Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2023

Mathematics

StemUp Model Solution

Unit M3
(With calculator)
Higher Tier



[GMC31]

GMC31

FRIDAY 19 MAY, 9.15 am–11.15 am

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page. **You are provided with Higher Tier Additional Support Materials for use with this paper.**
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all twenty-five** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

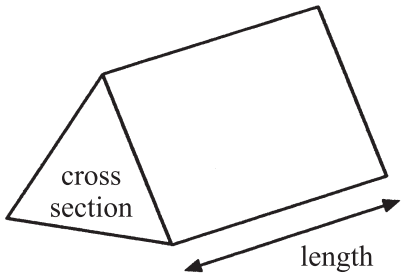
INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

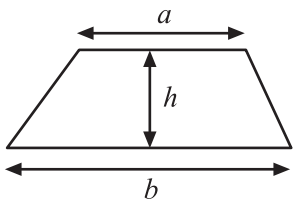
13340

Formula Sheet

Volume of prism = area of cross section × length

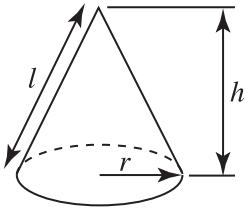


Area of trapezium = $\frac{1}{2}(a+b)h$



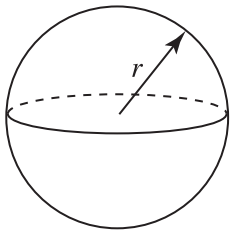
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl

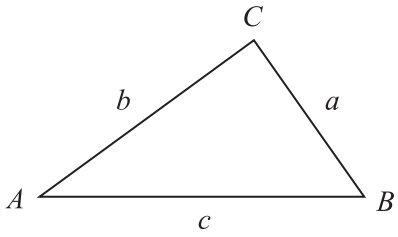


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

13340

1 Debbie works as a salesperson for an insurance company.

Her rate of pay is £11.50 per hour.

Every time she sells an insurance policy she receives a bonus of £7.25

Last week Debbie worked these hours.

Monday	8am – 4pm
Tuesday	8am – 4pm
Wednesday	8am – 4pm
Thursday	8am – 4pm
Friday	8am – 12 noon
Saturday	Off
Sunday	Off

Her total pay for the week was £537.25

How many insurance policies did she sell?

Hours = 8 + 8 + 8 + 8 + 4 = 36 hours

Basic pay = 36 × £11.50 = £414

Bonus = £537.25 − £414 = £123.25

Policies = £123.25 ÷ £7.25 = 17

Answer 17 [4]

[Turn over

2 A survey was issued online, by post, by text and face to face.

The number of responses received for each method are shown in the table.

Method	No of surveys issued	No of responses
Online	240	120
Post	100	30
Text	300	150
Face to face	160	120

Frank thinks that you are most likely to get a response from surveys issued by text.

Is he right?

You must show working to justify your answer.

Online: $\frac{120}{240} = \frac{1}{2} = 50\%$

Post: $\frac{30}{100} = \frac{3}{10} = 30\%$

Text: $\frac{150}{300} = \frac{1}{2} = 50\%$

Face to face: $\frac{120}{160} = \frac{3}{4} = 75\%$

Answer No because face to face has the highest response rate (75%), higher than text (50%). [4]

3

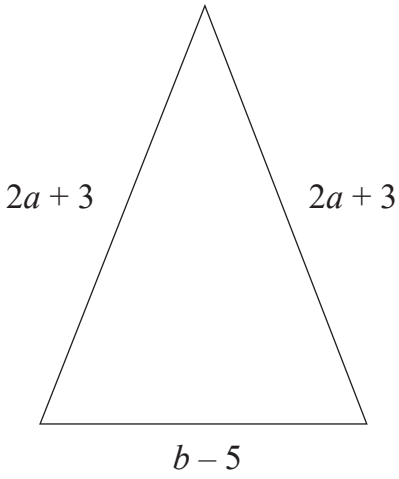


diagram not drawn accurately

(a) Write an expression for the perimeter of the triangle.

Simplify your answer.

