Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2022

Mathematics

StemUp Model Solution

Unit M4
(With calculator)
Higher Tier



[GMC41]

GMC41

TUESDAY 24 MAY, 9.15am–11.15am

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page. **You are provided with Higher Tier Additional Support Materials for use with this paper.**
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all twenty-one** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

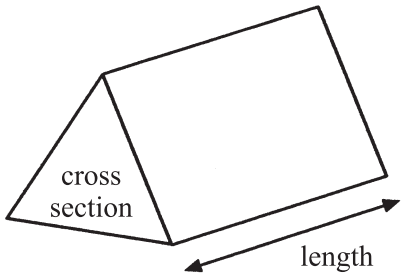
INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

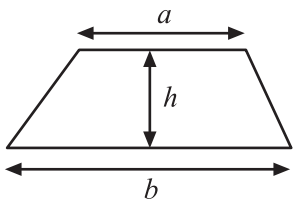
12990.06 **R**

Formula Sheet

Volume of prism = area of cross section × length

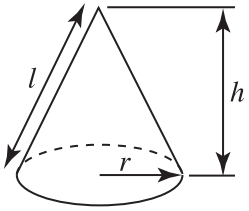


Area of trapezium = $\frac{1}{2}(a+b)h$



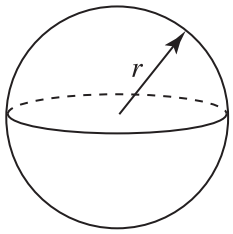
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl

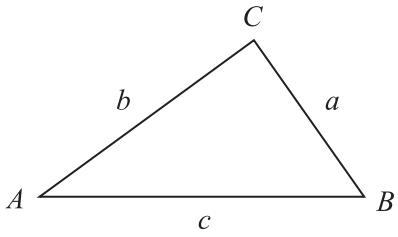


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

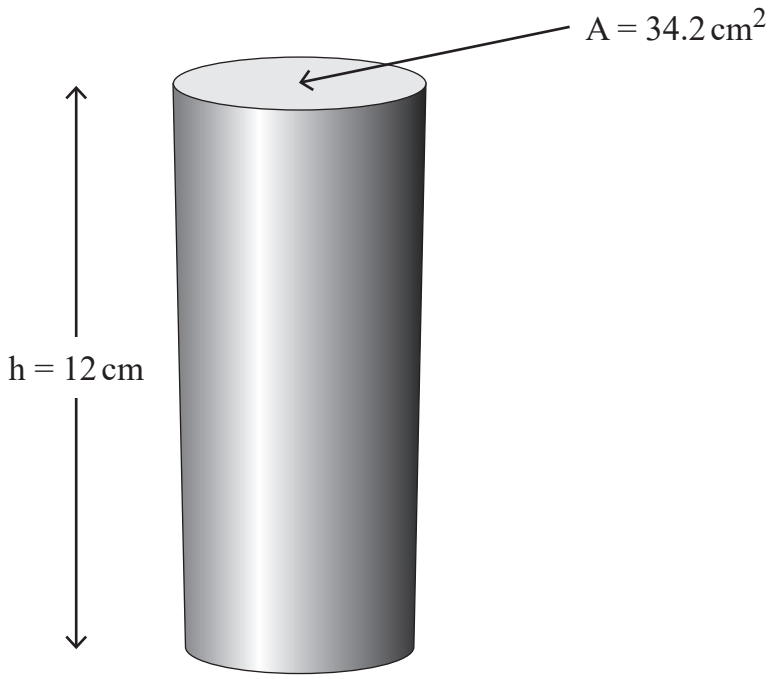
Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

1 A solid cylinder has a height of 12 cm and a circular cross-sectional area of 34.2 cm^2

The density is 0.83 g/cm^3

Find the mass of the cylinder.



Volume = A × h

= 34.2 × 12

= 410.4 cm^3

Mass = density × volume

= 0.83 × 410.4

= 340.632 g

Answer 340.632 g [3]

[Turn over

2 Shares were bought with an original value of £1600

The value increased by 5% each year.

Calculate the value after three years.

Year 1: 5% of 1600 = £80 1600 + 80 = £1680

Year 2: 5% of 1680 = £84 1680 + 84 = £1764

Year 3: 5% of 1764 = £88.20 1764 + 88.20 = £1852.20

Check: $1600 \times 1.05^3 = 1852.2$

Answer £ 1852.20 [3]

3 The number of flights delayed at an airport were recorded over a 24-hour period.

The number of minutes they were delayed is shown in the table below.

Minutes delayed (t)	**Number of flights**	Mid-point	mid × freq
$0 < t \leq 20$	31	10	310
$20 < t \leq 40$	19	30	570
$40 < t \leq 60$	10	50	500
$60 < t \leq 80$	14	70	980
$80 < t \leq 100$	12	90	1080
$100 < t \leq 120$	6	110	660

(a) Calculate an estimate for the mean number of minutes the flights were delayed.

