Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2022

# Mathematics

StemUp Model Solution

Unit M3
**(With calculator)**
Higher Tier



**[GMC31]**

*GMC31*

**TUESDAY 24 MAY, 9.15am–11.15am**

## TIME

2 hours.

## INSTRUCTIONS TO CANDIDATES

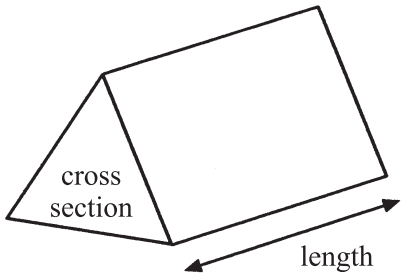
Write your Centre Number and Candidate Number in the spaces provided at the top of this page. **You are provided with Higher Tier Additional Support Materials for use with this paper.**
**You must answer the questions in the spaces provided.**
**Do not write outside the boxed area on each page or on blank pages.**
Complete in black ink only. **Do not write with a gel pen.**
Answer **all twenty-seven** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

## INFORMATION FOR CANDIDATES

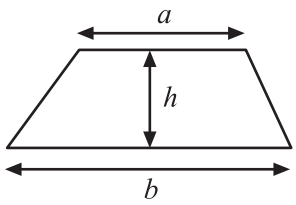
The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

# Formula Sheet

**Volume of prism** = area of cross section × length

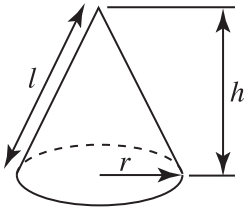



**Area of trapezium** = $\frac{1}{2}(a+b)h$



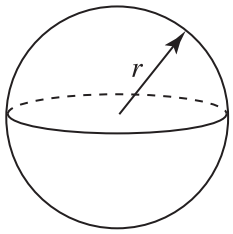

**Volume of cone** = $\frac{1}{3}\pi r^2 h$

**Curved surface area of cone** = $\pi rl$

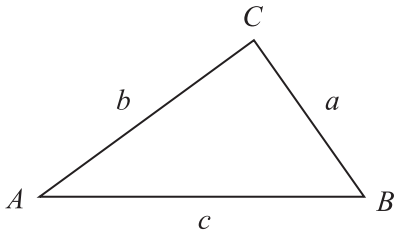



**Volume of sphere** = $\frac{4}{3}\pi r^3$

**Surface area of sphere** = $4\pi r^2$




**In any triangle *ABC***




**Quadratic Equation**

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Sine Rule:** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine Rule:** $a^2 = b^2 + c^2 - 2bc\cos A$

**Area of triangle** = $\frac{1}{2}ab\sin C$

**1**



*Source: © Getty Images*

Belfast Zoo has 120 different species of animals.

One quarter of the species are birds.

40% of the species are mammals.

What **fraction** of the species are neither birds nor mammals?

$\frac{1}{4}$ of 120 = 120 ÷ 4 = 30 birds

40% of 120 = 0.4 × 120 = 48 mammals

Neither: 120 − (30 + 48) = 120 − 78 = 42

Fraction = $\frac{42}{120} = \frac{7}{20}$

Answer $\frac{7}{20}$ [4]

