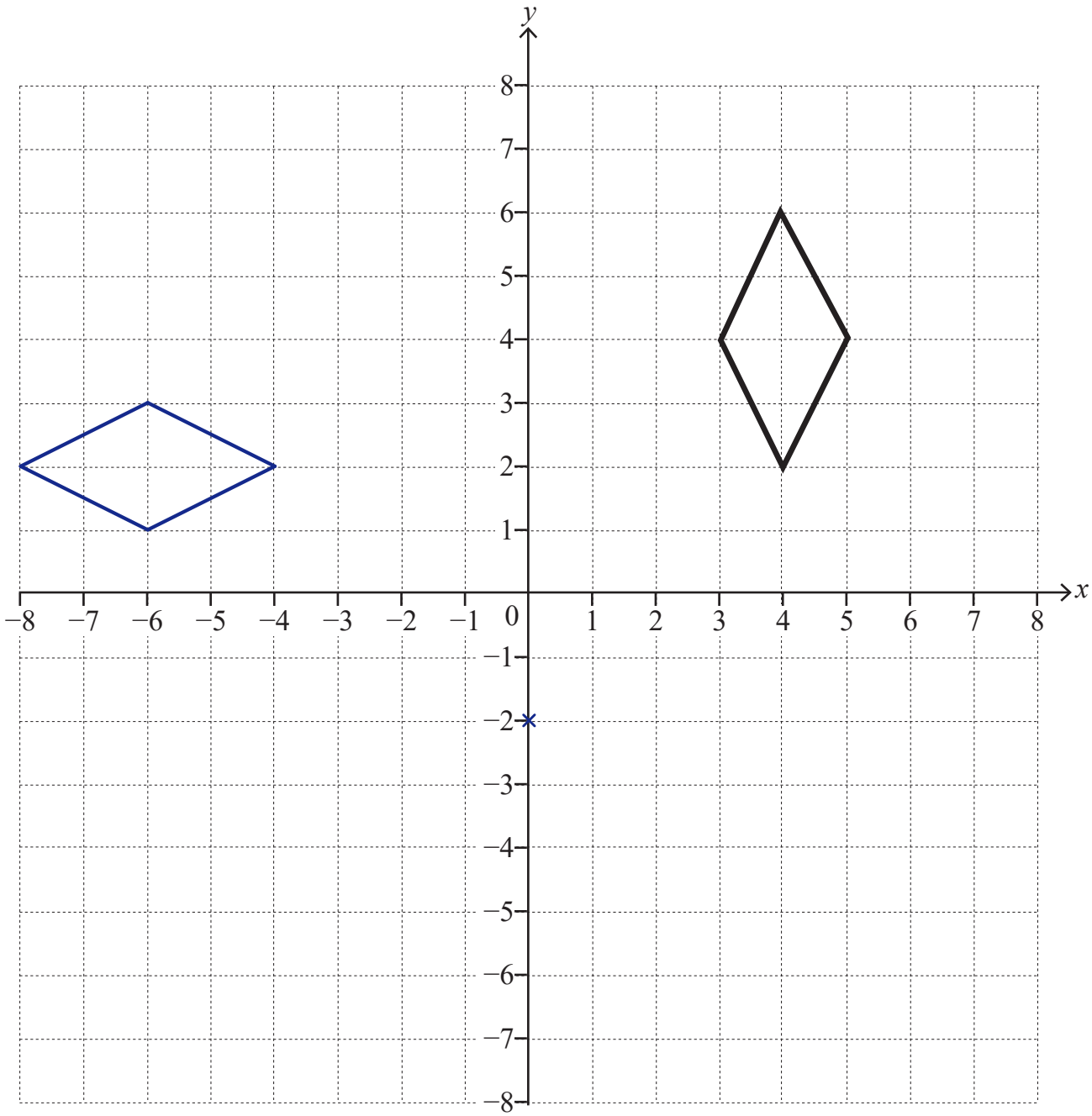
**(b)** Rearrange $h - 3m = y$ to make $m$ the subject.