$\Sigma fx = 310 + 570 + 500 + 980 + 1080 + 660 = 4100$

$\Sigma f = 31 + 19 + 10 + 14 + 12 + 6 = 92$

$\text{Mean} = \frac{4100}{92} = 44.56521739...$

$= 44.6$ minutes

Answer 44.6 minutes [4]

(b) Write down the modal class interval.

Answer $0 < t \leq 20$ [1]

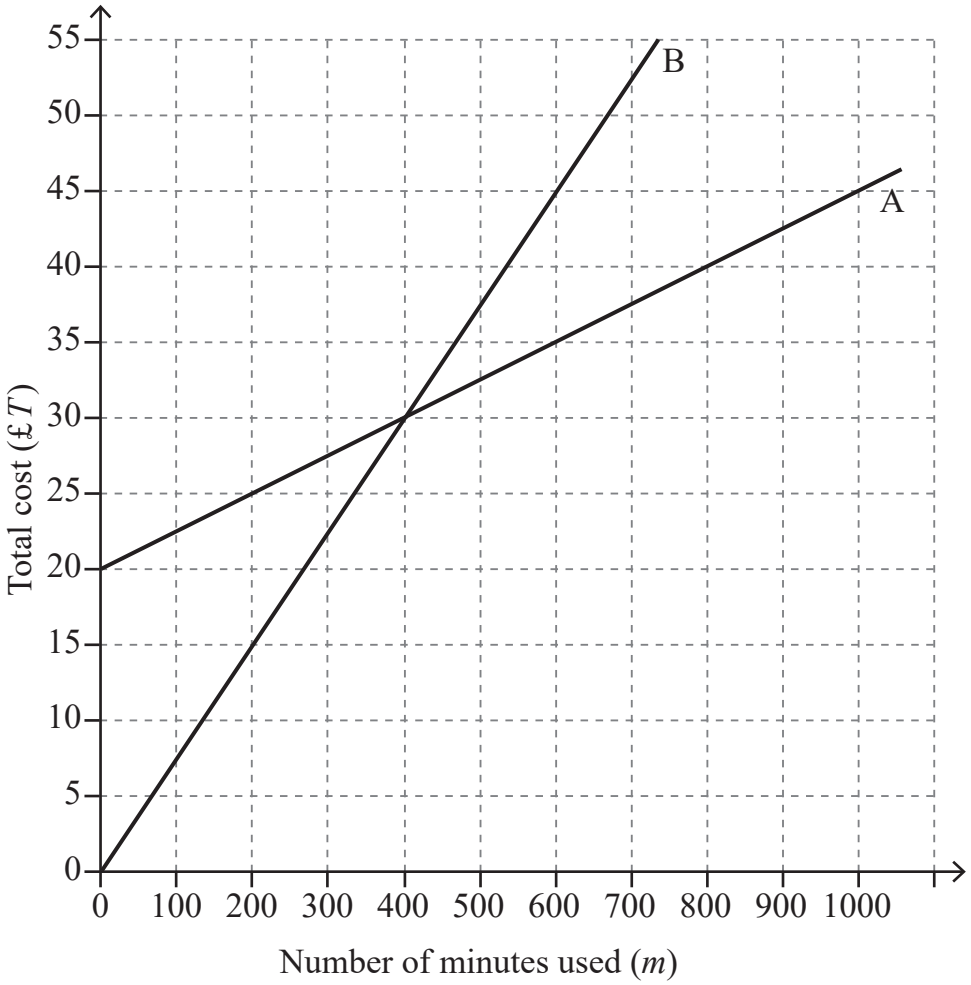
(c) Write down the class interval in which the median lies.

92 flights, median is the 46.5th value: $31 + 19 = 50 \geq 46.5$

Answer $20 < t \leq 40$ [1]

[Turn over

4 The graph below illustrates two different monthly tariffs, A and B, for a mobile phone.



Tariff A has a fixed charge per month plus a charge per minute for phone calls.

(a) What is the fixed charge per month?

Answer £ 20 [1]

(b) What is the charge per minute for tariff A?

From 0 to 400 min: £20 → £30
£10 = 1000p for 400 min
1000 ÷ 400 = 2.5p

Answer 2.5 p [2]

(c) Hence write a formula for the total cost T of tariff A for m minutes.

Answer T = 20 + 0.025m [2]

2.5p = £0.025 per minute, fixed charge £20

5 Expand and simplify $(x - 3)(x + 5)$

$(x - 3)(x + 5)$

$= x^2 + 5x - 3x - 15$

$= x^2 + 2x - 15$

Answer $x^2 + 2x - 15$ [2]

6 Two cyclists start from the same location.

One cyclist travels due North at an average speed of 22 km / h and the other travels due East at an average speed of 26 km / h.

How far apart are the cyclists after 2 hours?