**[Turn over**

**2** Work out the size of angle $y$ in the diagram below.

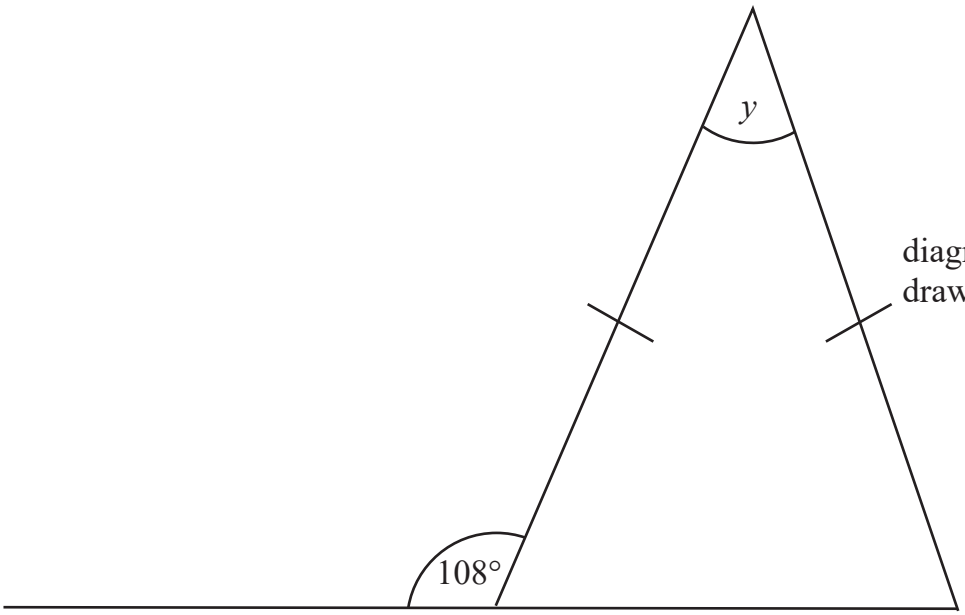



diagram not drawn accurately

Angles on a straight line: 180 − 108 = 72°

Isosceles, so both base angles are 72°

Angles in a triangle: y = 180 − (72 + 72) = 36°

Answer $y$ = 36 ° [3]

**3** **(a)** Which of these is correct?

Circle the answer.

$2^3 = 6$ $3^3 = 81$ $5^3 = 125$

[1]

**(b)** **(i)** Calculate $8.5^3$

8.5 × 8.5 × 8.5 = 614.125

Answer 614.125 [1]

**(ii)** Round this answer to 1 significant figure.

Answer 600 [1]

**[Turn over**

**4** A building firm needs to put fencing around the perimeter of a rectangular site.

The dimensions of the site are shown in the diagram.

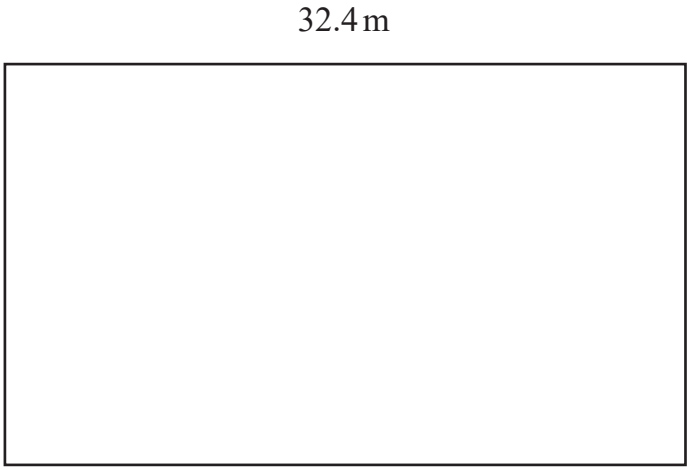



diagram not drawn to scale

**(a)** Calculate the total perimeter of the site.

32.4 + 22.8 + 32.4 + 22.8
= 2 × 55.2 = 110.4 m

Answer 110.4 m [1]

**(b)** Each section of fencing is 180 cm wide.

How many sections are needed to go around the perimeter of the site?

110.4 m = 11040 cm
11040 ÷ 180 = 61.333...
so 62 sections are needed

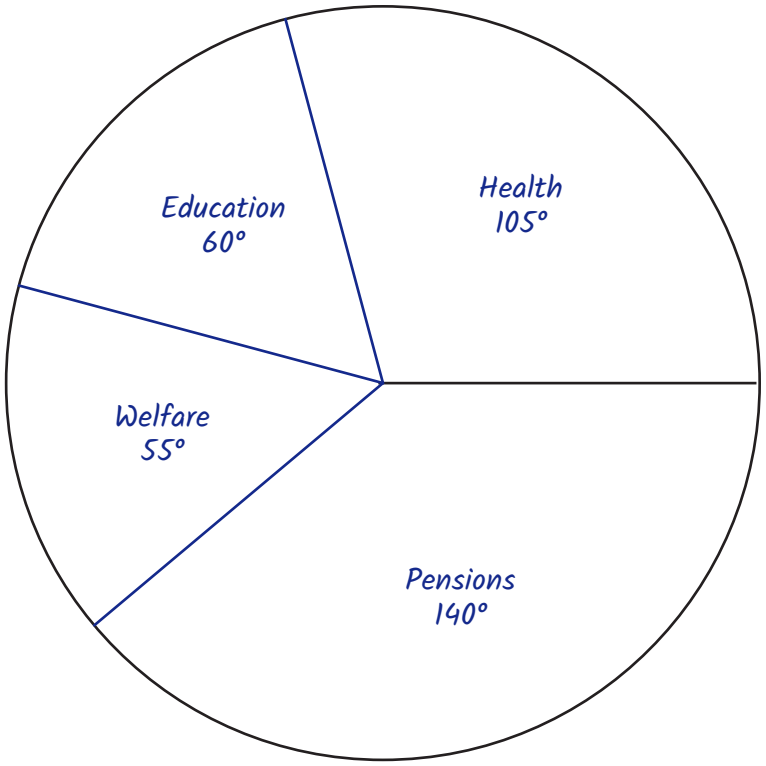
Answer 62 [2]