Perimeter = 2a + 3 + 2a + 3 + b – 5

Answer 4a + b + 1 [2]

(b) The perimeter of the triangle is 30

You are told that $b = 9$

Work out the value of a

4a + 9 + 1 = 30
4a + 10 = 30
4a = 20
a = 20 ÷ 4 = 5

Answer $a =$ 5 [3]

[Turn over

4 A class of 30 pupils chose their favourite drink.

40% chose cola

$\frac{3}{10}$ chose milk

3 chose water

the rest chose other

(a) How many pupils chose other?

Cola = 40% × 30 = 12, Milk = 3/10 × 30 = 9

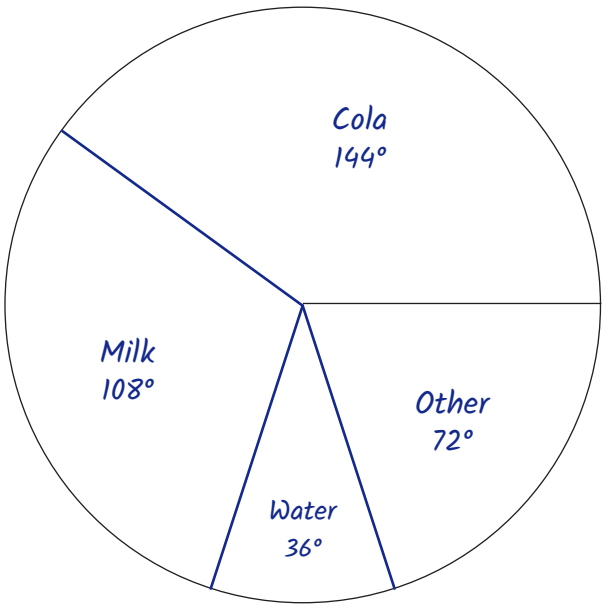
Other = 30 − 12 − 9 − 3 = 6

Answer 6 [2]

Drink	Number of pupils	Angle
Cola	12	12 × 12 = 144°
Milk	9	9 × 12 = 108°
Water	3	3 × 12 = 36°
Other	6	6 × 12 = 72°

(b) Use the information to draw a pie chart for the 30 pupils.

360° ÷ 30 = 12° per pupil



[4]

5 The table shows part of a train timetable from Edinburgh to St Andrews.

The Express trains travel directly. The Standard trains stop at other stations.

	Express	**Standard**	**Express**	**Standard**	**Express**
Edinburgh	1318	1343	1424	1441	1520
Haymarket		1406		1504	
Kirkcaldy		1418		1516	
Ladybank		1423		1521	
St Andrews	1403	1439	1509	1537	1605

(a) Alex arrives at Edinburgh Airport at 1306

It takes him 26 minutes to collect his luggage.

By taxi, he arrives at Edinburgh Train Station 18 minutes later.

How long will he have to wait at the station for the next train to St Andrews?

1306 + 26 min = 1332
1332 + 18 min = 1350
Next train 1424: 1350 to 1424 = 34 minutes

Answer 34 minutes [3]

(b) The distance between Edinburgh and St Andrews is 54 miles.

Calculate the average speed at which the Express train travels between Edinburgh and St Andrews.

Express: 1424 to 1509 = 45 min = 0.75 hour
Speed = 54 ÷ 0.75 = 72 miles/hr

Answer 72 miles/hr [3]

[Turn over

6 The diagram shows a net which is to be folded to make an open box.

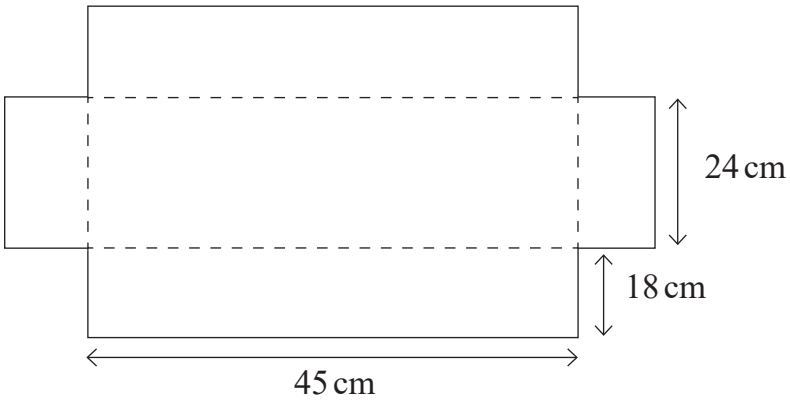


diagram not drawn accurately

(a) Calculate the volume of the box.

