Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2023

Mathematics

StemUp Model Solution

Unit M8 Paper 1
(Non-Calculator)
Higher Tier



[GMC81]

GMC81

WEDNESDAY 7 JUNE, 9.15 am–10.30 am

TIME

1 hour 15 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page. **You are provided with Higher Tier Additional Support Materials for use with this paper.**
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all twelve** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **must not** use a calculator for this paper.

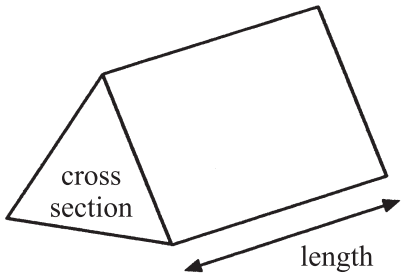
INFORMATION FOR CANDIDATES

The total mark for this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a ruler, compasses and a protractor.
The Formula Sheet is on page 2.

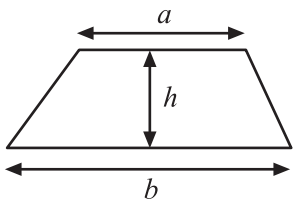
13348.07 **R**

Formula Sheet

Volume of prism = area of cross section × length

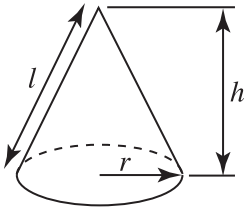


Area of trapezium = $\frac{1}{2}(a+b)h$



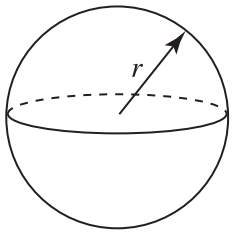
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl

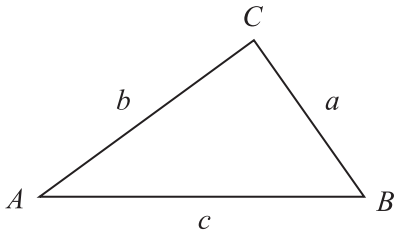


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

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1 Tony owns a pizza takeaway.

Every time he sells 100 pizzas, he records the number that were pepperoni.

The table shows some of his data.

Number of pizzas sold	Number of pepperoni pizzas	Relative frequency of pepperoni pizzas
100	17	0.17
100	23	0.2
100	23	0.21
100	25	0.22
100	27	0.23

(a) Complete the table.

Row 4: $17 + 23 + 23 + 25 = 88$, $88 \div 400 = 0.22$

Row 5: $500 \times 0.23 = 115$

$115 - 88 = 27$

[4]

(b) Tony uses his data to predict next year's pizza sales.

He thinks he will sell 2300 pepperoni pizzas next year.

How many pizzas is he expecting to sell in total?

$2300 \div 0.23 = 230\,000 \div 23 = 10\,000$

Answer 10 000 [1]

