Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2022

Mathematics

StemUp Model Solution

Unit M8 Paper 2
(With calculator)
Higher Tier



[GMC82]

GMC82

MONDAY 13 JUNE, 10.45am–12 NOON

TIME

1 hour 15 minutes.

INSTRUCTIONS TO CANDIDATES

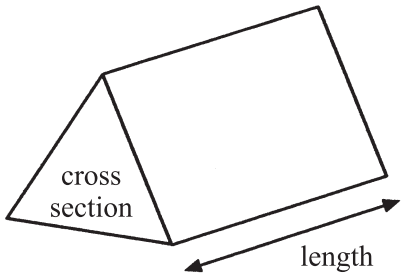
Write your Centre Number and Candidate Number in the spaces provided at the top of this page. **You are provided with Higher Tier Additional Support Materials for use with this paper.**
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all eleven** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

INFORMATION FOR CANDIDATES

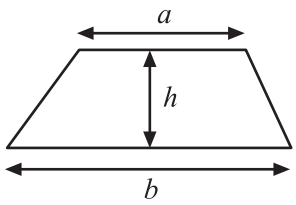
The total mark for this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

Formula Sheet

Volume of prism = area of cross section × length

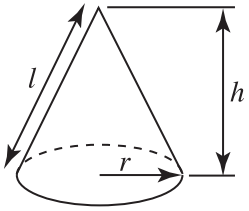


Area of trapezium = $\frac{1}{2}(a+b)h$



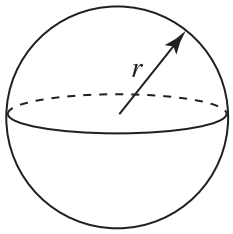
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

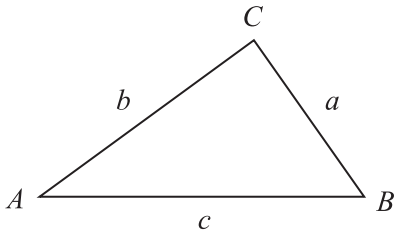


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

1 The equation $x^3 - 3x = 34$ has a solution between 1 and 5

Use a trial and improvement method to find this solution.

Give your answer correct to one decimal place.

You must show ALL your working.

x	$x^3 - 3x$	result
3	18	too small
4	52	too big
3.5	32.375	too small
3.6	35.856	too big
3.55	34.088875	too big

3.55 is too big, so x lies between 3.5 and 3.55
x = 3.5 (to 1 d.p.)

Answer x = 3.5 [4]

[Turn over

2 **(a)** When a spinner was spun 50 times, it landed on red 12 times.
When spun another 50 times, it landed on red another 9 times.

Calculate the relative frequency for red

(i) after the first 50 spins,

Answer $\frac{12}{50} = 0.24$ [1]

(ii) after the 100 spins. $12 + 9 = 21$ reds in 100 spins

Answer $\frac{21}{100} = 0.21$ [1]

(b) When spun another 50 times the spinner landed on red another 9 times.

What value of relative frequency gives the best estimate of the probability of this spinner landing on red?

Explain your answer.

$12 + 9 + 9 = 30$ reds in 150 spins

Answer $\frac{30}{150} = 0.2$ because it uses the most spins (150) and the bigger the sample size, the more reliable the estimate. [2]

(c) The spinner landed on yellow 40 times in the first 100 spins.

(i) Estimate the probability for this spinner landing on yellow.

Answer $\frac{40}{100} = 0.4$ [1]

(ii) Estimate how many times the spinner will land on yellow in 300 spins.

$\frac{40}{100} \times 300 = 120$

Answer 120 [1]

3 A **solid** cone has a vertical height of 12 cm, a slant height of 13 cm and a base radius of 5 cm.

Work out the **total** surface area of the cone, leaving your answer in terms of π