Volume = 45 × 24 × 18
= 19440

Answer 19440 cm^3 [2]

(b) Calculate the length of the longest straight line that can be drawn on the base of the box.

$d^2 = 45^2 + 24^2$
$d^2 = 2025 + 576 = 2601$
$d = \sqrt{2601} = 51$

Answer 51 cm [3]

7 Greg bought a mobile phone for £180

A year later he sold it for £54

What was Greg's percentage loss on the phone?

Loss = £180 − £54 = £126

$\%\ \text{loss} = \frac{126}{180} \times 100 = 70\%$

Answer 70 % [3]

8 Solve the equation $4(y + 2) = 22$

$4y + 8 = 22$

$4y = 14$

$y = 14 \div 4 = 3.5$

Answer $y =$ 3.5 [3]

[Turn over

9 A local sports club runs a tuck shop during matches.

Before last week's match the club bought

25 packs, each containing 6 drinks, costing £3 per pack,

15 packs, each containing 12 bags of crisps, costing £2.40 per pack,

35 packs, each containing 5 chocolate bars, costing £1 per pack.

(a) Calculate the total cost to the club.

Drinks: 25 × £3 = £75

Crisps: 15 × £2.40 = £36

Chocolate: 35 × £1 = £35

Total = £75 + £36 + £35 = £146

Answer £ 146 [2]

(b)

Tuck shop prices

Drinks	80p each
Crisps	50p per bag
Chocolate Bars	30p each

Special Deal ***1 drink, 1 bag of crisps, 1 chocolate bar for £1.50***

During the match, 115 people bought the Special Deal.

By the end of the match the tuck shop had sold **all** the drinks, crisps and chocolate bars.

What was the tuck shop's profit for that day?

Drinks: 25 × 6 = 150, Crisps: 15 × 12 = 180, Bars: 35 × 5 = 175

Special Deal: 115 × £1.50 = £172.50

Drinks left: 150 − 115 = 35, 35 × 80p = £28

Crisps left: 180 − 115 = 65, 65 × 50p = £32.50

Bars left: 175 − 115 = 60, 60 × 30p = £18

Income = £172.50 + £28 + £32.50 + £18 = £251

Profit = £251 − £146 = £105

Answer £ 105 [4]

[Turn over

10

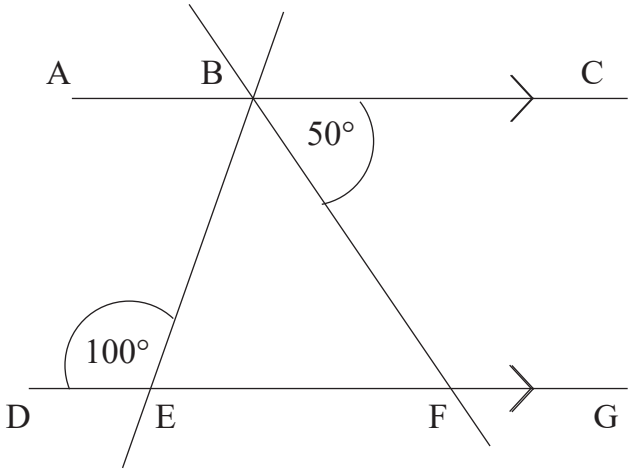


diagram not drawn accurately

AC and DG are parallel lines.

Angle CBF = 50° and angle BED = 100°

What type of triangle is BEF?

Give a reason for each angle found.