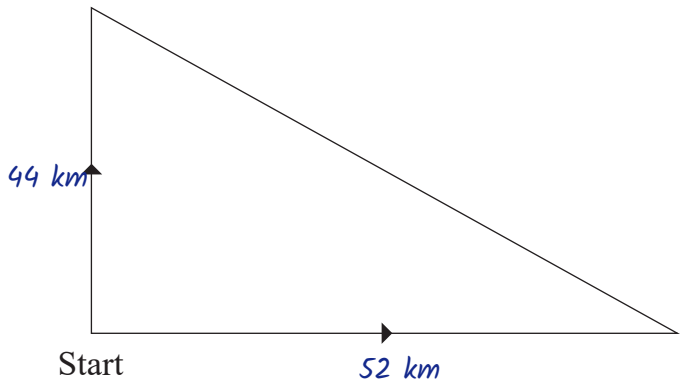


diagram not drawn accurately

North: $22 \times 2 = 44$ km, East: $26 \times 2 = 52$ km

$d^2 = 44^2 + 52^2 = 1936 + 2704 = 4640$

$d = \sqrt{4640} = 68.11754546...$

Answer 68.1 km [4]

7 Simplify $5 - \frac{x+10}{2}$

$5 - \frac{x+10}{2} = \frac{10}{2} - \frac{x+10}{2} = \frac{10-x-10}{2}$

Answer $\frac{-x}{2}$ [2]

8 A factory produces three types of lightbulbs: clear, pearl and colour.

In a week 45 360 clear bulbs are produced.

Pearl and colour make up 36% of the total produced each week.

There are four times as many pearl produced as colour.

How many pearl are produced?

Clear = 100% − 36% = 64%

64% = 45 360

1% = 45 360 ÷ 64 = 708.75

100% = 70 875 bulbs in total

36% = 708.75 × 36 = 25 515 (pearl and colour)

pearl : colour = 4 : 1

Pearl = $\frac{4}{5}$ × 25 515 = 20 412

Answer 20 412 [4]

[Turn over

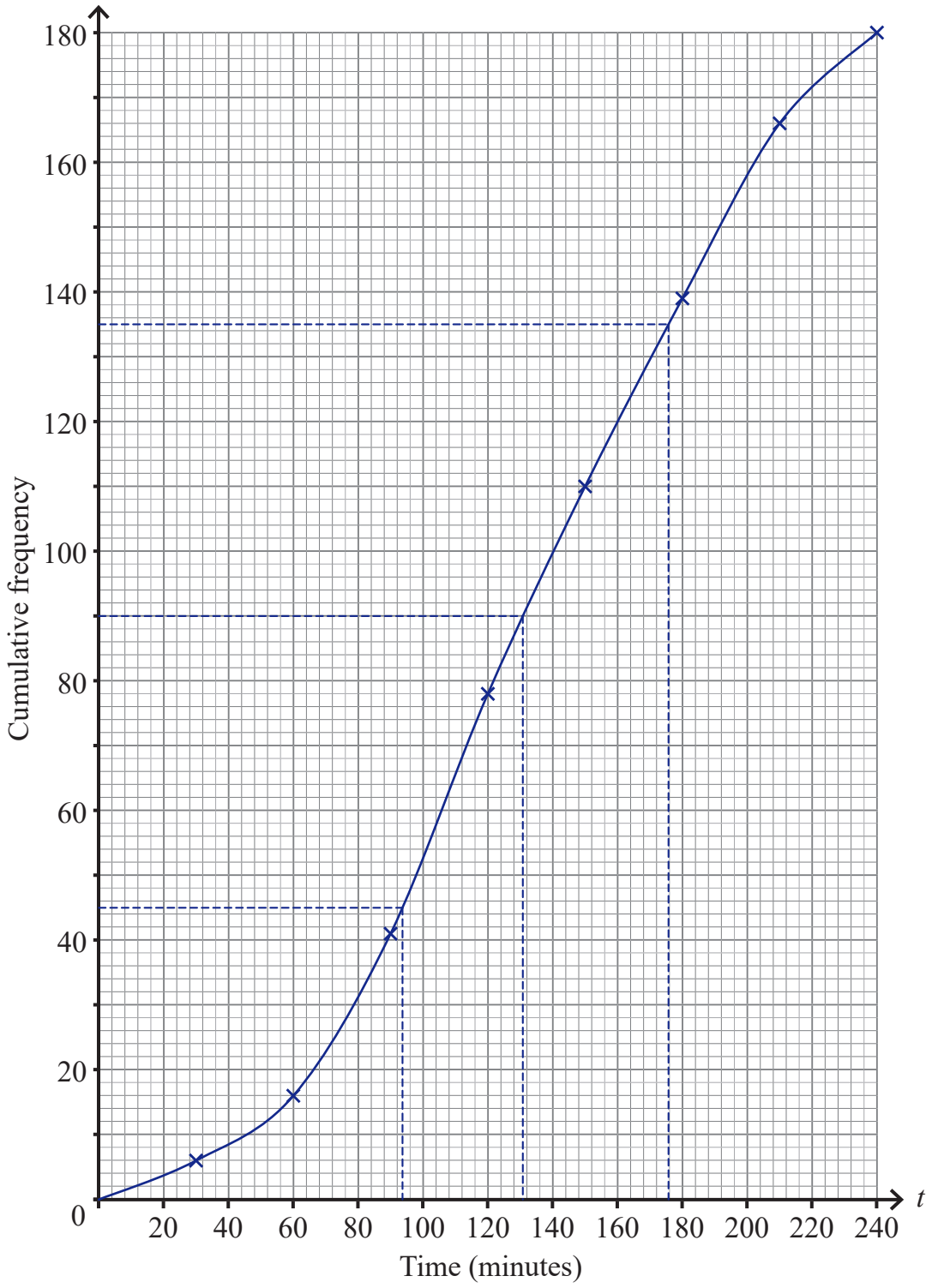
9 180 Year 11 pupils in Glasgow High School were asked to record how much time they spent on their phones one Saturday.