Total surface area = curved surface + base

$= \pi rl + \pi r^2$

$= \pi \times 5 \times 13 + \pi \times 5^2$

$= 65\pi + 25\pi$

$= 90\pi$

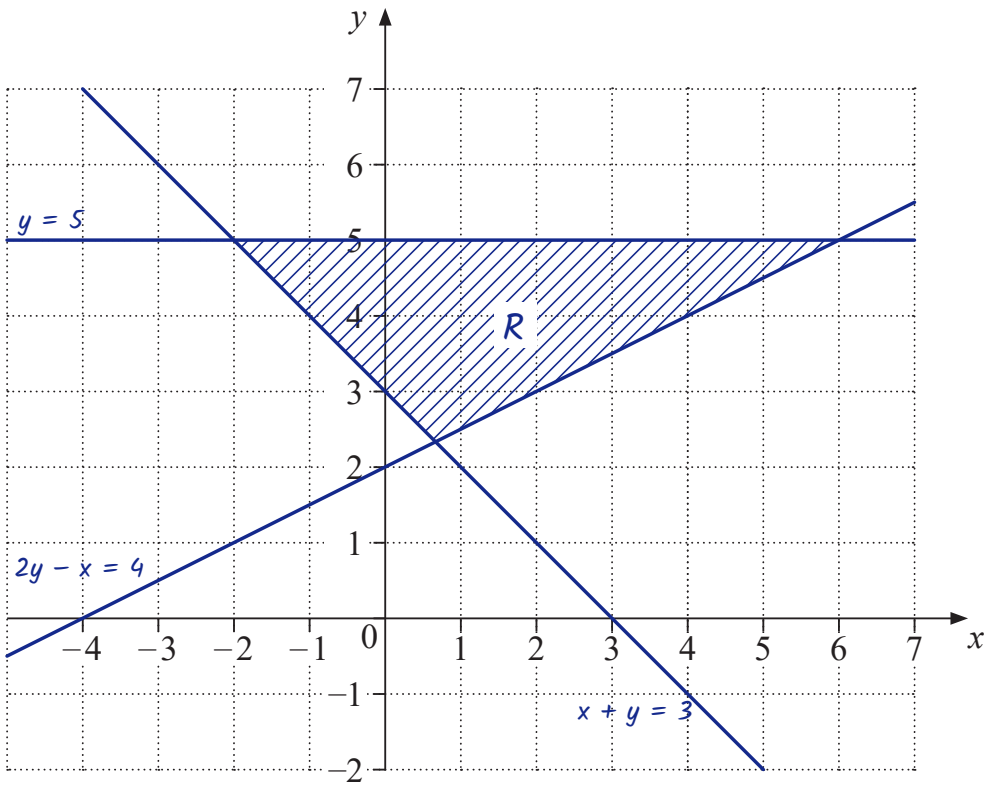
Answer 90π cm^2 [2]

[Turn over

4 The following linear inequalities represent a closed region.

$$x + y \geqslant 3$$
$$2y - x \geqslant 4$$
$$y \leqslant 5$$

(a) By drawing straight lines and shading, show this region on the grid below. Label the region R.



[3]

(b) Hence work out the maximum value of $3x + 2y$ in this region.

Vertices: (−2, 5), (6, 5), (0.67, 2.33)

At (6, 5): 3 × 6 + 2 × 5 = 18 + 10 = 28

At (−2, 5): −6 + 10 = 4; at (0.67, 2.33): 2 + 4.67 = 6.67

Answer 28 [2]

5 A book club offers 26 **different** books for sale.

Five of the books cost £3 each
Six of the books cost £4 each
Seven of the books cost £6 each
Eight of the books cost £7 each.

Abi wants to buy two books for exactly £10

How many combinations of books can she choose from?