$h - y = 3m$
$m = (h - y) \div 3$

Answer $m = \frac{h - y}{3}$ [2]

**[Turn over**

**3** Rotate the shape 90° anticlockwise about (0, −2)




[2]

**4** A regular polygon has exterior angles of size 15°

**(a)** How many sides has the polygon?

360 ÷ 15 = 24

Answer 24 [2]

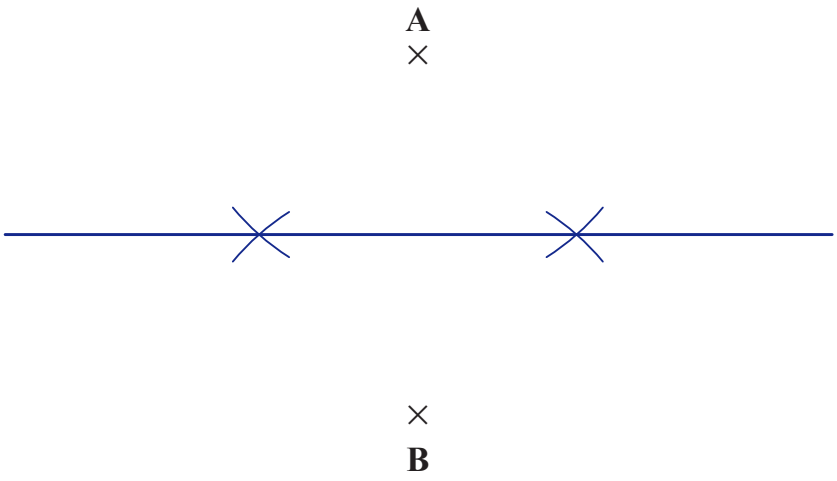
**(b)** Bailey thinks all regular pentagons are congruent.

Is he correct?

Circle your answer.

yes (no) more information needed [1]

**5** Draw the locus of all points which are the same distance from **A** and **B**.

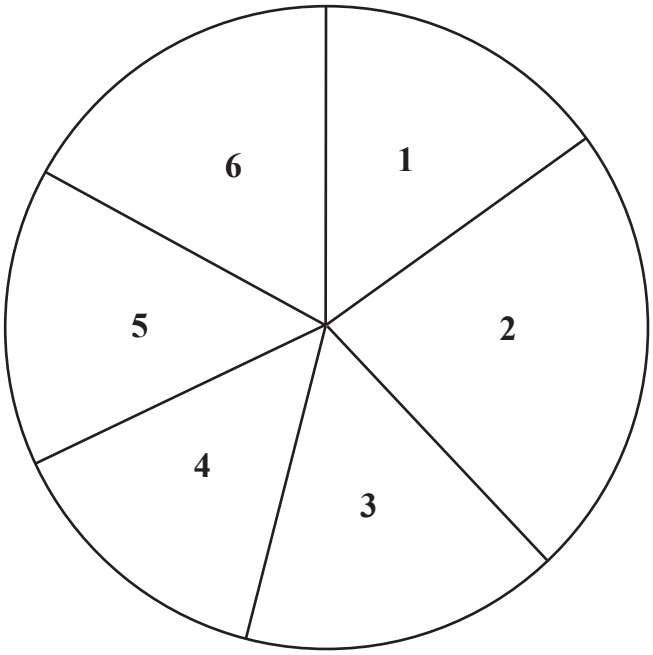



[2]

**[Turn over**

**6** Georgia rolls a dice a number of times and records the outcome each time.

She displays her results in this pie chart.