The results are shown in the table below.

Time, t (minutes)	Number of pupils	Time, t mins ($\leq$)	Cumulative frequency
$0 < t \leq 30$	6	30	6
$30 < t \leq 60$	10	60	16
$60 < t \leq 90$	25	90	41
$90 < t \leq 120$	37	120	78
$120 < t \leq 150$	32	150	110
$150 < t \leq 180$	29	180	139
$180 < t \leq 210$	27	210	166
$210 < t \leq 240$	14	240	180

(a) Complete the cumulative frequency column in the table. [1]

(b) Plot a cumulative frequency graph on the given axes.



[3]

[Turn over

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(c) Use your graph to estimate

(i) the median,

½ × 180 = 90th pupil → 131

Answer 131 minutes [1]

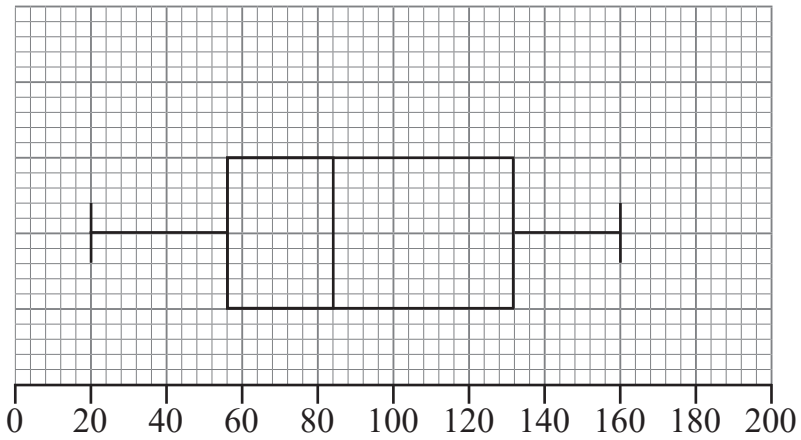
(ii) the inter-quartile range.

UQ (135th) = 176, LQ (45th) = 94, IQR = 176 − 94 = 82

Answer 82 minutes [2]

The staff at the same school were also asked to record the time they spent on their phones on the same Saturday.

Their results are recorded on the box-plot diagram.



(d) Compare the results for the pupils and staff.

1. On average pupils spent more time (median 131 v 84 minutes).
2. Pupils' times are more spread out (IQR 82 v 76 minutes).
3. Max for pupils is a lot more than staff (240 v 160 mins). [3]

10 Solve $\frac{1}{4}(y+3)-2=\frac{1}{2}(3-2y)$

Multiply every term by 4:

$(y + 3) - 8 = 2(3 - 2y)$

$y + 3 - 8 = 6 - 4y$

$y - 5 = 6 - 4y$

$y + 4y = 6 + 5$

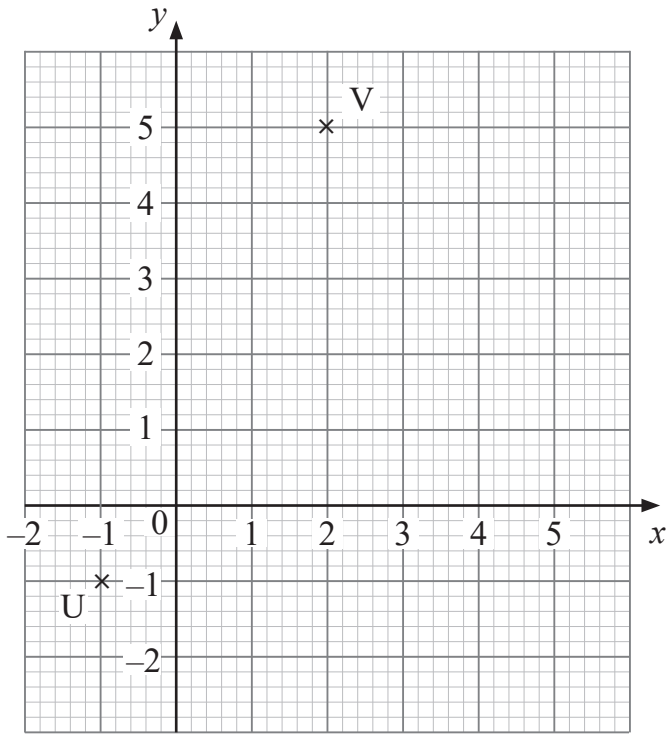
$5y = 11$

$y = \frac{11}{5} = 2\frac{1}{5} = 2.2$

Answer $y =$ 2.2 [4]