£3 + £7: 5 × 8 = 40 combinations

£4 + £6: 6 × 7 = 42 combinations

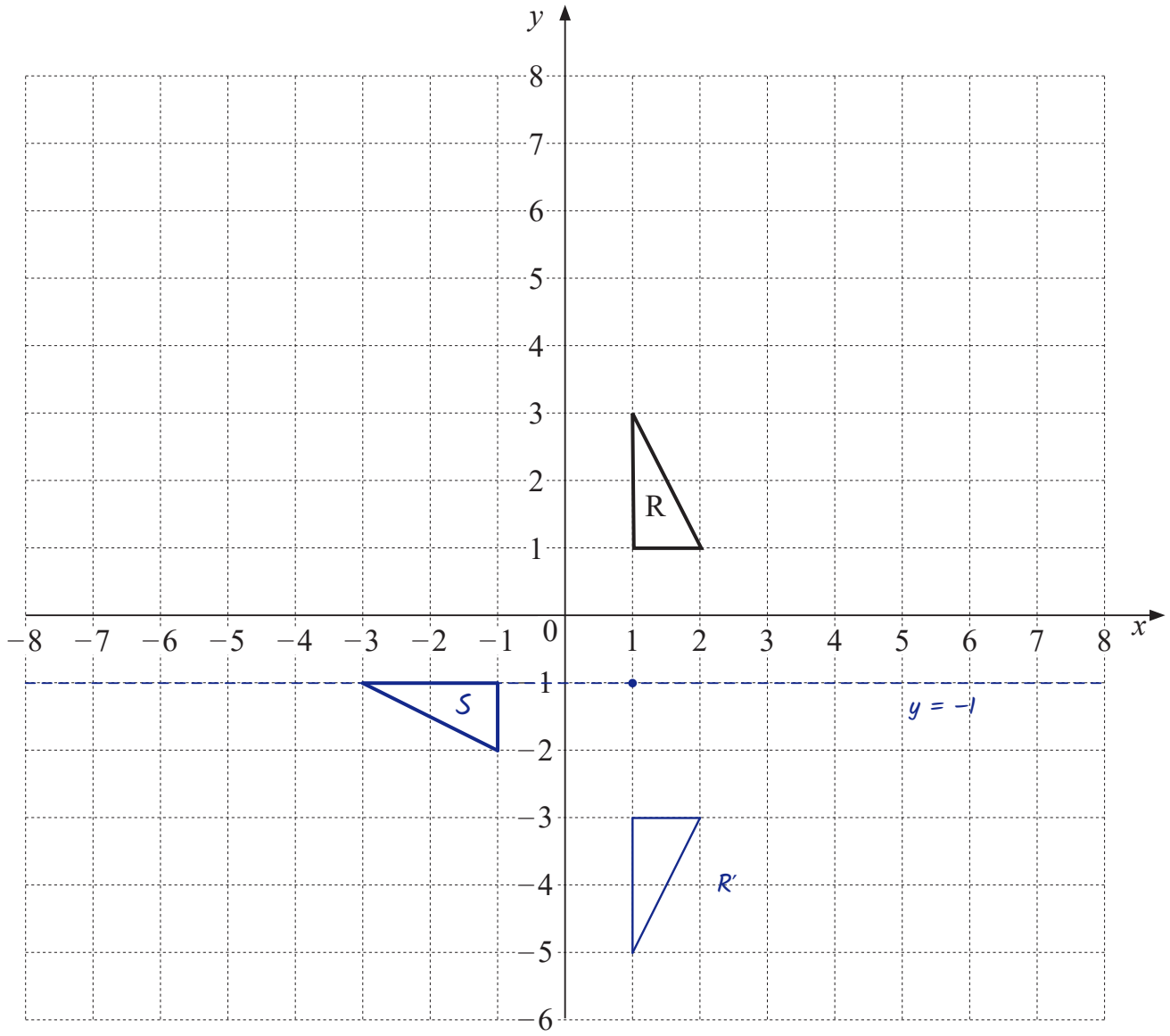
(£5 + £5 is not possible)

Total = 40 + 42 = 82

Answer 82 [2]

[Turn over

6



(a) Triangle R is reflected in the line $y = -1$ and its image rotated 90° clockwise about the point (1, −1)

Draw the final image after these two transformations and label it S. [4]

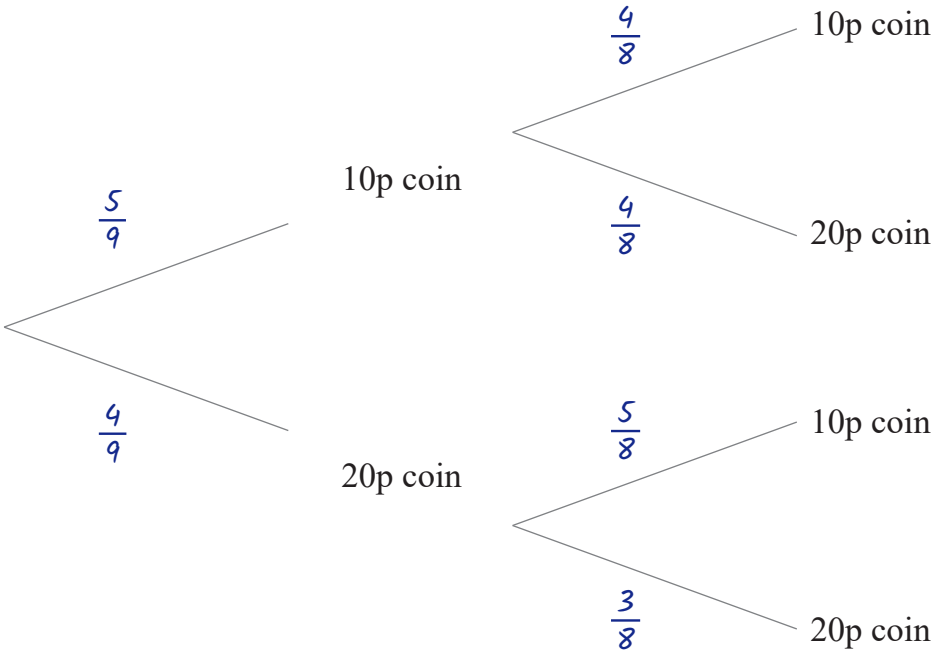
(b) Describe fully the single transformation which maps R to S.