[Turn over

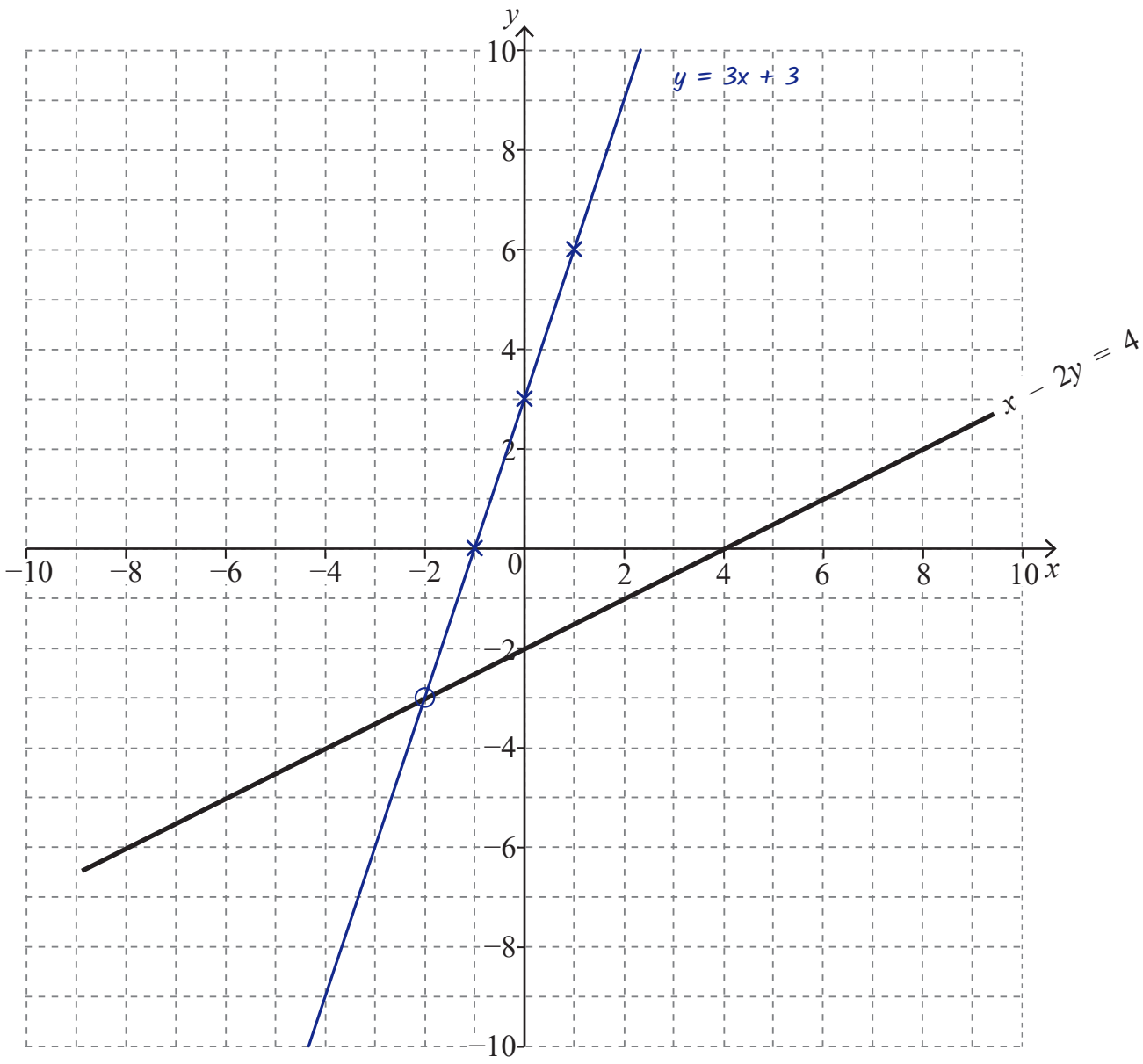
2 By drawing a suitable line on the grid, solve the simultaneous equations

$$x - 2y = 4$$

$$y = 3x + 3$$

y = 3x + 3: x = −1, y = 0; x = 0, y = 3; x = 1, y = 6

Lines cross at (−2, −3)



Answer $x =$ *−2* $y =$ *−3* [4]

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3 **(a)** The first four terms of a sequence are

$$1, \quad 4, \quad 7, \quad 10$$

What is the n^{th} term for this sequence?

difference 3 → 3n, 1 − 3 = −2

Answer *3n − 2* [2]

(b) Hence find the n^{th} term for the sequence below.

$$\frac{1}{1}, \quad \frac{4}{4}, \quad \frac{9}{7}, \quad \frac{16}{10}$$

numerators: n^2
denominators: 3n − 2

Answer $\frac{n^2}{3n-2}$ [2]

4 In standard form

$$(2 \times 10^x) \times (3 \times 10^y) = 6 \times 10^{11}$$

$$(3 \times 10^y) \div (2 \times 10^x) = 1.5 \times 10^3$$

Find the values of x and y.

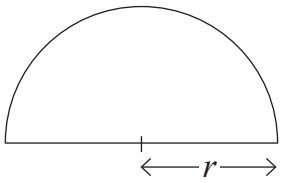
Show your working.

