Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2022

Mathematics

StemUp Model Solution

Unit M8 Paper 1
(Non-Calculator)
Higher Tier



[GMC81]

GMC81

MONDAY 13 JUNE, 9.15am–10.30am

TIME

1 hour 15 minutes.

INSTRUCTIONS TO CANDIDATES

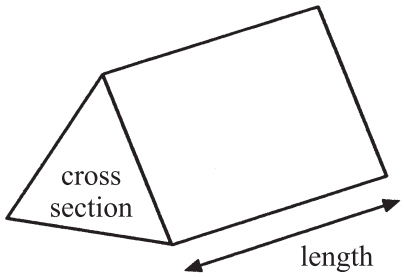
Write your Centre Number and Candidate Number in the spaces provided at the top of this page. **You are provided with Higher Tier Additional Support Materials for use with this paper.**
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all fifteen** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **must not** use a calculator for this paper.

INFORMATION FOR CANDIDATES

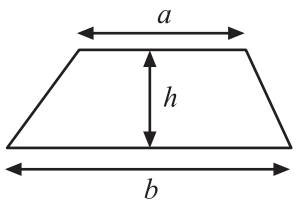
The total mark for this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a ruler, compasses and a protractor.
The Formula Sheet is on page 2.

Formula Sheet

Volume of prism = area of cross section × length

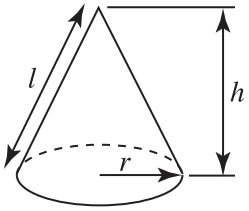


Area of trapezium = $\frac{1}{2}(a+b)h$



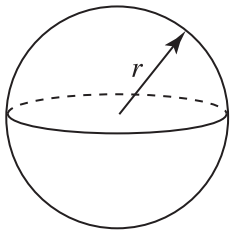
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl

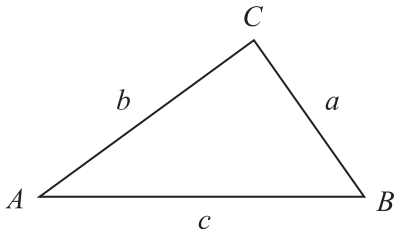


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

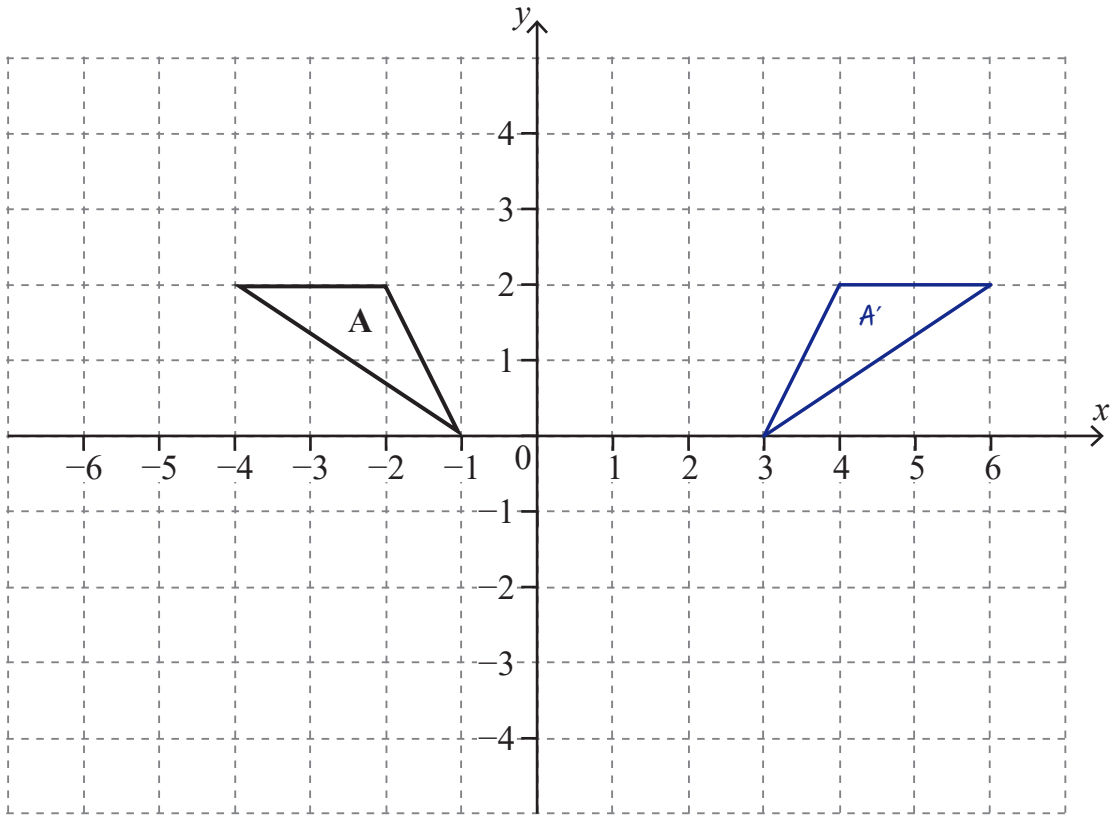
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