Answer *Reflection in the line y = −x* [2]

7 A box contains five 10p coins and four 20p coins.

One coin is taken at random from the box and not replaced. A second coin is taken at random from the box.

(a) Complete the tree diagram below.



[3]

(b) What is the probability that the two coins taken have the same value as each other?

$P(10p, 10p) + P(20p, 20p) = \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{3}{8}$

$= \frac{20}{72} + \frac{12}{72} = \frac{32}{72}$

Answer $\frac{4}{9}$ [2]

(c) What is the probability that at least 30p is taken?

$P(\text{at least } 30p) = 1 - P(10p, 10p) = 1 - \frac{5}{9} \times \frac{4}{8}$

$= 1 - \frac{20}{72} = \frac{52}{72}$

Answer $\frac{13}{18}$ [2]

[Turn over

8 A container of radioactive waste, with an initial radioactivity of 100, is buried deep underground.

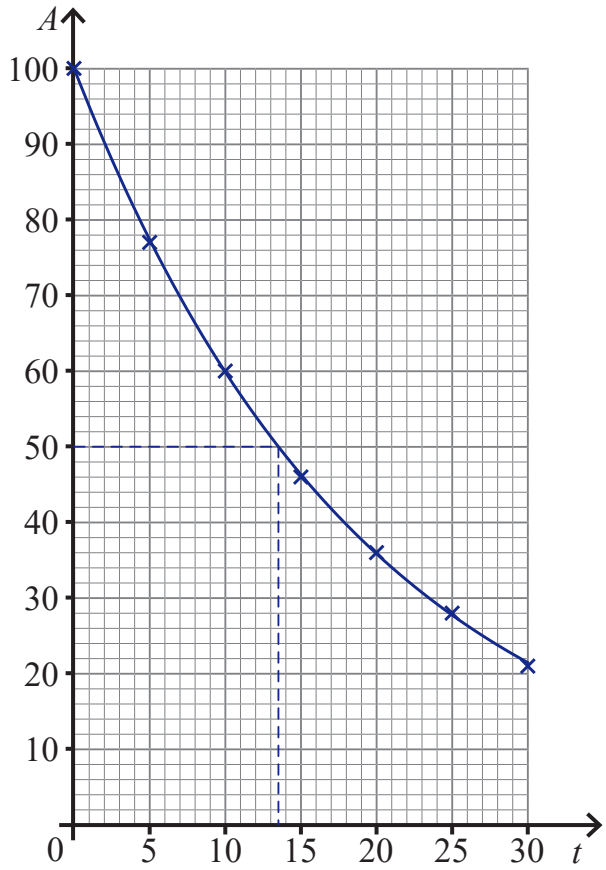
It decays by 5% each year.

After t years, its radioactivity A is given by the equation

$$A = 100 \times 0.95^t$$

(a) Complete the following table and use it to draw the graph of A against t on the grid below.

t	0	5	10	15	20	25	30
A	100	77	60	46	36	28	21



[4]

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(b) The half-life of a radioactive substance is defined as the time taken for the radioactivity of the material to decay to half its initial value.

Use your graph to estimate the expected half-life of this container of radioactive waste.