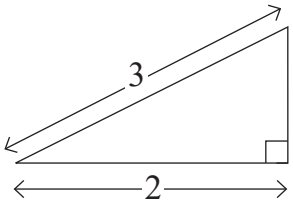
$6 \times 10^{x+y} = 6 \times 10^{11}$ so $x + y = 11$
$1.5 \times 10^{y-x} = 1.5 \times 10^3$ so $y - x = 3$
add: $2y = 14$, $y = 7$
$x = 11 - 7 = 4$

Answer $x =$ *4* $y =$ *7* [4]

[Turn over

5





The semicircle and the triangle have the same perimeter.

Find the exact value of r, giving your answer in terms of π.

Semicircle perimeter = $\frac{1}{2} \times 2\pi r + 2r = \pi r + 2r$

Triangle height = $\sqrt{(3^2 - 2^2)} = \sqrt{(9 - 4)} = \sqrt{5}$

Triangle perimeter = $2 + 3 + \sqrt{5} = 5 + \sqrt{5}$

$\pi r + 2r = 5 + \sqrt{5}$

$r(\pi + 2) = 5 + \sqrt{5}$

$r = \dfrac{5 + \sqrt{5}}{\pi + 2}$

Answer $r =$ $\dfrac{5 + \sqrt{5}}{\pi + 2}$ [4]

6 **(a)** A dice is rolled once and a coin is flipped once.

Here are 4 events.

Event A – the roll of the dice is even

Event B – the roll of the dice is odd

Event C – the flipped coin lands heads

Event D – the flipped coin lands tails

(i) Write down two of these events that are independent.

Answer Event ___A___ and Event ___C___ [1]

(ii) Write down two of these events that are mutually exclusive.

Answer Event ___A___ and Event ___B___ [1]

(b) Two events, P and Q, are mutually exclusive.

What is the probability that either event P or event Q occurs?

Circle your answer.

(Probability (P) + Probability (Q))

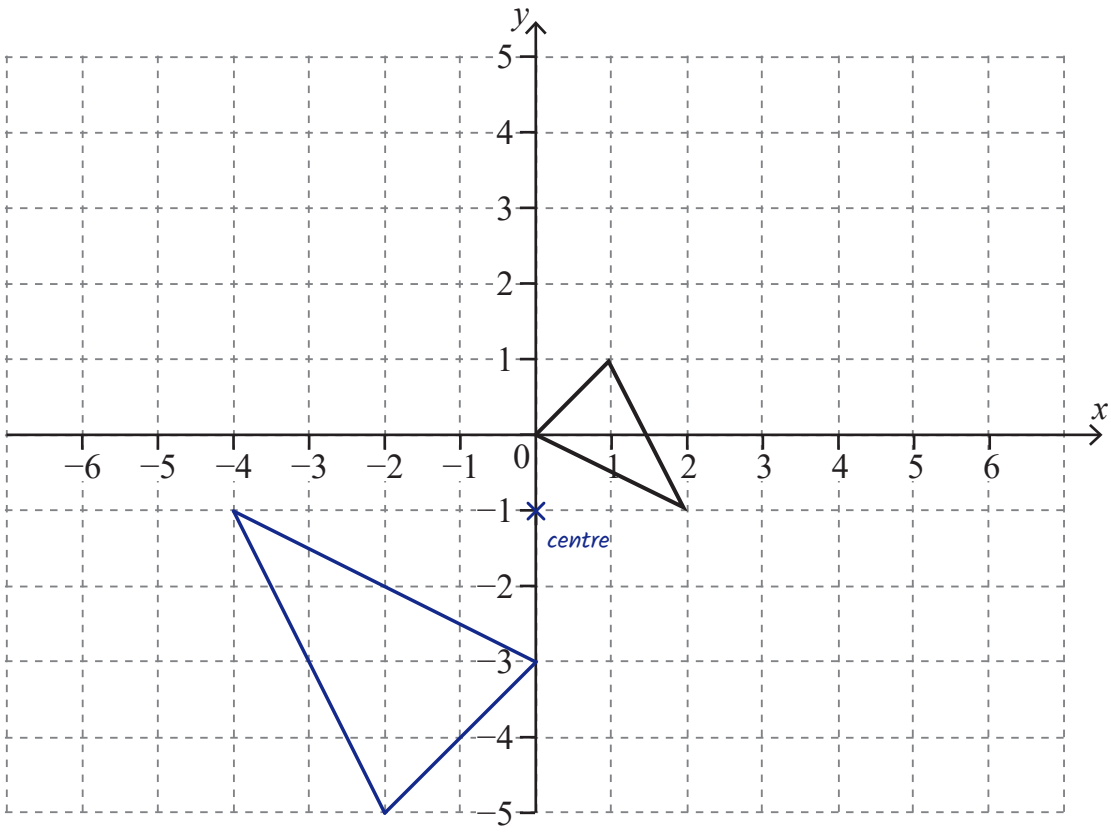
Probability (P) × Probability (Q)