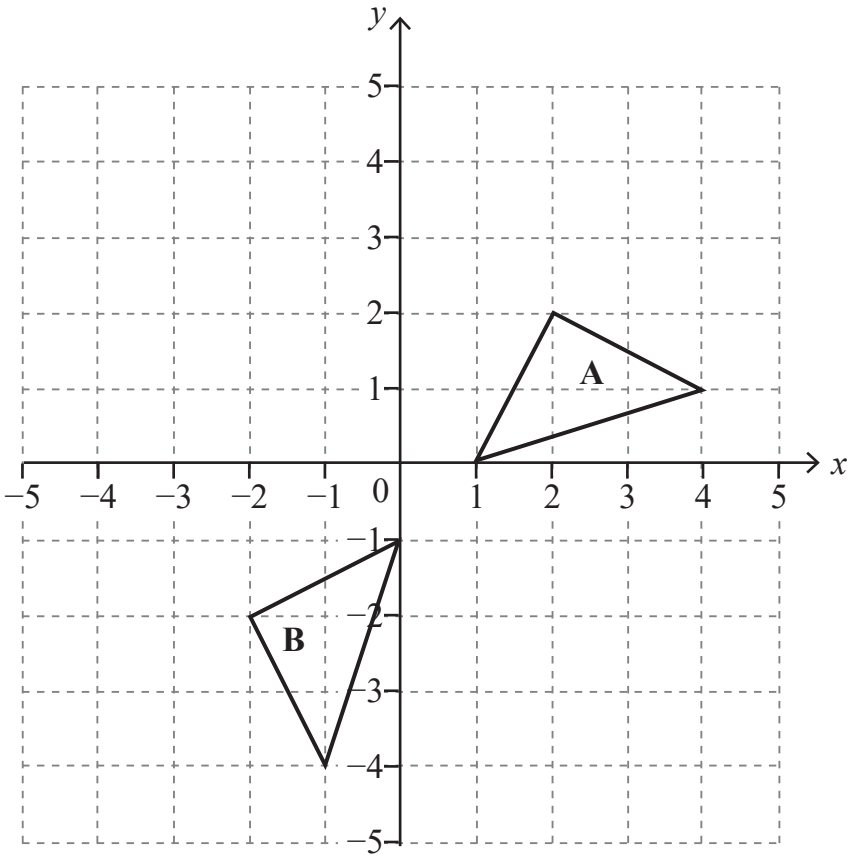

Alice thinks Georgia's dice is biased.

Bob disagrees.

What information is needed to decide who is correct?

Answer *the number of times she rolled the dice* [1]

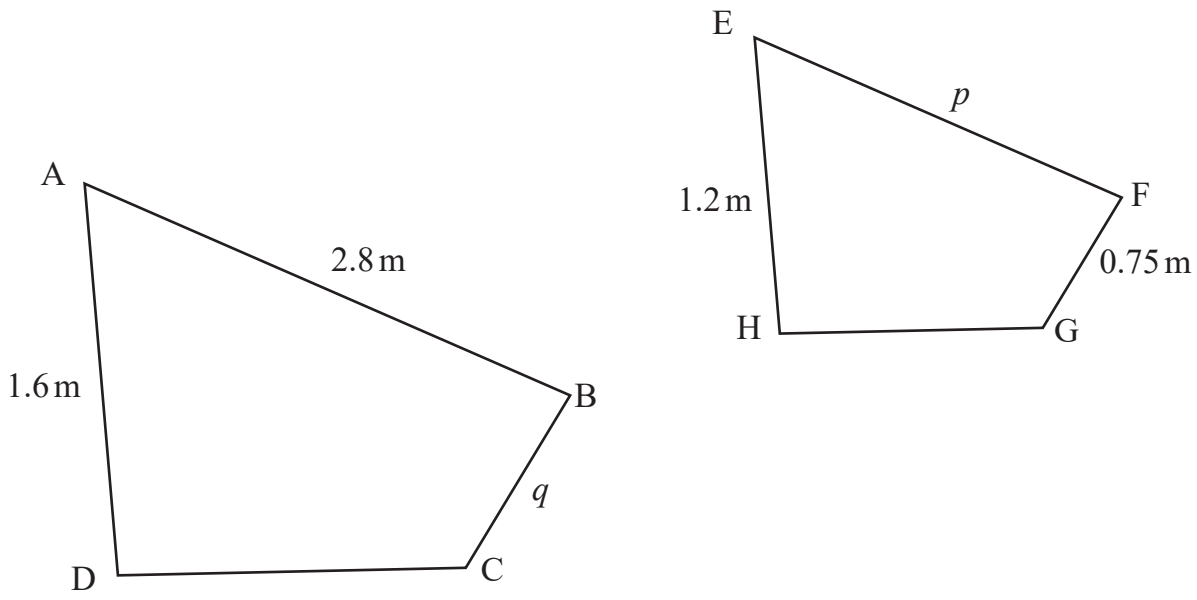
7 **(a)** Describe fully the single transformation that maps triangle **A** to triangle **B**.




Answer Reflection in the line $y = -x$ [2]

**[Turn over**

**(b)** Two similar shapes are shown.




Diagrams are not drawn to scale.

**(i)** Find the value of $p$.

scale factor = 1.2 ÷ 1.6 = 0.75
p = 2.8 × 0.75 = 2.1

Answer 2.1 m [2]

**(ii)** Find the value of $q$.

q = 0.75 ÷ 0.75 = 1

Answer 1 m [1]

**8** When a body is moving through the air, the air resistance $R$ Newtons (N) is proportional to the square of the velocity $v$ m/s.