[Turn over

11 U has coordinates (–1, –1) and V has coordinates (2, 5) as shown.



(a) Find the equation of the straight line joining U and V.

gradient $m = \frac{5-(-1)}{2-(-1)} = \frac{6}{3} = 2$

$y = 2x + c$, through (2, 5): $5 = 4 + c$, $c = 1$

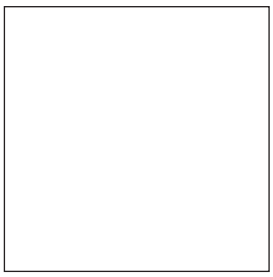
Answer $y = 2x + 1$ [3]

(b) Find the equation of the line perpendicular to UV that passes through the point (0, –1).

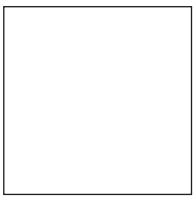
perpendicular gradient $= -\frac{1}{2}$, $c = -1$

Answer $y = -\frac{1}{2}x - 1$ [2]

12



$4n + 3$



$3n + 4$

Find an expression in n for the difference between the areas of the two squares.

Give your answer in fully factorised form.

Show your working.

Difference $= (4n + 3)^2 - (3n + 4)^2$

$= 16n^2 + 24n + 9 - (9n^2 + 24n + 16)$

$= 16n^2 + 24n + 9 - 9n^2 - 24n - 16$

$= 7n^2 - 7$

$= 7(n^2 - 1)$

$= 7(n - 1)(n + 1)$

Answer $7(n - 1)(n + 1)$ [4]

[Turn over

13 The diagram shows two vertical masts A and B.
A is 210 m high and B is 1.5 times the height of A.
The angle of depression from the top of B to the top of A is 25°

Calculate the horizontal distance between the two masts.

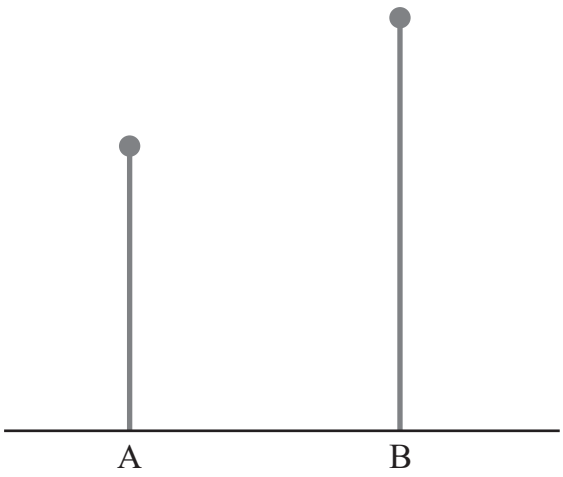


diagram not drawn accurately

Height of B = 1.5 × 210 = 315 m

Difference in height = 315 − 210 = 105 m

$\tan 25° = \frac{105}{x}$

$x = \frac{105}{\tan 25°}$

x = 225.1749...

x = 225.2 m

Answer 225.2 m [4]

14 Solve $(x-2)^2 = 10$ giving your answers correct to 2 decimal places.

$(x-2)^2 = 10$

$x - 2 = \pm\sqrt{10}$

$x = 2 \pm \sqrt{10}$

$x = 2 + 3.16227766... = 5.16227766...$

$x = 2 - 3.16227766... = -1.16227766...$

Answer 5.16 or −1.16 [4]