**5** The table below shows how an organisation spent its annual budget.

| Spending area | £ (millions) | Angle |
|---|---|---|
| Health | 21 | 21 × 5 = 105° |
| Education | 12 | 12 × 5 = 60° |
| Welfare | 11 | 11 × 5 = 55° |
| Pensions | 28 | 28 × 5 = 140° |

Draw a labelled pie chart to show this information.

Total = 21 + 12 + 11 + 28 = 72
360 ÷ 72 = 5° per £1 million




[4]

**[Turn over**

**6** Debbie earns £9.50 an hour.

She is paid **double time** for overtime.

Robbie earns £12 an hour.

He is paid overtime at a rate of **time and a half**.

They both worked 6 hours on Saturday at their normal rate and 2 hours overtime.

Who earned more and how much more?

Debbie: 6 × 9.50 = £57

overtime 9.50 × 2 = £19 an hour, 2 × 19 = £38

total = 57 + 38 = £95

Robbie: 6 × 12 = £72

overtime 12 × 1.5 = £18 an hour, 2 × 18 = £36

total = 72 + 36 = £108

108 − 95 = £13

Answer Robbie earned £ 13 more [3]

7 A club shop sold T-shirts and polo shirts in sizes small, medium and large.

They sold

- 4 small T-shirts
- 9 large polo shirts
- 8 medium T-shirts
- a total of 21 polo shirts
- a total of 11 small items
- a total of 15 large items

Use this information to complete the two-way table below.

| | T-shirt | Polo shirt | Total |
|---|---|---|---|
| **Small** | 4 | 7 | 11 |
| **Medium** | 8 | 5 | 13 |
| **Large** | 6 | 9 | 15 |
| **Total** | 18 | 21 | 39 |

[3]

Small polo = 11 − 4 = 7, Large T-shirt = 15 − 9 = 6

Medium polo = 21 − 7 − 9 = 5, T-shirts = 4 + 8 + 6 = 18

Medium total = 8 + 5 = 13, Total = 18 + 21 = 39

**[Turn over**

**8** Clare buys some buns and a gift box for them.

The buns cost 55p each.

The gift box costs 80p.

She pays with a £10 note and receives £2.60 change.

How many buns did Clare buy?

Show all your working.

£10 − £2.60 = £7.40 spent

£7.40 − £0.80 (box) = £6.60 on buns

660p ÷ 55p = 12

Answer 12 [3]

**9** Jenny has $p$ paintbrushes and $c$ crayons.

**(a)** Kev has twice as many paintbrushes and three times as many crayons as Jenny.

Write an expression for Kev's total, in terms of $p$ and $c$.

Answer $2p + 3c$ [1]

**(b)** Lily has the same number of paintbrushes and three-quarters of the number of crayons that Jenny has.

Write an expression for Lily's total, in terms of $p$ and $c$.

Answer $p + \frac{3}{4}c$ [1]

**(c)** Dan has 3 fewer paintbrushes and 4 more crayons than Jenny.

Write an expression for Dan's total, in terms of $p$ and $c$.

Give your answer in its simplest form.

$(p - 3) + (c + 4)$

Answer $p + c + 1$ [1]

**[Turn over**

**10** Gail wants to do some research into people's spending on food.

**(a)** She decides to use the following question in her survey.

How much money do you spend on food?

£0–10 ☐ £20–30 ☐ £40–50 ☐ over £50 ☐

Give two possible reasons why people may be confused by this question.

Reason 1 *No time period is given (per week? per month?)*

Reason 2 *Gaps between the boxes, e.g. where does £35 or £10.50 go?*

[2]

**(b)** Gail decides to survey 10 people.

Why might this lead her to make unreliable conclusions?

Answer *Her sample size of 10 is too small to represent everyone* [1]

**11** I spent $2\frac{1}{2}$ hours on my homework last night.

One third of that time was spent on Geography and half of the time spent on Geography was on map work.

What **fraction** of my homework time was spent on map work?