more information is needed

[1]

[Turn over

7



Enlarge the shape by scale factor −2, using centre (0, −1) [3]

Centre (0, −1), scale factor −2:

(0, 0) → (0, −3), (1, 1) → (−2, −5), (2, −1) → (−4, −1)

8 Lucy and Martin play in different chess leagues.

The probability that Lucy wins her next chess game is $\frac{2}{3}$

The probability that Martin wins his next chess game is $\frac{2}{5}$

What is the probability that only one of them wins their next chess game?

$P(L \text{ wins, } M \text{ loses}) = \frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$

$P(L \text{ loses, } M \text{ wins}) = \frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

$P(\text{only one wins}) = \frac{6}{15} + \frac{2}{15} = \frac{8}{15}$

Answer $\frac{8}{15}$ [3]

[Turn over

9 A cuboid has volume $150\,cm^3$

The height of a similar cuboid is twice the height of the original cuboid.

Calculate the volume of the larger cuboid.

Length scale factor = 2, volume scale factor = $2^3 = 8$

$150 \times 8 = 1200$

Answer 1200 cm^3 [2]

10 $(2 + a\sqrt{3})^2 = b + 20\sqrt{3}$

Work out the values of the integers a and b.

Show all your working clearly.

$(2 + a\sqrt{3})^2 = (2 + a\sqrt{3})(2 + a\sqrt{3})$

$= 4 + 2a\sqrt{3} + 2a\sqrt{3} + 3a^2$

$= 4 + 3a^2 + 4a\sqrt{3}$

$4a\sqrt{3} = 20\sqrt{3}$ so $4a = 20$, $a = 5$

$b = 4 + 3 \times 5^2 = 4 + 75 = 79$

Answer $a =$ 5 , $b =$ 79 [4]

11 A box contains 4 red pens, 2 blue pens and 1 green pen.

Two pens are taken at random from the box.

(a) Show that the probability they are the same colour is $\frac{1}{3}$

$P(RR) = \frac{4}{7} \times \frac{3}{6} = \frac{12}{42}$ $P(BB) = \frac{2}{7} \times \frac{1}{6} = \frac{2}{42}$

$P(GG) = 0$ (only one green pen)

$P(\text{same colour}) = \frac{12}{42} + \frac{2}{42} = \frac{14}{42} = \frac{1}{3}$

[4]

(b) What is the probability that neither of the two pens is green?

$P(\text{neither green}) = P(\text{not G}) \times P(\text{not G})$

$= \frac{6}{7} \times \frac{5}{6} = \frac{30}{42} = \frac{5}{7}$

Answer $\frac{5}{7}$ [2]

[Turn over

12 A circle has equation $x^2 + y^2 = 25$

PQ is a chord of the circle.

PQ has equation $y = -2x - 5$

(a) Find the coordinates of P and Q.

$x^2 + (-2x - 5)^2 = 25$

$x^2 + 4x^2 + 20x + 25 = 25$

$5x^2 + 20x = 0$

$5x(x + 4) = 0$

$x = 0$ or $x = -4$

$x = 0$: $y = -2(0) - 5 = -5$

$x = -4$: $y = -2(-4) - 5 = 3$

Answer P(0, −5) and Q(−4, 3) [5]

(b) Find the equation of the diameter of the circle which is perpendicular to PQ.