1



Reflect triangle **A** in the line $x = 1$ [2]

[Turn over

2 **(a)** What is the main difference between the Binary number system and the Decimal number system?

Answer Binary is base 2 (only digits 0 and 1), decimal is base 10 [1]

(b) Write the binary number 111111 as a decimal number.

32 16 8 4 2 1
1 1 1 1 1 1
32 + 16 + 8 + 4 + 2 + 1 = 63

Answer 63 [1]

(c) Write the decimal number 87 as a binary number.

87 = 64 + 16 + 4 + 2 + 1

64 32 16 8 4 2 1 → 1 0 1 0 1 1 1

Answer 1010111 [1]

3 **(a)** Simplify

(i) $w^3 \times w^2$

$w^{3+2} = w^5$

Answer w^5 [1]

(ii) $\frac{y^6}{y^2}$

$y^{6-2} = y^4$

Answer y^4 [1]

(b) Work out the n^{th} term of the sequence

7, 14, 21, 28, 35 …

goes up in 7s, so nth term = 7n

Answer $7n$ [1]

(c) Work out the value of

(i) 5^{-2}

$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Answer $\frac{1}{25}$ [1]

(ii) $1^5 + 6^0$

$1^5 + 6^0 = 1 + 1 = 2$

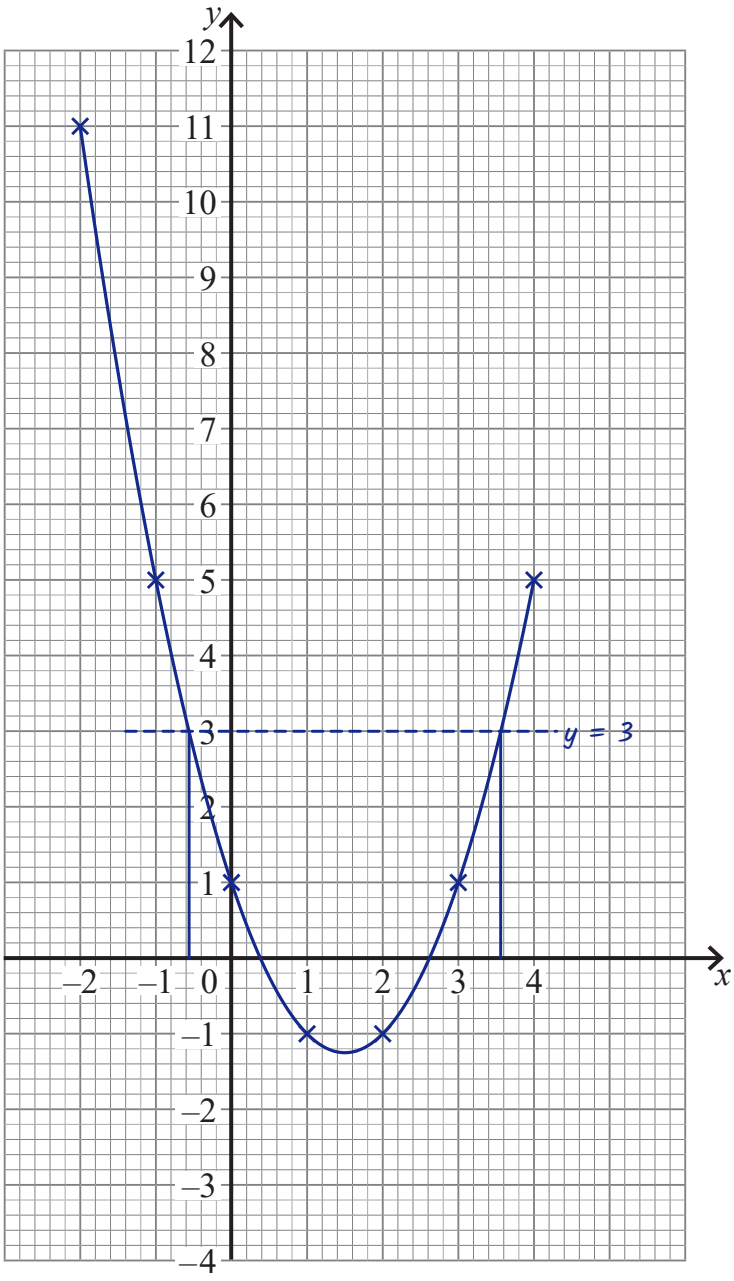
Answer 2 [1]

[Turn over

4 The following table gives some values for the quadratic equation $y = x^2 - 3x + 1$

x	–2	–1	0	1	2	3	4
y	11	5	1	–1	–1	1	5

(a) On the grid below, draw the graph of $y = x^2 - 3x + 1$ for values of x between –2 and 4



[2]

(b) Use your graph to estimate the values of x for which $y = 3$

Draw the line $y = 3$ and read off where it meets the curve:

$x \approx -0.6$ and $x \approx 3.6$

Answer $x =$ _____ -0.6 and 3.6 _____ [2]

[Turn over

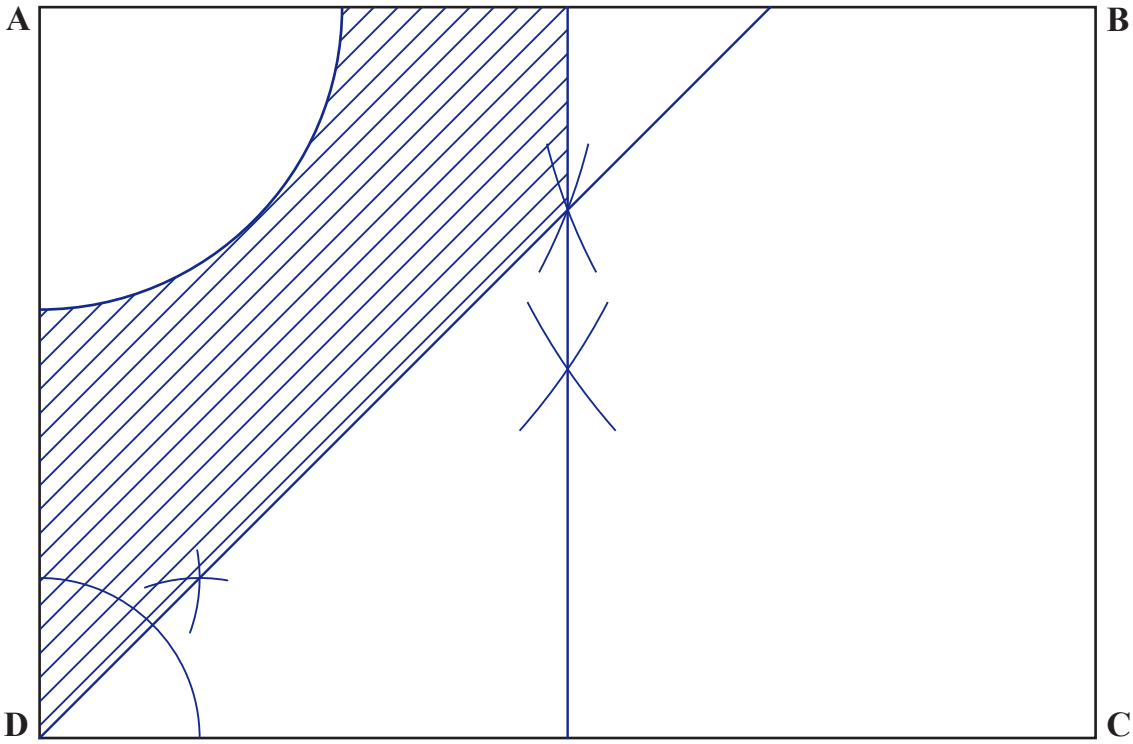
5 Toby walks his dog in the field **ABCD** so that he is always:

more than 40 m from **A**;

nearer to **A** than **B**;

nearer to **DA** than **DC**.

Shade the area where Toby walks his dog.



Scale of diagram: 1 cm = 10 m

[4]

6 A tent in the shape of a cone has a perpendicular height of 7 m and a volume of 220 m^3

Using $\pi = \frac{22}{7}$, work out the base radius of this tent.

Write your answer in surd form.

$V = \frac{1}{3}\pi r^2 h$

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 7 = 220$

$\frac{22}{3} r^2 = 220$

$r^2 = 220 \times \frac{3}{22} = 30$

$r = \sqrt{30}$

Answer $\sqrt{30}$ m [3]

7 $m = 4.5 \times 10^7$ $n = 5 \times 10^{-3}$

Work out the value of $\frac{m}{n}$, giving your answer in standard form.

$\frac{m}{n} = \frac{4.5 \times 10^7}{5 \times 10^{-3}}$

$= (4.5 \div 5) \times 10^{7-(-3)}$

$= 0.9 \times 10^{10}$

$= 9 \times 10^9$

Answer 9×10^9 [2]

8 Make q the subject of the formula

$$p = \frac{q}{r - q}$$

$p(r - q) = q$

$pr - pq = q$

$pr = q + pq$

$pr = q(1 + p)$

$q = \frac{pr}{1 + p}$

Answer $q = \frac{pr}{1 + p}$ [3]

9 x is a number with a value between 0 and 1

From the following list

$$x^{-3} \qquad \frac{1}{x} \qquad x \qquad x^{0.5}$$

(i) which would have the lowest value,

e.g. $x = 0.25$: $x^{-3} = 64$, $1/x = 4$, $x = 0.25$, $x^{0.5} = 0.5$

so $x < x^{0.5} < 1/x < x^{-3}$

Answer x [1]

(ii) which would have the biggest value?

Answer x^{-3} [1]

10 $\frac{3}{4\sqrt{5}}$ can be expressed in the form $a\sqrt{5}$

Find the value of a

$$\frac{3}{4\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{4 \times 5} = \frac{3\sqrt{5}}{20}$$

so $a = \frac{3}{20}$

Answer $a =$ $\frac{3}{20}$ [2]

[Turn over

11 A scale model of a car is made.

The length of the model is $\frac{1}{10}$ of the length of the car.

The volume of the petrol tank of the car is 50 000 cm^3

What is the volume of the petrol tank of the model?