$2\frac{1}{2}$ hours = 150 minutes

$\frac{1}{3}$ of 150 = 50 minutes on Geography

$\frac{1}{2}$ of 50 = 25 minutes on map work

$\frac{25}{150} = \frac{1}{6}$

Answer $\frac{1}{6}$ [3]

**12** Solve $5(p - 3) = 20$

$5p - 15 = 20$

$5p = 35$

$p = 35 \div 5 = 7$

Answer $p =$ 7 [3]

**[Turn over**

**13** The diagram shows a "figure of 8" shaped race track, which has a semi-circular section at either end.

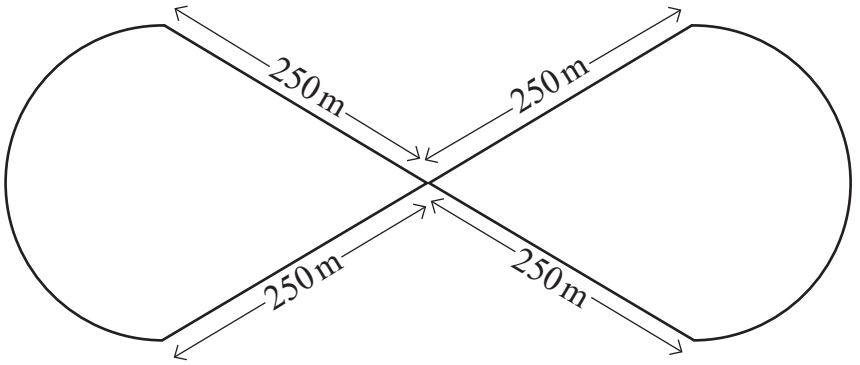



diagram not drawn accurately

**(a)** Calculate the total distance travelled during one complete lap of the track.

Two semicircles = one full circle, diameter 250 m

Circumference = π × 250 = 785.3981... m

Straight parts = 4 × 250 = 1000 m

Total = 1000 + 785.3981... = 1785.3981... m

Answer 1785.40 m [3]

**(b)** A horse runs 2000 metres at an average speed of 14.5 m/s.

How long does this take?

Give your answer in minutes and seconds, to the nearest second.

time = distance ÷ speed

= 2000 ÷ 14.5 = 137.9310... seconds

137.93 − 120 = 17.93 seconds

= 2 minutes 17.93... s = 2 minutes 18 seconds

Answer 2 minutes 18 seconds [3]

**14** Factorise

**(a)** $12x + 20$

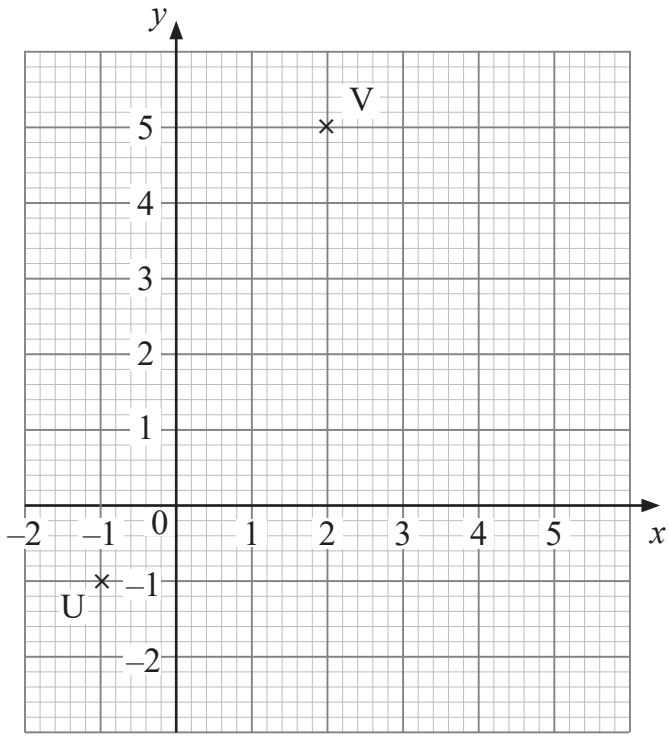
Answer $4(3x + 5)$ [1]

**(b)** $p - p^2$

Answer $p(1 - p)$ [1]

**[Turn over**

**15** U has coordinates (–1, –1) and V has coordinates (2, 5) as shown.




**(a)** Write down the coordinates of the midpoint of the line joining U and V.

$\frac{-1+2}{2} = 0.5 \quad \frac{-1+5}{2} = 2$

Answer ( $\frac{1}{2}$ , 2 ) [2]