$A = 50$ when $t \approx 13.5$

Answer 13.5 years [1]

9 Simplify

(a) $(3a^5 b^{-1})^3$

$(3a^5b^{-1})^3 = 3^3 \times a^{5\times3} \times b^{-1\times3}$

$= 27a^{15}b^{-3}$

Answer $27a^{15}b^{-3}$ [2]

(b) $\dfrac{x^{\frac{1}{2}}}{x^{\frac{3}{2}}}$ $= x^{1/2 - 3/2} = x^{-1}$

Answer x^{-1} [1]

[Turn over

10 B is 30 km East and 20 km North of A.
C is 40 km from B and 60 km from A.

Calculate the bearing of C from B.

A solution by scale drawing will not be accepted.

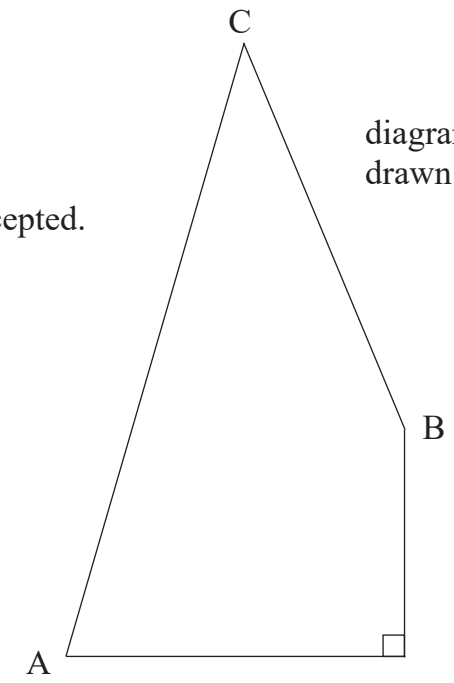


diagram not drawn accurately

$AB^2 = 20^2 + 30^2 = 1300$

$AB = \sqrt{1300} = 36.0555\ldots$ km

Cosine rule in triangle ABC:

$$\cos B = \frac{AB^2 + BC^2 - AC^2}{2 \times AB \times BC}$$

$$\cos B = \frac{1300 + 1600 - 3600}{2 \times 40 \times 36.0555\ldots}$$

$\cos B = -0.242681\ldots$

angle $ABC = 104.0448\ldots°$

Angle between south and BA: $\tan^{-1}\frac{30}{20} = 56.3099\ldots°$

Bearing of A from B $= 180° + 56.3099\ldots° = 236.3099\ldots°$

Bearing of C from B $= 236.3099\ldots° + 104.0448\ldots°$

$= 340.3547\ldots°$

$= 340.4°$ (to 1 d.p.)

Answer 340.4 ° [7]

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11 **A** y is directly proportional to x

B y is directly proportional to x^2

C y is directly proportional to $\sqrt{x}$

D y is indirectly proportional to x

E y is indirectly proportional to x^2

(a) Which of the cases **A**, **B**, **C**, **D** or **E** leads to each statement below?

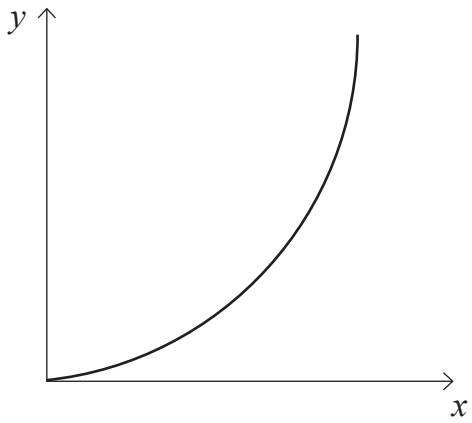
If x is halved, y is halved, Answer A

If x is halved, y is doubled, Answer D

If x is halved, y is four times as large. Answer E

[2]

(b) Which of **A**, **B**, **C**, **D** or **E** is represented by the sketch graph below?



Answer B [1]

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Question Number	**Marks**
1	
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Total Marks	

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General Certificate of Secondary Education
Summer 2022

GCSE Mathematics

HIGHER TIER
ADDITIONAL SUPPORT MATERIALS
(For use in Summer 2022)

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HIGHER TIER ADDITIONAL SUPPORT MATERIALS (Summer 2022)

Numbers

Lowest common multiple (LCM): The lowest common multiple is the lowest multiple shared by 2 or more numbers.