Volume scale factor = $(\frac{1}{10})^3 = \frac{1}{1000}$

$\frac{1}{1000} \times 50\,000 = 50$

Answer 50 cm^3 [2]

12 Express the recurring decimal $0.2\dot{6}\dot{9}$ as a fraction in its simplest form.

$x = 0.2696969...$

$10x = 2.696969...$

$1000x = 269.696969...$

$1000x - 10x = 269.69... - 2.69...$

$990x = 267$

$x = \frac{267}{990} = \frac{89}{330}$

Answer $\frac{89}{330}$ [3]

13 A circle of radius r cm has circumference C cm and area A cm^2

Circle whether each value is rational or irrational in the table.

r	C	A
$\frac{3}{\pi}$	(rational) / irrational	rational / (irrational)
$\sqrt{\frac{3}{\pi}}$	rational / (irrational)	(rational) / irrational
$3\sqrt{\pi}$	rational / (irrational)	rational / (irrational)

[4]

$r = 3/\pi$: $C = 2\pi \times 3/\pi = 6$, $A = \pi \times 9/\pi^2 = 9/\pi$

$r = \sqrt{(3/\pi)}$: $C = 2\pi\sqrt{(3/\pi)} = 2\sqrt{(3\pi)}$, $A = \pi \times 3/\pi = 3$

$r = 3\sqrt{\pi}$: $C = 6\pi\sqrt{\pi}$, $A = \pi \times 9\pi = 9\pi^2$

[Turn over

14 A cuboid has sides of length

$$3 + 2\sqrt{3}, \qquad 3 + \sqrt{3}, \qquad 9 - \sqrt{3}$$

Find the length of the space diagonal, giving your answer in the form $a\sqrt{b}$

$d^2 = (3 + 2\sqrt{3})^2 + (3 + \sqrt{3})^2 + (9 - \sqrt{3})^2$

$(3 + 2\sqrt{3})^2 = 9 + 12\sqrt{3} + 12 = 21 + 12\sqrt{3}$

$(3 + \sqrt{3})^2 = 9 + 6\sqrt{3} + 3 = 12 + 6\sqrt{3}$

$(9 - \sqrt{3})^2 = 81 - 18\sqrt{3} + 3 = 84 - 18\sqrt{3}$

$d^2 = 21 + 12 + 84 + 12\sqrt{3} + 6\sqrt{3} - 18\sqrt{3}$

$d^2 = 117$

$d = \sqrt{117} = \sqrt{(9 \times 13)} = 3\sqrt{13}$

Answer $3\sqrt{13}$ [5]

15 A bag contains x green and 5 red beads. Two beads are taken without replacement from the bag at random.

(a) What is the probability of taking 2 green beads if $x = 3$?

$\frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$

Answer $\frac{3}{28}$ [2]

(b) What is the probability of taking 2 green beads if x has the larger value of 10?

$\frac{10}{15} \times \frac{9}{14} = \frac{90}{210} = \frac{3}{7}$

Answer $\frac{3}{7}$ [1]

(c) Hence find the smallest value of x for which the probability of taking 2 green beads is greater than $\frac{1}{2}$

$x = 11$: $\frac{11}{16} \times \frac{10}{15} = \frac{110}{240} = \frac{11}{24} < \frac{1}{2}$

$x = 12$: $\frac{12}{17} \times \frac{11}{16} = \frac{132}{272} = \frac{33}{68} < \frac{34}{68} = \frac{1}{2}$

$x = 13$: $\frac{13}{18} \times \frac{12}{17} = \frac{156}{306} = \frac{26}{51} > \frac{25.5}{51} = \frac{1}{2}$

Smallest value is $x = 13$

Answer 13 [3]

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THIS IS THE END OF THE QUESTION PAPER

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Question Number	**Marks**
1	
2	
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15	
Total Marks	

Examiner Number

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Summer 2022

GCSE Mathematics

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Numbers

Lowest common multiple (LCM): The lowest common multiple is the lowest multiple shared by 2 or more numbers.

Trial and Improvement

This is a method of trying different values in an equation until you get a suitable solution (e.g to 1 decimal place).