**(b)** Find the equation of the straight line joining U and V.

$\text{gradient} = \frac{5-(-1)}{2-(-1)} = \frac{6}{3} = 2$

$y = 2x + c$, through $(2, 5)$: $5 = 2 \times 2 + c$

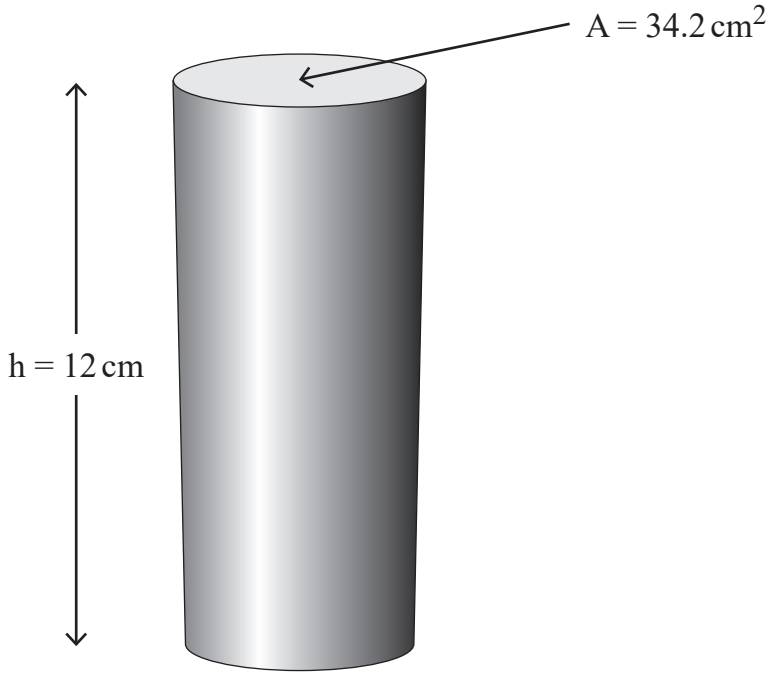
$c = 1$

Answer $y = 2x + 1$ [3]

**16** A solid cylinder has a height of 12 cm and a circular cross-sectional area of 34.2 $cm^2$

The density is 0.83 $g/cm^3$

Find the mass of the cylinder.




Volume = area × height

= 34.2 × 12 = 410.4 $cm^3$

Mass = density × volume

= 0.83 × 410.4 = 340.632 g

Answer 340.632 g [3]

**[Turn over**

**17** Shares were bought with an original value of £1600

The value increased by 5% each year.

Calculate the value after three years.

Year 1: 5% of 1600 = £80, 1600 + 80 = £1680

Year 2: 5% of 1680 = £84, 1680 + 84 = £1764

Year 3: 5% of 1764 = £88.20, 1764 + 88.20 = £1852.20

(Check: $1600 \times 1.05^3 = 1852.2$)

Answer £ 1852.20 [3]

**18** P is the smallest number which has prime factors of 2, 3 and 5

Q is the number 24

Work out the LCM (lowest common multiple) of P and Q.

$P = 2 \times 3 \times 5 = 30$

Multiples of 30: 30, 60, 90, 120, 150, ...

Multiples of 24: 24, 48, 72, 96, 120, ...

LCM = 120

Answer 120 [3]