Angle BFE = 50° (alternate angles, AC parallel to DG)

Angle BEF = 180° – 100° = 80° (angles on a straight line add to 180°)

Angle EBF = 180° – 50° – 80° = 50° (angles in a triangle add to 180°)

Angles BFE and EBF are both 50°, so triangle BEF is isosceles

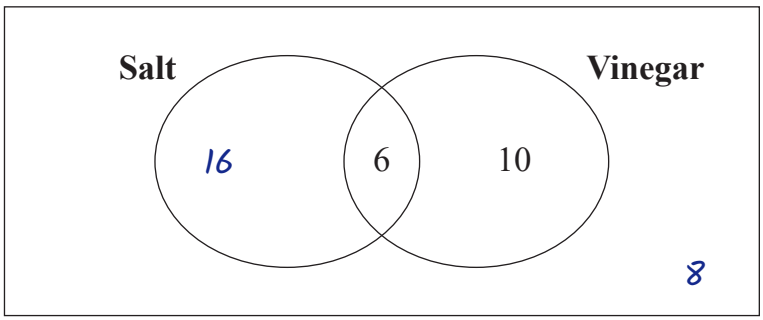
Answer *isosceles* [3]

11 40 customers order chips in a takeaway.

6 customers take **both** salt and vinegar on their chips.

10 customers take vinegar **only**.

This information is shown on the Venn diagram.



22 customers take salt on their chips.

Use the Venn diagram to work out how many customers take neither salt nor vinegar on their chips.

Salt only = 22 − 6 = 16

16 + 6 + 10 = 32

Neither = 40 − 32 = 8

Answer 8 [3]

[Turn over

12 The ages of 21 workers in an office are recorded as

28	32	38	40	42	49	51	23	26	27	53
49	45	36	38	37	62	23	46	58	47	

(a) Draw a stem and leaf diagram to display this set of data.

2	3 3 6 7 8
3	2 6 7 8 8
4	0 2 5 6 7 9 9
5	1 3 8
6	2

Key: 2 | 3 = 23 years

[3]

(b) A new worker joins the office. He is aged 34

What effect will this have on the median age?

Answer The median will be lowered (from 40 to 39). [1]

13 Colm wants to put £4500 into a savings account for 2 years.

He can choose one of the following options.

Option A	**Option B**
3.5% compound interest per year for 2 years	5% compound interest per year for the first year followed by 2% compound interest per year for the second year

Which option is better and by how much?

Show your working.

Option A:

Year 1: 4500 × 0.035 = 157.50, 4500 + 157.50 = 4657.50

Year 2: 4657.50 × 0.035 = 163.0125, 4657.50 + 163.0125 = 4820.5125

Amount = £4820.51 (interest £320.51)

Option B:

Year 1: 4500 × 0.05 = 225, 4500 + 225 = 4725

Year 2: 4725 × 0.02 = 94.50, 4725 + 94.50 = 4819.50

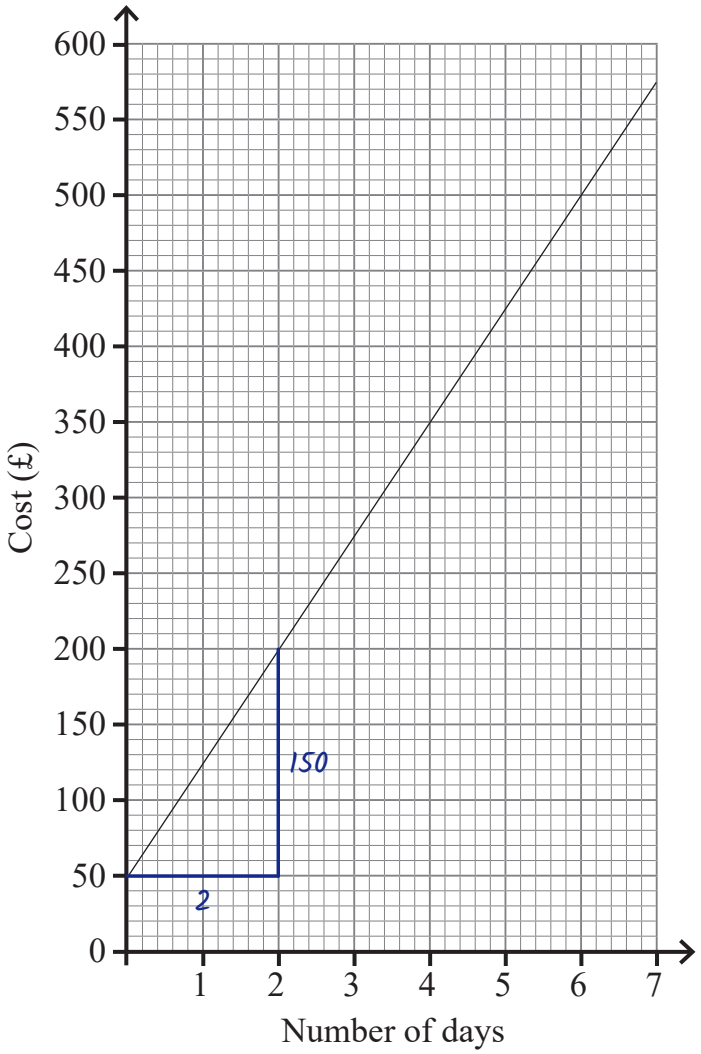
Amount = £4819.50 (interest £319.50)