[Turn over

15

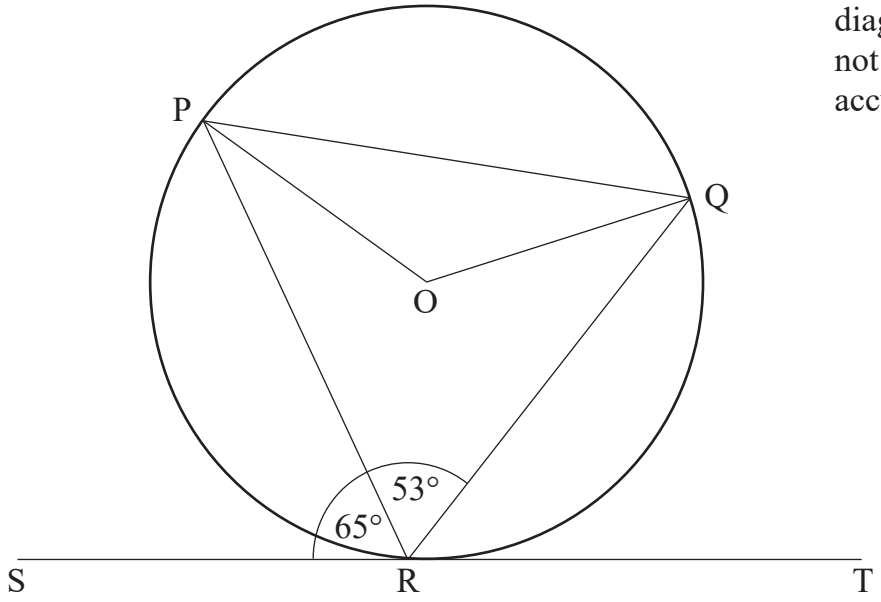


diagram not drawn accurately

P, Q and R are points on the circumference of a circle, centre O.
The line ST is a tangent to the circle.
Angle PRS = 65°
Angle PRQ = 53°

(a) Calculate the size of angle POQ, giving a reason for your answer.

Answer 106 ° because the angle at the centre is twice the angle at the circumference (2 × 53 = 106) [2]

(b) Calculate the size of angle OQR, giving reasons for each step of your answer.

Angle PQR = 65° (alternate segment theorem)

Triangle POQ is isosceles (OP = OQ are radii), so

angle PQO = (180 − 106) ÷ 2 = 37° (base angles of isosceles triangle)

Angle OQR = angle PQR − angle PQO

= 65 − 37 = 28°

Answer 28 ° [3]

[Turn over

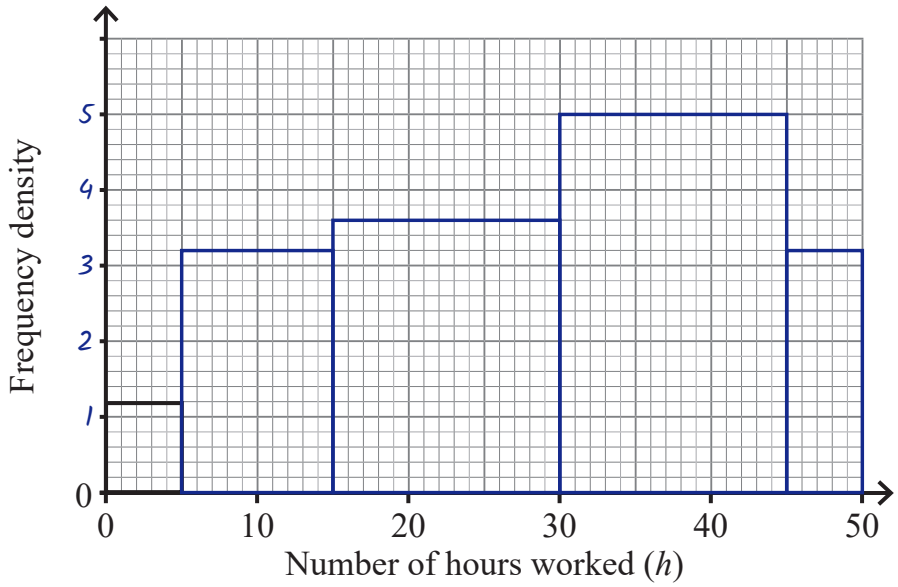
16 The table gives information about the number of hours worked by employees of a factory in one week.

No employees worked more than 50 hours.

Number of hours (h)	Frequency
$0 < h \leq 5$	6
$5 < h \leq 15$	32
$15 < h \leq 30$	54
$30 < h \leq 45$	75
$45 < h \leq 50$	16

fd = f ÷ width
6 ÷ 5 = 1.2
32 ÷ 10 = 3.2
54 ÷ 15 = 3.6
75 ÷ 15 = 5
16 ÷ 5 = 3.2

(a) Complete the histogram for this information.



[4]

(b) To investigate the link between job satisfaction and number of hours worked, explain why a stratified sample is the best method to use here.

Answer the sample is representative of the data: each group of hours worked is sampled in proportion to its size [1]

(c) A stratified sample of 30 employees is taken from this data.

Calculate an estimate for how many employees in the stratified sample worked more than 36 hours.

Total = 6 + 32 + 54 + 75 + 16 = 183

More than 36 hours in $30 < h \leq 45$: $\frac{9}{15} \times 75 = 45$

Plus all of $45 < h \leq 50$: $45 + 16 = 61$

$\frac{61}{183} \times 30 = 10$

Answer 10 [3]

17 Factorise

(a) $25 - 0.01t^2$

$25 - 0.01t^2 = 5^2 - (0.1t)^2$

$= (5 - 0.1t)(5 + 0.1t)$

Answer $(5 - 0.1t)(5 + 0.1t)$ [2]

(b) $ab - cd - ca + db$

$ab - ca + db - cd$

$= a(b - c) + d(b - c)$

Answer $(a + d)(b - c)$ [2]

[Turn over

18 An employee of a company earns £260 per week for a basic 40 hour week.
She is offered a 10% rise in her hourly rate.
She is also offered a percentage reduction in her working hours.
The combined impact of these changes is a 4.5% increase in her total weekly pay.