**19** Expand and simplify $(x - 3)(x + 5)$

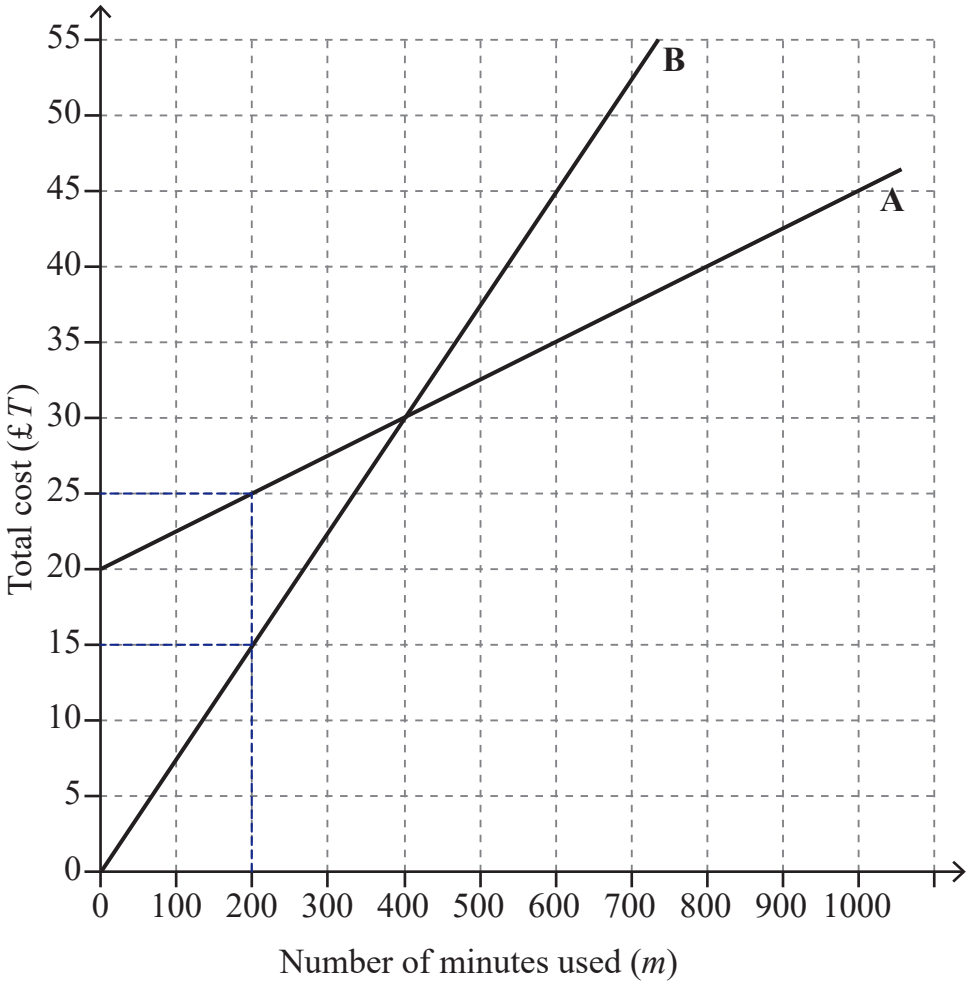
$(x - 3)(x + 5) = x^2 + 5x - 3x - 15$

$= x^2 + 2x - 15$

Answer $x^2 + 2x - 15$ [2]

**[Turn over**

**20** The graph below illustrates two different monthly tariffs, A and B, for a mobile phone.




**(a)** Max uses 200 minutes per month.

Which tariff is cheaper for him and by how much?

At 200 minutes: A = £25, B = £15, 25 − 15 = £10

Answer Tariff ___B___ is cheaper by £ ___10___ [1]

**(b)** Tariff **A** has a fixed charge per month plus a charge per minute for phone calls.

**(i)** What is the fixed charge per month?

Answer £ 20 [1]

**(ii)** What is the charge per minute?

A goes from £20 to £30 in 400 mins

£10 = 1000p, 1000 ÷ 400 = 2.5p

Answer 2.5 p [2]

**(iii)** Hence write a formula for the total cost $T$ of tariff **A** for $m$ minutes.

2.5p = £0.025 per minute

Answer $T = 20 + 0.025m$ [2]

**[Turn over**

**21** The number of flights delayed at an airport were recorded over a 24-hour period.

The number of minutes they were delayed is shown in the table below.

| **Minutes delayed ($t$)** | **Number of flights** | Mid-point ($x$) | $f \times x$ |
|---|---|---|---|
| $0 < t \leq 20$ | 31 | 10 | 10 × 31 = 310 |
| $20 < t \leq 40$ | 19 | 30 | 30 × 19 = 570 |
| $40 < t \leq 60$ | 10 | 50 | 50 × 10 = 500 |
| $60 < t \leq 80$ | 14 | 70 | 70 × 14 = 980 |
| $80 < t \leq 100$ | 12 | 90 | 90 × 12 = 1080 |
| $100 < t \leq 120$ | 6 | 110 | 110 × 6 = 660 |

**(a)** Calculate an estimate for the mean number of minutes the flights were delayed.

$\Sigma fx = 310 + 570 + 500 + 980 + 1080 + 660 = 4100$

$\Sigma f = 31 + 19 + 10 + 14 + 12 + 6 = 92$

Mean = 4100 ÷ 92 = 44.5652...