Difference = £4820.51 − £4819.50 = £1.01

Answer Option A by £ 1.01 [5]

[Turn over

14 The graph shows the costs of hiring a mini digger for up to seven days, including the delivery charge.



(a) Use the graph to find

(i) the delivery charge,

Answer £ 50 [1]

(ii) the gradient of the line.

$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{200 - 50}{2 - 0} = \frac{150}{2} = 75$

Answer 75 [2]

(b) What does the gradient represent when hiring the mini digger?

Answer The cost per day to hire the mini digger, £75 a day [1]

15 Expand and simplify $2y(3y - 7) - 8y$

$= 6y^2 - 14y - 8y$

Answer $6y^2 - 22y$ [3]

[Turn over

16 A circle of diameter 12 cm just fits inside a semicircle as shown.

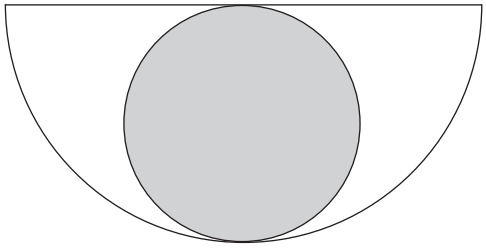


diagram not drawn accurately

Show that the shaded area and unshaded area are exactly the same.

You must show all your work clearly.

Radius of circle = 12 ÷ 2 = 6 cm

Shaded area = π × 6^2 = 36π (= 113.097... cm^2)

The circle just fits, so radius of semicircle = 12 cm

Semicircle area = ½ × π × 12^2 = 72π (= 226.194... cm^2)

Unshaded area = 72π − 36π = 36π (= 113.097... cm^2)

Shaded = 36π = Unshaded, so the areas are exactly the same

[4]

17 The times which members at a gym spend on a treadmill are recorded in the table.

Time (t mins)	Frequency	Mid-point	Mid × f
$0 < t \leq 15$	8	7.5	7.5 × 8 = 60
$15 < t \leq 30$	3	22.5	22.5 × 3 = 67.5
$30 < t \leq 45$	5	37.5	37.5 × 5 = 187.5
$45 < t \leq 60$	4	52.5	52.5 × 4 = 210

(a) Estimate the mean time spent on the treadmill.

$\Sigma fx = 60 + 67.5 + 187.5 + 210 = 525$

$\Sigma f = 8 + 3 + 5 + 4 = 20$

$\text{Mean} = \frac{525}{20} = 26.25$

Answer 26.25 mins [4]

(b) Explain why your answer in **(a)** is only an estimate of the mean time.

The data is grouped, so we do not know the exact times;

the mid-points are used instead. [1]

[Turn over

18 Barry sold his car for £10 225

The car had depreciated by 18.2 % since Barry bought it originally.

How much did Barry pay for the car originally?

100% − 18.2% = 81.8%
81.8% = £10 225
1% = 10 225 ÷ 81.8 = 125
100% = 125 × 100 = £12 500

Answer £ 12 500 [3]

19 Solve $\frac{1}{3}(2x-5) + \frac{3}{4}(3x+1) = \frac{5}{6}$

Multiply every term by 12:

$4(2x-5) + 9(3x+1) = 10$

$8x - 20 + 27x + 9 = 10$

$35x - 11 = 10$

$35x = 21$

$x = \frac{21}{35} = \frac{3}{5}$

Answer $x =$ $\frac{3}{5}$ [4]

20 Write 4725 as a product of prime factors.

Write your answer in index notation.