At a velocity of 100 m/s, the air resistance is 50 N.

**(a)** Find $R$ in terms of $v$.

$R = kv^2$
$50 = k \times 100^2$
$50 = 10000k$
$k = 50 \div 10000 = 0.005$

Answer $R = 0.005v^2$ [2]

**(b)** Find $R$ when $v = 120$ m/s.

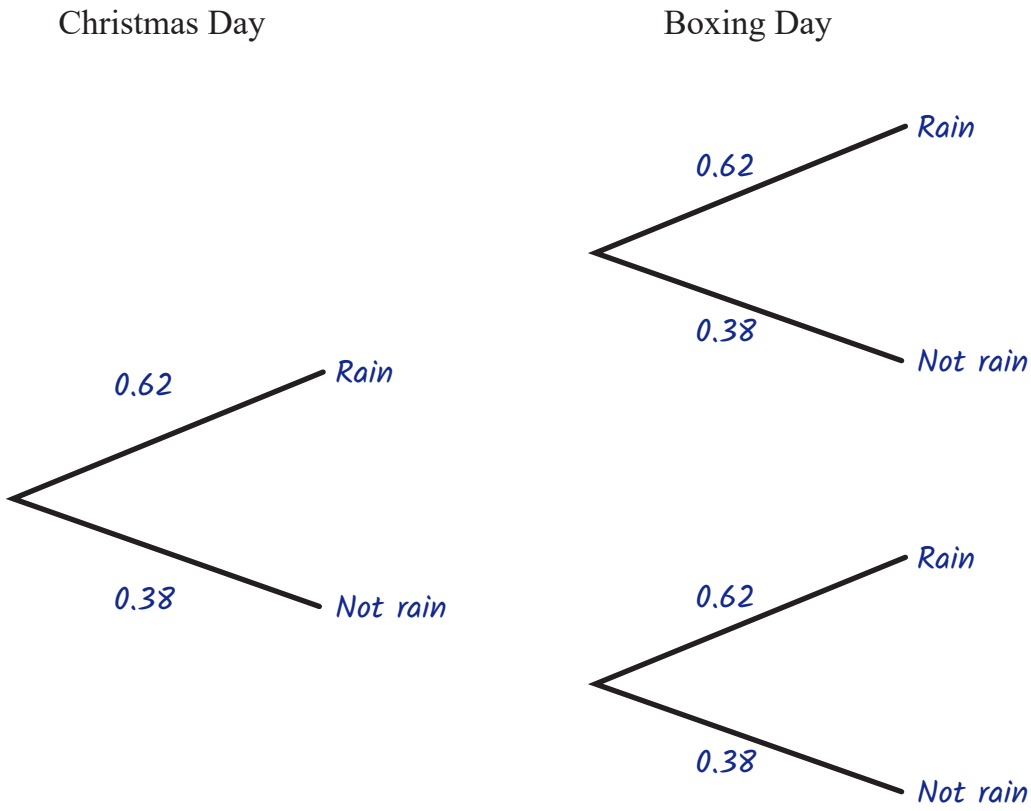
$R = 0.005 \times 120^2$
$R = 0.005 \times 14400 = 72$

Answer 72 N [1]

**[Turn over**

**9** The probability that it rains in Ireland on any given day in December is 0.62

**(a)** Complete the tree diagram to show this information for Christmas Day and Boxing Day.




[2]

**(b)** What is the probability that it rains on exactly one of these days?

P(R, NR) + P(NR, R)
= 0.62 × 0.38 + 0.38 × 0.62
= 0.2356 + 0.2356 = 0.4712

Answer 0.4712 [2]

**10**

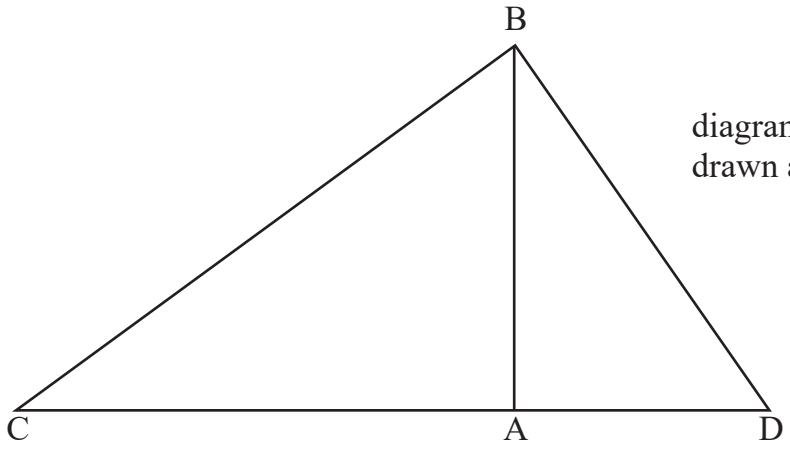



diagram not drawn accurately

AB is a vertical mast and CAD is horizontal.