Answer 44.57 minutes [4]

**(b)** Write down the modal class interval.

Answer $0 < t \leq 20$ [1]

**(c)** Write down the class interval in which the median lies.

92 flights: median is the 46.5th value; 31 + 19 = 50

Answer $20 < t \leq 40$ [1]

**22** Two cyclists start from the same location.

One cyclist travels due North at an average speed of 22 km / h and the other travels due East at an average speed of 26 km / h.

How far apart are the cyclists after 2 hours?

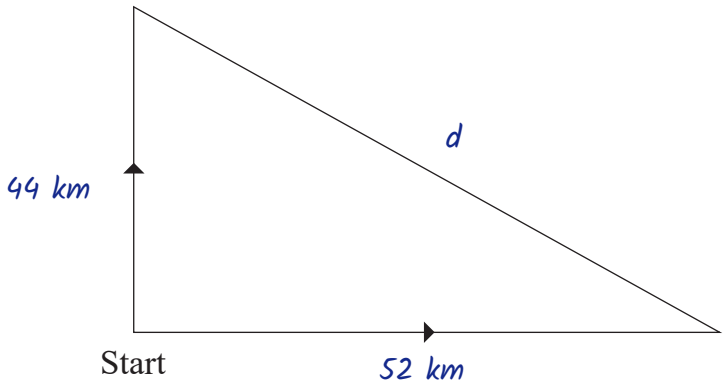



diagram not drawn accurately

North: 22 × 2 = 44 km, East: 26 × 2 = 52 km

$d^2 = 44^2 + 52^2 = 1936 + 2704 = 4640$

$d = \sqrt{4640} = 68.1175... = 68.1$ km

Answer 68.1 km [4]

**[Turn over**

**23** A factory produces three types of lightbulbs: clear, pearl and colour.

In a week 45 360 clear bulbs are produced.

Pearl and colour make up 36% of the total produced each week.

There are four times as many pearl produced as colour.

How many pearl are produced?

Clear = 100 − 36 = 64% of the total

64% = 45360, so 1% = 45360 ÷ 64 = 708.75

Pearl and colour = 36% = 36 × 708.75 = 25515

Pearl : colour = 4 : 1, so pearl = 4/5 of 25515

25515 ÷ 5 × 4 = 20412

Answer 20412 [4]

**24** A rectangle has been recorded as having a length of 80 m, correct to the nearest 10 m, and a width of 30 m, correct to the nearest m.




Jane says the area could be 2400 $m^2$

Steve says the area could be 1875 $m^2$

Paula says the area could be 2212.5 $m^2$

Which of the three is definitely not correct and what mistake has been made?

**Explain your reasoning clearly.**

Length: $75 \leq L < 85$, Width: $29.5 \leq W < 30.5$

Minimum area = $75 \times 29.5 = 2212.5\ m^2$

Maximum area = $85 \times 30.5 = 2592.5\ m^2$

Jane (2400) and Paula (2212.5) are within this range.

Steve is definitely not correct: $1875 = 75 \times 25$,

he used 25 as the lower bound of the width, as if it

were to the nearest 10 m. The lower bound is 29.5 m,

so the area cannot be less than $2212.5\ m^2$.

[2]

**[Turn over**

**25** Solve $\frac{1}{4}(y+3)-2=\frac{1}{2}(3-2y)$

Multiply both sides by 4:

$(y + 3) - 8 = 2(3 - 2y)$

$y + 3 - 8 = 6 - 4y$

$y + 4y = 6 - 3 + 8$

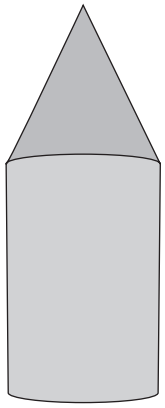
$5y = 11$

$y = \frac{11}{5} = 2\frac{1}{5}$

Answer $y =$ $\frac{11}{5}$ [4]

**26** The roof on a tower is in the shape of a cone with a diameter of 7.5 m and a height of 5 m.

Calculate the volume of this cone.