Gradient of PQ = −2, perpendicular gradient = $\frac{1}{2}$

Diameter passes through centre (0, 0), so c = 0

Answer $y = \frac{1}{2}x$ [3]

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Question Number	**Marks**
1	
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12	
Total Marks	

Examiner Number

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General Certificate of Secondary Education
Summer 2023

GCSE Mathematics

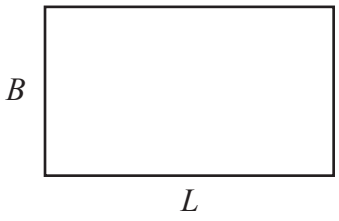
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$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

Perimeter, Area and Volume

The perimeter of a polygon is the distance around the outside of the polygon.



The area of a rectangle is found by multiplying the length of the rectangle by the breadth.

$A = L \times B$ where L is length and B is breadth.

The volume of a cuboid is found by multiplying the length by the breadth by the height of the cuboid.

$V = L \times B \times H$ where V is volume, L is length, B is breadth and H is height.

The area of a circle is $A = \pi r^2$ where r is the radius of the circle.

The circumference (perimeter) of a circle is $C = 2\pi r$ where r is the radius of the circle. An alternative formula is $C = \pi d$ where d is the diameter of the circle.

Mid point of a line

If (x_1, y_1) and (x_2, y_2) are the end points of a line, then the coordinates of the midpoint M of the line are

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Gradient of a line

If (x_1, y_1) and (x_2, y_2) are two points on a line, then the gradient m of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Lines

Parallel lines have the same gradient.

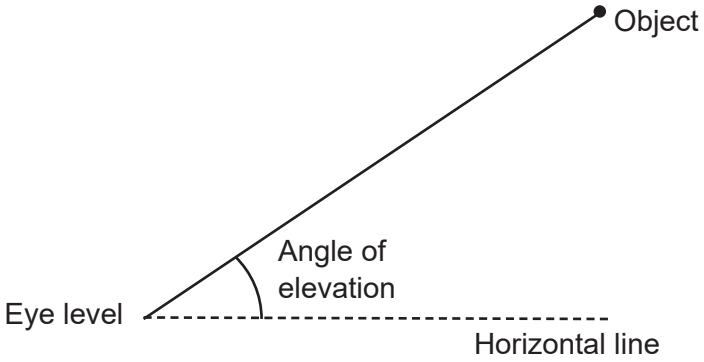
If a straight line has gradient m, then a line which is perpendicular to this line has a gradient $-\frac{1}{m}$

Geometry and Angles

There are 180° on a straight line.
There are 180° inside a triangle.
An isosceles triangle is a triangle with 2 equal sides and 2 equal angles.
The sum of all the angles inside a polygon is given by $180(n - 2)$ where n is the number of sides in the polygon.

Angle of elevation

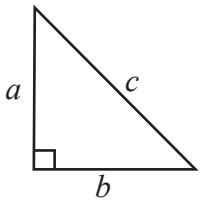
If a person stands and looks up at an object, the **angle of elevation** is the angle between the horizontal line of sight and the object.



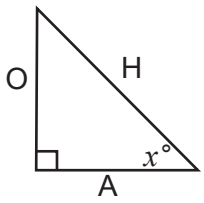
Pythagoras' Theorem

If a, b and c are the sides of a right angled triangle shown below, then

$$a^2 + b^2 = c^2$$



Trigonometric ratios in right angled triangles



$\sin x° = \frac{O}{H}$ $\cos x° = \frac{A}{H}$ $\tan x° = \frac{O}{A}$

Tangent/Radius property

The tangent to a circle is perpendicular to the radius at the point of contact with the circle.

Alternate Segment Theorem

In a circle, the angle between a chord and a tangent through one of the end points of the chord is equal to the angle in the alternate segment.

Mean

The mean of a set of data is the sum of all the data values divided by the number of data values.

Estimate for the mean of a grouped frequency distribution

Estimated mean = sum of (mid interval values multiplied by their frequency) divided by the sum of all the frequencies.

Pie Chart

In a pie chart, the total angle that corresponds to the entire data set is 360°

Probability

The sum of the probabilities of all outcomes equals 1

Frequency density in histograms

$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$