The angle of elevation of B from C is 36°

The angle of elevation of B from D is 48°

The distance CD is 100 metres.

Calculate the height of AB.

Angle CBD = 180° − 36° − 48° = 96°

$$\frac{BC}{\sin 48°} = \frac{100}{\sin 96°}$$

BC = 100 × sin 48° ÷ sin 96° = 74.7238...

In triangle ABC: sin 36° = AB ÷ BC

AB = 74.7238... × sin 36°

AB = 43.92156... = 43.9 m

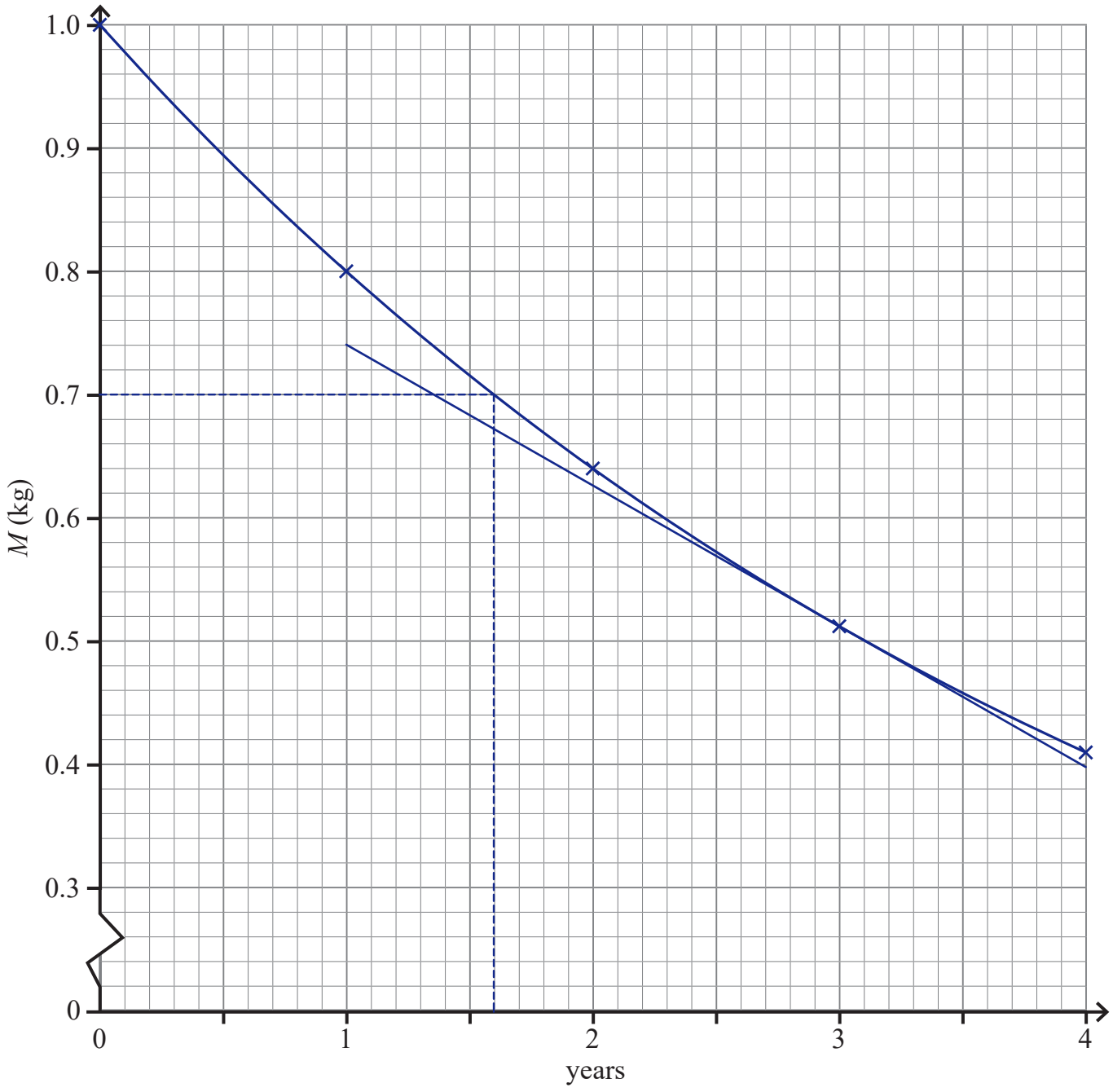
Answer 43.9 m [5]