Metric units

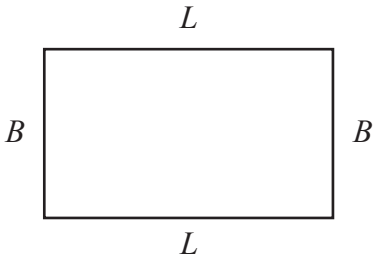
$1 \text{ ml} = 1 \text{ cm}^3$

Compound Measures

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

Perimeter, Area and Volume



The perimeter of a rectangle is the distance around the outside of the rectangle. It is found by adding the lengths of the 4 sides of the rectangle.

$P = 2L + 2B$ where P is perimeter, L is length and B is breadth.

The area of a rectangle is found by multiplying the length of the rectangle by the breadth.

$A = L \times B$ where L is length and B is breadth.

The volume of a cuboid is found by multiplying the length by the breadth by the height of the cuboid.

$V = L \times B \times H$ where V is volume, L is length, B is breadth and H is height.

The area of a circle is $A = \pi r^2$ where r is the radius of the circle.

The circumference (perimeter) of a circle is $C = 2\pi r$ where r is the radius of the circle. An alternative formula is $C = \pi d$ where d is the diameter of the circle.

Mid point of a line

If (x_1, y_1) and (x_2, y_2) are the end points of a line, then the coordinates of the midpoint M of the line are

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Gradient of a line

If (x_1, y_1) and (x_2, y_2) are two points on a line, then the gradient m of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Lines

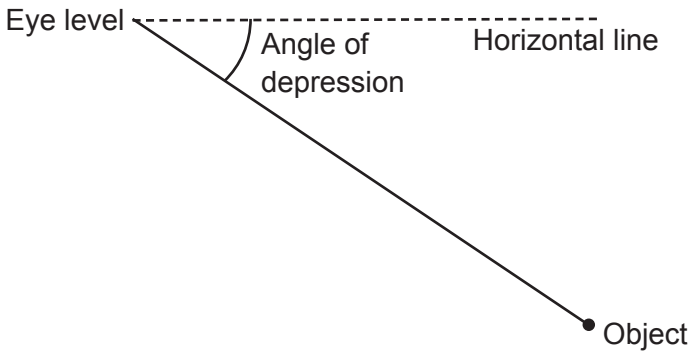
Parallel lines have the same gradient.

If a straight line has gradient m, then a line which is perpendicular to this line has a gradient $-\frac{1}{m}$

Geometry and Angles

There are 180° on a straight line.
There are 180° inside a triangle.
An isosceles triangle is a triangle with 2 equal sides and 2 equal angles.
The sum of all the angles inside a polygon is given by $180(n - 2)$ where n is the number of sides in the polygon.

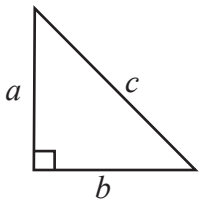
Angle of depression
If a person stands and looks down at an object, the **angle of depression** is the angle between the horizontal line of sight and the object.



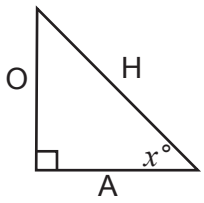
Pythagoras' Theorem

If *a, b* and *c* are the sides of a right angled triangle shown below, then

$$a^2 + b^2 = c^2$$



Trigonometric ratios in right angled triangles



$\sin x° = \frac{O}{H}$ $\cos x° = \frac{A}{H}$ $\tan x° = \frac{O}{A}$

Tangent/Radius property

The tangent to a circle is perpendicular to the radius at the point of contact with the circle.

Alternate Segment Theorem
In a circle, the angle between a chord and a tangent through one of the end points of the chord is equal to the angle in the alternate segment.

Range
The range of a set of data is the difference between the largest value and the smallest value in the data set.

Mean

The mean of a set of data is the sum of all the data values divided by the number of data values.

Estimate for the mean of a grouped frequency distribution

Estimated mean = sum of (mid interval values multiplied by their frequency) divided by the sum of all the frequencies.

Pie Chart

In a pie chart, the total angle that corresponds to the entire data set is 360°

Probability

The sum of the probabilities of all outcomes equals 1

Frequency density in histograms

$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$

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