Calculate the percentage reduction in her working hours.

Hourly rate = 260 ÷ 40 = £6.50

New rate = 6.50 × 1.10 = 6.50 + 0.65 = £7.15

New weekly pay = 260 × 1.045 = £271.70

New hours = 271.70 ÷ 7.15 = 38 hours

Reduction = 40 − 38 = 2 hours

Percentage reduction = $\frac{2}{40}$ × 100 = 5%

Answer 5 % [4]

19 A lampshade is made by removing a small cone from a larger cone as shown in the diagram.

The volume of the lampshade is 16 625 cm^3

Find the value of r.

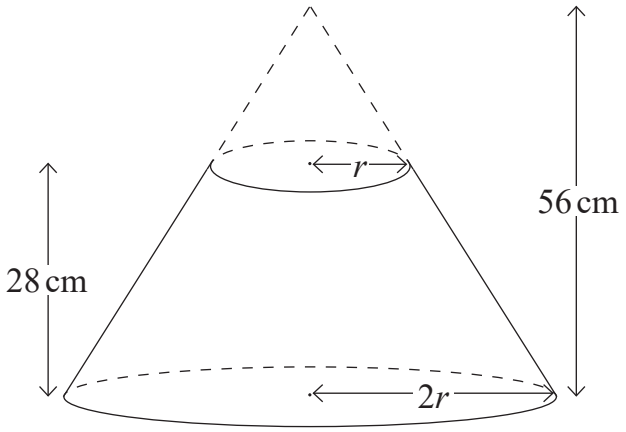


diagram not drawn accurately

$V = \frac{1}{3} \times \pi \times (2r)^2 \times 56 - \frac{1}{3} \times \pi \times r^2 \times 28$

$V = \frac{224\pi}{3} r^2 - \frac{28\pi}{3} r^2 = \frac{196\pi}{3} r^2$

$V = 205.2507202... \, r^2$

$205.2507202... \, r^2 = 16\,625$

$r^2 = 16\,625 \div 205.2507202... = 81$

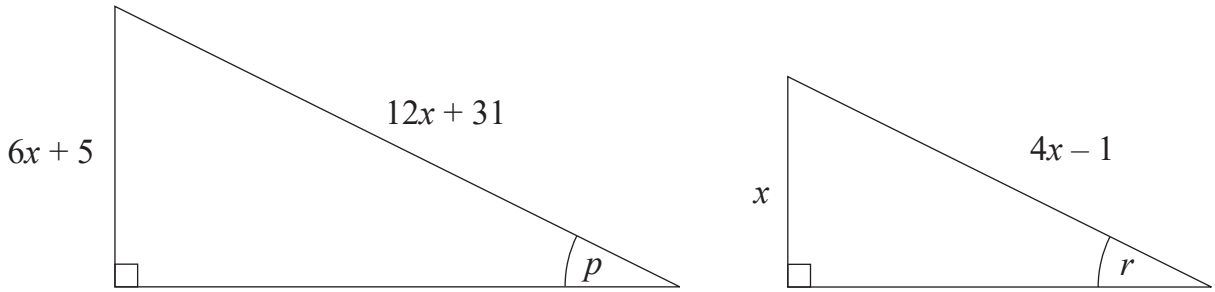
$r = \sqrt{81} = 9$

Answer $r =$ 9 cm [5]

[Turn over

20

diagram not drawn accurately



(a) If $\sin p = \sin r$, show that $12x^2 - 17x - 5 = 0$

$\sin p = \frac{6x + 5}{12x + 31}$ $\sin r = \frac{x}{4x - 1}$

$\frac{6x + 5}{12x + 31} = \frac{x}{4x - 1}$

$(6x + 5)(4x - 1) = x(12x + 31)$

$24x^2 - 6x + 20x - 5 = 12x^2 + 31x$

$24x^2 - 12x^2 + 14x - 31x - 5 = 0$

$12x^2 - 17x - 5 = 0$

[4]

(b) Hence calculate the size of angle p

$(4x + 1)(3x - 5) = 0$

$x = -1/4$ or $x = 5/3$

x cannot be negative (it is a length), so $x = 5/3$

$\sin p = \frac{6 \times 5/3 + 5}{12 \times 5/3 + 31} = \frac{15}{51}$

$p = \sin^{-1}\frac{15}{51} = 17.10261...$

Answer $p =$ 17.1 ° [4]

21 Given that $\frac{2}{x+a} - \frac{5}{x-a} = 1$ has a solution $x = 3$, find the possible values of a.

$x = 3$: $\frac{2}{3+a} - \frac{5}{3-a} = 1$

$\frac{2(3-a) - 5(3+a)}{(3+a)(3-a)} = 1$

$2(3-a) - 5(3+a) = (3+a)(3-a)$

$6 - 2a - 15 - 5a = 9 - a^2$

$-9 - 7a = 9 - a^2$

$a^2 - 7a - 18 = 0$

$(a-9)(a+2) = 0$

$a = 9$ or $a = -2$

Answer $a = 9$ or $a = -2$ [6]

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Examiner Number

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General Certificate of Secondary Education
Summer 2022

GCSE Mathematics

HIGHER TIER
ADDITIONAL SUPPORT MATERIALS
(For use in Summer 2022)

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HIGHER TIER ADDITIONAL SUPPORT MATERIALS (Summer 2022)

Numbers

Lowest common multiple (LCM): The lowest common multiple is the lowest multiple shared by 2 or more numbers.