**[Turn over**

**11** The mass $M$kg of a decaying substance, $x$ years after its manufacture, is given by the formula

$$M = (0.8)^x$$

**(a)** Sketch a graph to show the mass over the first 4 years. [4]







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**(b)** By drawing appropriate lines on the graph, find

**(i)** after how many months the mass was 70% of its original value,

*M = 0.7 when x = 1.6 years, 1.6 × 12 = 19.2*

Answer *19.2* months [2]

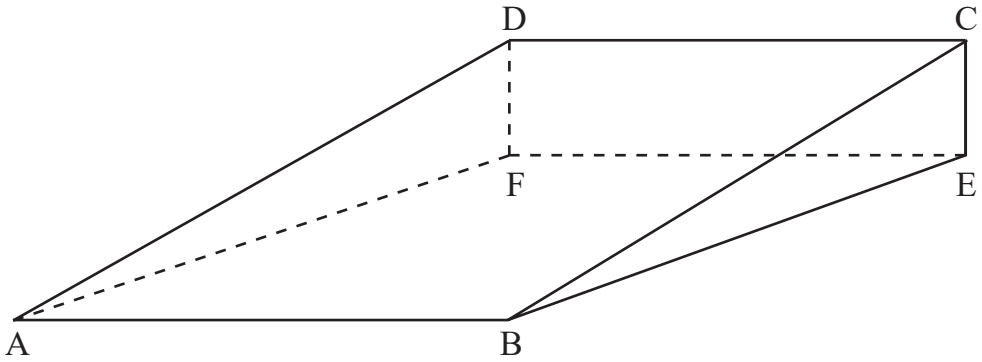
**(ii)** the rate of decay when $x = 3$

Answer *0.113* kg/year [2]

*Tangent at x = 3 passes through (1, 0.74) and (4, 0.40)*
*gradient = (0.40 − 0.74) ÷ (4 − 1) = −0.34 ÷ 3 = −0.113*
*so mass decreases at 0.113 kg per year*

**[Turn over**

**12**




The rectangular ramp ABCD is inclined to the horizontal rectangular base ABEF.

CDFE is vertical. AB = 6 m, CE = 4 m and BE = 9 m.

**(a)** Calculate the length of the line AC.

$AC^2 = AB^2 + BE^2 + CE^2$
$AC^2 = 6^2 + 4^2 + 9^2 = 133$
$AC = \sqrt{133} = 11.53256259...$

Answer 11.5 m [2]

**(b)** Calculate the angle between AC and the base.

Angle between AC and the base is angle CAE
$\sin CAE = CE \div AC = 4 \div 11.53256259...$
$CAE = \sin^{-1}(0.34684...) = 20.2944...°$

Answer 20.3 ° [3]

**13** $P$ is inversely proportional to the square of $x$.

When the value of $x$ is increased from $a$ to $(a + 1)$, the value of $P$ is halved.

Find the exact values of $a$.

$P = \frac{k}{x^2}$ so $P = \frac{k}{a^2}$ and $\frac{P}{2} = \frac{k}{(a+1)^2}$

$k = Pa^2$ and $k = \frac{P(a+1)^2}{2}$

$Pa^2 = \frac{P(a+1)^2}{2}$

$2a^2 = (a+1)^2$

$2a^2 = a^2 + 2a + 1$

$a^2 - 2a - 1 = 0$

$a = \frac{2 \pm \sqrt{(4+4)}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$

$a = 1 \pm \sqrt{2}$

Answer $a = 1 + \sqrt{2}$ or $a = 1 - \sqrt{2}$ [6]

**THIS IS THE END OF THE QUESTION PAPER**

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| For Examiner's use only | |
|---|---|
| **Question Number** | **Marks** |
| 1 | |
| 2 | |
| 3 | |
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| 7 | |
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| **Total Marks** | |