Trial and Improvement

This is a method of trying different values in an equation until you get a suitable solution (e.g to 1 decimal place).

Metric units

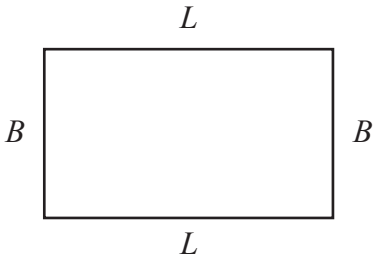
$1 \text{ ml} = 1 \text{ cm}^3$

Compound Measures

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

Perimeter, Area and Volume



The perimeter of a rectangle is the distance around the outside of the rectangle. It is found by adding the lengths of the 4 sides of the rectangle.

$P = 2L + 2B$ where P is perimeter, L is length and B is breadth.

The area of a rectangle is found by multiplying the length of the rectangle by the breadth.

$A = L \times B$ where L is length and B is breadth.

The volume of a cuboid is found by multiplying the length by the breadth by the height of the cuboid.

$V = L \times B \times H$ where V is volume, L is length, B is breadth and H is height.

The area of a circle is $A = \pi r^2$ where r is the radius of the circle.

The circumference (perimeter) of a circle is $C = 2\pi r$ where r is the radius of the circle. An alternative formula is $C = \pi d$ where d is the diameter of the circle.

Mid point of a line

If (x_1, y_1) and (x_2, y_2) are the end points of a line, then the coordinates of the midpoint M of the line are

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Gradient of a line

If (x_1, y_1) and (x_2, y_2) are two points on a line, then the gradient m of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Lines

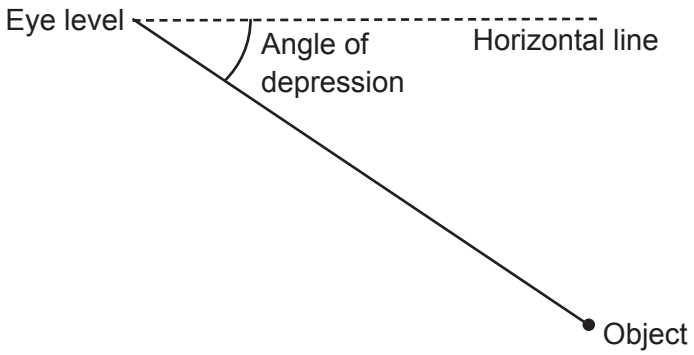
Parallel lines have the same gradient.

If a straight line has gradient m, then a line which is perpendicular to this line has a gradient $-\frac{1}{m}$

Geometry and Angles

There are 180° on a straight line.
There are 180° inside a triangle.
An isosceles triangle is a triangle with 2 equal sides and 2 equal angles.
The sum of all the angles inside a polygon is given by $180(n - 2)$ where n is the number of sides in the polygon.

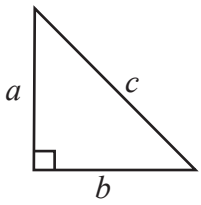
Angle of depression
If a person stands and looks down at an object, the **angle of depression** is the angle between the horizontal line of sight and the object.



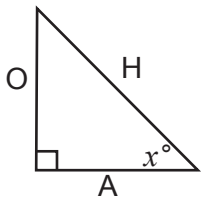
Pythagoras' Theorem

If *a, b* and *c* are the sides of a right angled triangle shown below, then

$$a^2 + b^2 = c^2$$



Trigonometric ratios in right angled triangles



$\sin x° = \frac{O}{H}$ $\cos x° = \frac{A}{H}$ $\tan x° = \frac{O}{A}$

Tangent/Radius property

The tangent to a circle is perpendicular to the radius at the point of contact with the circle.

Alternate Segment Theorem
In a circle, the angle between a chord and a tangent through one of the end points of the chord is equal to the angle in the alternate segment.

Range
The range of a set of data is the difference between the largest value and the smallest value in the data set.

Mean

The mean of a set of data is the sum of all the data values divided by the number of data values.

Estimate for the mean of a grouped frequency distribution

Estimated mean = sum of (mid interval values multiplied by their frequency) divided by the sum of all the frequencies.

Pie Chart

In a pie chart, the total angle that corresponds to the entire data set is 360°

Probability

The sum of the probabilities of all outcomes equals 1

Frequency density in histograms

$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$

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