4725 ÷ 3 = 1575
1575 ÷ 3 = 525
525 ÷ 3 = 175
175 ÷ 5 = 35
35 ÷ 5 = 7
7 ÷ 7 = 1

4725 = 3 × 3 × 3 × 5 × 5 × 7

Answer $3^3 \times 5^2 \times 7$ [2]

21 **(a)** Factorise $x^2 + 2x - 35$

Answer $(x + 7)(x - 5)$ [2]

(b) Hence, or otherwise, solve the equation $x^2 + 2x = 35$

$x^2 + 2x - 35 = 0$
$(x + 7)(x - 5) = 0$

Answer $x =$ -7 or 5 [1]

[Turn over

22 Find the equation of the line passing through the points (0, −2) and (6, 16)

Gradient $m = \frac{16 - (-2)}{6 - 0} = \frac{18}{6} = 3$

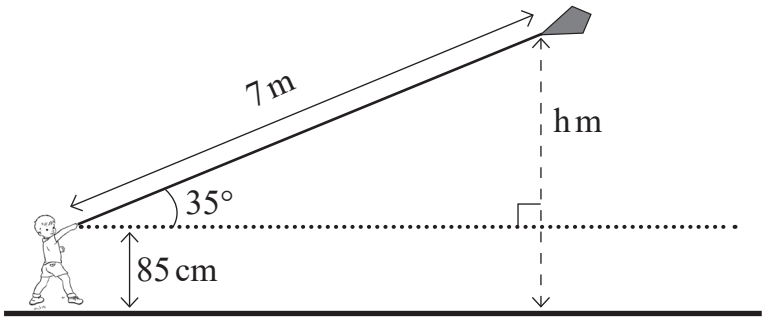
Passes through (0, −2), so y-intercept $c = -2$

$y = mx + c$

$y = 3x - 2$

Answer $y = 3x - 2$ [3]

23 A boy is flying a kite as shown in the diagram.



Calculate h, the height of the kite above the ground.

$\sin 35° = \frac{x}{7}$

$x = 7 \times \sin 35°$

$x = 4.015035054...$

$85 \text{ cm} = 0.85 \text{ m}$

$h = 4.015035054... + 0.85$

$h = 4.865035054... = 4.865 \text{ m}$

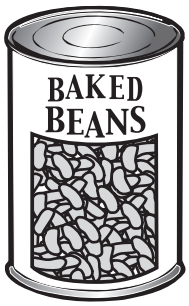
Answer 4.865 m [4]

[Turn over

24 A cylindrical can of beans has a radius of 3.4 cm and a height of 12 cm.

The entire curved surface area is covered by a label.

The label has a 1 cm overlap to allow for sticking.



Calculate the area of the label.

Circumference = $2 \times \pi \times 3.4 = 21.36283004...$

Length of label = $21.36283004... + 1 = 22.36283004...$

Area of label = $22.36283004... \times 12$

$= 268.3539605...$

$= 268.35 \text{ cm}^2$

Answer 268.35 cm^2 [3]

25 The following information is available relating to a data set on age.

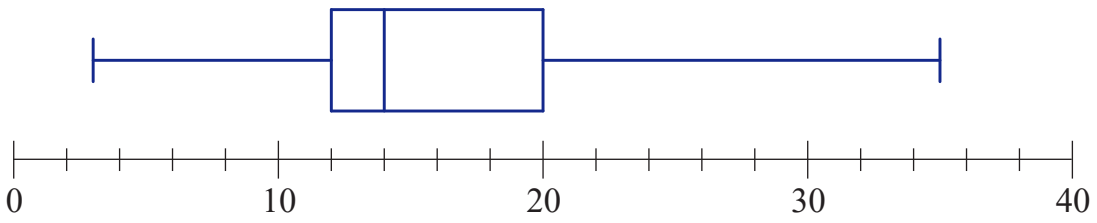
The median age is 14

The maximum age is 35
The range of ages is 32

The lower quartile is 12
The interquartile range (IQR) is 8

(a) Use all the above information to draw a box plot for the data set.