**Examiner Number**

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General Certificate of Secondary Education
Summer 2023

# GCSE Mathematics

# HIGHER TIER
# ADDITIONAL SUPPORT MATERIALS
# (For use in Summer 2023)

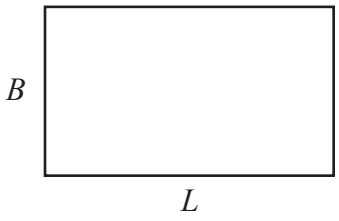


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$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

**Perimeter, Area and Volume**

The perimeter of a polygon is the distance around the outside of the polygon.




The area of a rectangle is found by multiplying the length of the rectangle by the breadth.

$A = L \times B$ where $L$ is length and $B$ is breadth.

The volume of a cuboid is found by multiplying the length by the breadth by the height of the cuboid.

$V = L \times B \times H$ where $V$ is volume, $L$ is length, $B$ is breadth and $H$ is height.

The area of a circle is $A = \pi r^2$ where $r$ is the radius of the circle.

The circumference (perimeter) of a circle is $C = 2\pi r$ where $r$ is the radius of the circle. An alternative formula is $C = \pi d$ where $d$ is the diameter of the circle.

**Mid point of a line**

If $(x_1, y_1)$ and $(x_2, y_2)$ are the end points of a line, then the coordinates of the midpoint $M$ of the line are

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Gradient of a line**

If $(x_1, y_1)$ and $(x_2, y_2)$ are two points on a line, then the gradient $m$ of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**Lines**

Parallel lines have the same gradient.

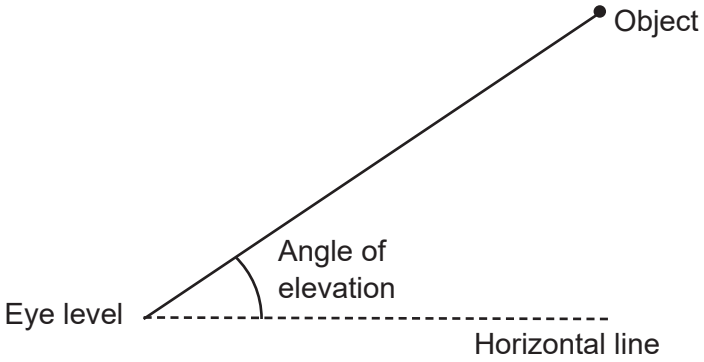
If a straight line has gradient $m$, then a line which is perpendicular to this line has a gradient $-\frac{1}{m}$

## Geometry and Angles

There are 180° on a straight line.
There are 180° inside a triangle.
An isosceles triangle is a triangle with 2 equal sides and 2 equal angles.
The sum of all the angles inside a polygon is given by $180(n - 2)$ where $n$ is the number of sides in the polygon.

## Angle of elevation

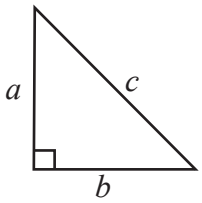
If a person stands and looks up at an object, the **angle of elevation** is the angle between the horizontal line of sight and the object.



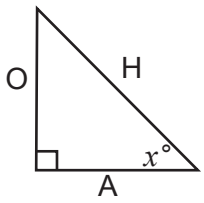

## Pythagoras' Theorem

If $a$, $b$ and $c$ are the sides of a right angled triangle shown below, then

$$a^2 + b^2 = c^2$$



**Trigonometric ratios in right angled triangles**




$\sin x° = \frac{O}{H}$ $\cos x° = \frac{A}{H}$ $\tan x° = \frac{O}{A}$

**Tangent/Radius property**

The tangent to a circle is perpendicular to the radius at the point of contact with the circle.

**Alternate Segment Theorem**

In a circle, the angle between a chord and a tangent through one of the end points of the chord is equal to the angle in the alternate segment.

**Mean**

The mean of a set of data is the sum of all the data values divided by the number of data values.

**Estimate for the mean of a grouped frequency distribution**

Estimated mean = sum of (mid interval values multiplied by their frequency) divided by the sum of all the frequencies.

**Pie Chart**

In a pie chart, the total angle that corresponds to the entire data set is 360°

**Probability**

The sum of the probabilities of all outcomes equals 1

**Frequency density in histograms**

$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$