Trial and Improvement

This is a method of trying different values in an equation until you get a suitable solution (e.g to 1 decimal place).

Metric units

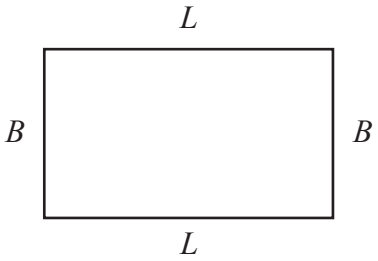
$1 \text{ ml} = 1 \text{ cm}^3$

Compound Measures

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

Perimeter, Area and Volume



The perimeter of a rectangle is the distance around the outside of the rectangle. It is found by adding the lengths of the 4 sides of the rectangle.

$P = 2L + 2B$ where P is perimeter, L is length and B is breadth.

The area of a rectangle is found by multiplying the length of the rectangle by the breadth.

$A = L \times B$ where L is length and B is breadth.

The volume of a cuboid is found by multiplying the length by the breadth by the height of the cuboid.

$V = L \times B \times H$ where V is volume, L is length, B is breadth and H is height.

The area of a circle is $A = \pi r^2$ where r is the radius of the circle.

The circumference (perimeter) of a circle is $C = 2\pi r$ where r is the radius of the circle. An alternative formula is $C = \pi d$ where d is the diameter of the circle.

Mid point of a line

If (x_1, y_1) and (x_2, y_2) are the end points of a line, then the coordinates of the midpoint M of the line are

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Gradient of a line

If (x_1, y_1) and (x_2, y_2) are two points on a line, then the gradient m of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Lines

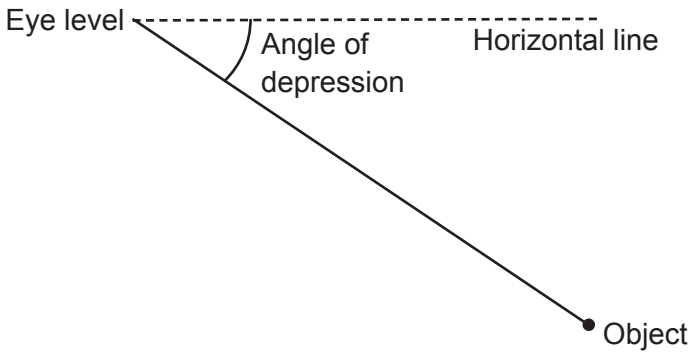
Parallel lines have the same gradient.

If a straight line has gradient m, then a line which is perpendicular to this line has a gradient $-\frac{1}{m}$

Geometry and Angles

There are 180° on a straight line.
There are 180° inside a triangle.
An isosceles triangle is a triangle with 2 equal sides and 2 equal angles.
The sum of all the angles inside a polygon is given by $180(n - 2)$ where n is the number of sides in the polygon.

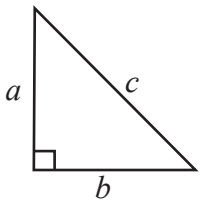
Angle of depression
If a person stands and looks down at an object, the **angle of depression** is the angle between the horizontal line of sight and the object.



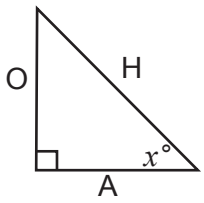
Pythagoras' Theorem

If *a, b* and *c* are the sides of a right angled triangle shown below, then

$$a^2 + b^2 = c^2$$



Trigonometric ratios in right angled triangles



$\sin x° = \frac{O}{H}$ $\cos x° = \frac{A}{H}$ $\tan x° = \frac{O}{A}$

Tangent/Radius property

The tangent to a circle is perpendicular to the radius at the point of contact with the circle.

Alternate Segment Theorem
In a circle, the angle between a chord and a tangent through one of the end points of the chord is equal to the angle in the alternate segment.

Range
The range of a set of data is the difference between the largest value and the smallest value in the data set.

Mean

The mean of a set of data is the sum of all the data values divided by the number of data values.

Estimate for the mean of a grouped frequency distribution

Estimated mean = sum of (mid interval values multiplied by their frequency) divided by the sum of all the frequencies.

Pie Chart

In a pie chart, the total angle that corresponds to the entire data set is 360°

Probability

The sum of the probabilities of all outcomes equals 1

Frequency density in histograms

$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$

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