UQ = 12 + 8 = 20, Min = 35 − 32 = 3



[4]

(b) Jane states that the majority of people in this data set are aged below 16
Is she correct?

Give a reason for your answer.

Answer Yes because 50% of people are below the median of 14, so more than 50% are below 16 (the majority). [1]

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General Certificate of Secondary Education
Summer 2023

GCSE Mathematics

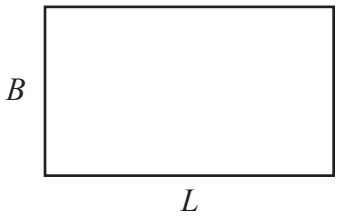
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$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

Perimeter, Area and Volume

The perimeter of a polygon is the distance around the outside of the polygon.



The area of a rectangle is found by multiplying the length of the rectangle by the breadth.

$A = L \times B$ where L is length and B is breadth.

The volume of a cuboid is found by multiplying the length by the breadth by the height of the cuboid.

$V = L \times B \times H$ where V is volume, L is length, B is breadth and H is height.

The area of a circle is $A = \pi r^2$ where r is the radius of the circle.

The circumference (perimeter) of a circle is $C = 2\pi r$ where r is the radius of the circle. An alternative formula is $C = \pi d$ where d is the diameter of the circle.

Mid point of a line

If (x_1, y_1) and (x_2, y_2) are the end points of a line, then the coordinates of the midpoint M of the line are

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Gradient of a line

If (x_1, y_1) and (x_2, y_2) are two points on a line, then the gradient m of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Lines

Parallel lines have the same gradient.

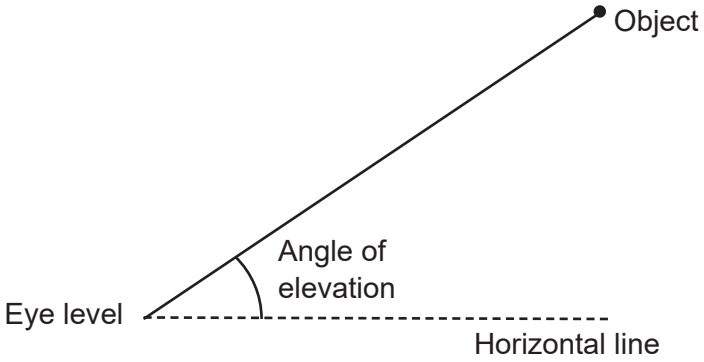
If a straight line has gradient m, then a line which is perpendicular to this line has a gradient $-\frac{1}{m}$

Geometry and Angles

There are 180° on a straight line.
There are 180° inside a triangle.
An isosceles triangle is a triangle with 2 equal sides and 2 equal angles.
The sum of all the angles inside a polygon is given by $180(n - 2)$ where n is the number of sides in the polygon.

Angle of elevation

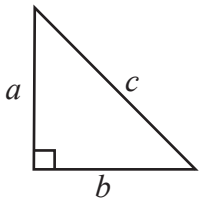
If a person stands and looks up at an object, the **angle of elevation** is the angle between the horizontal line of sight and the object.



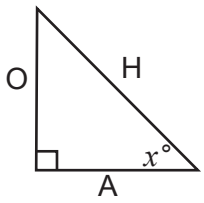
Pythagoras' Theorem

If a, b and c are the sides of a right angled triangle shown below, then

$$a^2 + b^2 = c^2$$



Trigonometric ratios in right angled triangles



$\sin x° = \frac{O}{H}$ $\cos x° = \frac{A}{H}$ $\tan x° = \frac{O}{A}$

Tangent/Radius property

The tangent to a circle is perpendicular to the radius at the point of contact with the circle.

Alternate Segment Theorem

In a circle, the angle between a chord and a tangent through one of the end points of the chord is equal to the angle in the alternate segment.

Mean

The mean of a set of data is the sum of all the data values divided by the number of data values.

Estimate for the mean of a grouped frequency distribution

Estimated mean = sum of (mid interval values multiplied by their frequency) divided by the sum of all the frequencies.

Pie Chart

In a pie chart, the total angle that corresponds to the entire data set is 360°

Probability

The sum of the probabilities of all outcomes equals 1

Frequency density in histograms

$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$