$r = 7.5 \div 2 = 3.75$ m

$V = \frac{1}{3} \times \pi \times 3.75^2 \times 5$

$= 73.6310...$ m$^3$

Answer 73.6 m$^3$ [2]

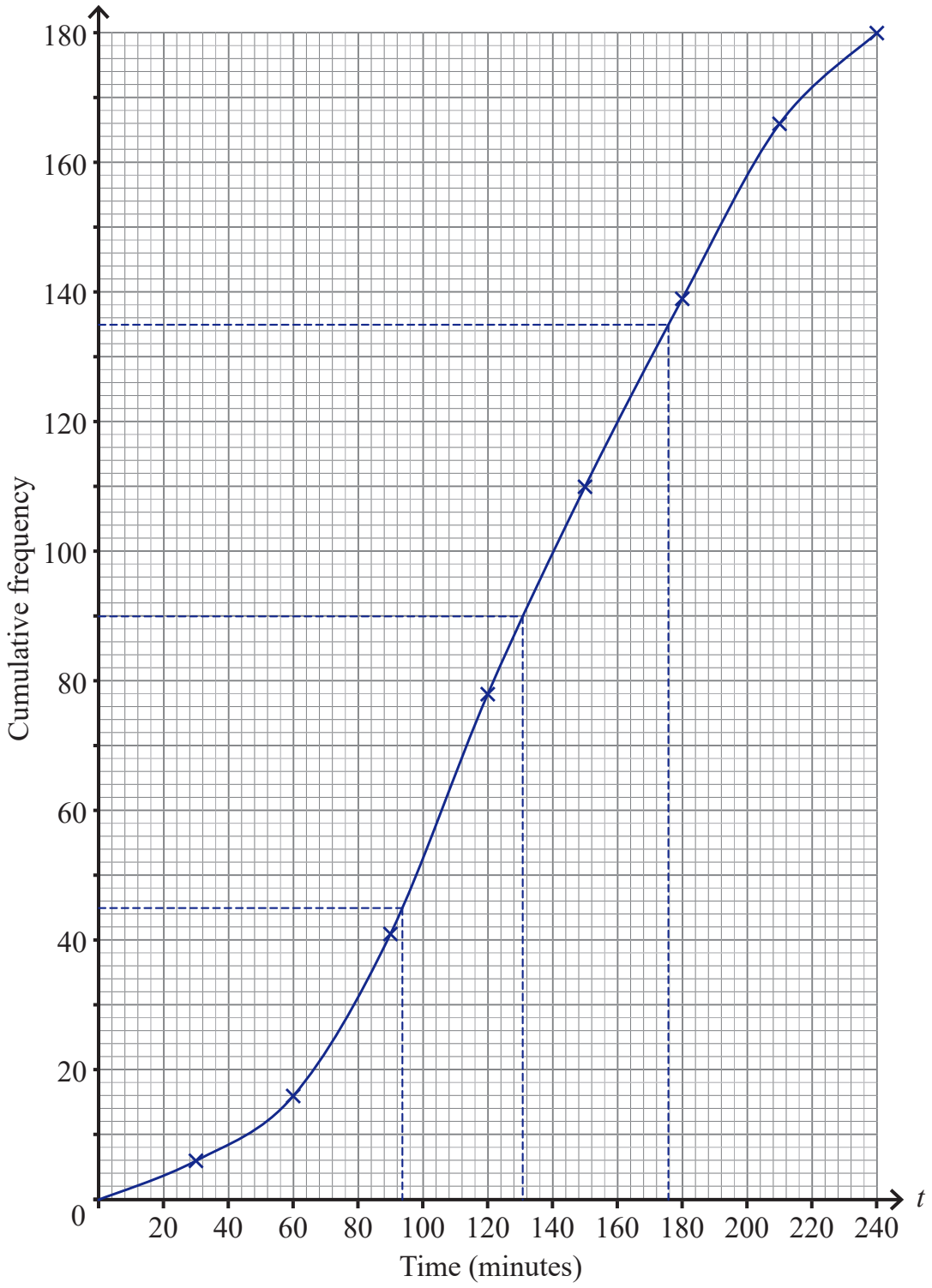
**27** 180 Year 11 pupils in Glasgow High School were asked to record how much time they spent on their phones one Saturday.

The results are shown in the table below.

| Time, $t$ (minutes) | Number of pupils | Time, $t$ mins ($\leq$) | Cumulative frequency |
|---|---|---|---|
| $0 < t \leq 30$ | 6 | 30 | 6 |
| $30 < t \leq 60$ | 10 | 60 | 16 |
| $60 < t \leq 90$ | 25 | 90 | 41 |
| $90 < t \leq 120$ | 37 | 120 | 78 |
| $120 < t \leq 150$ | 32 | 150 | 110 |
| $150 < t \leq 180$ | 29 | 180 | 139 |
| $180 < t \leq 210$ | 27 | 210 | 166 |
| $210 < t \leq 240$ | 14 | 240 | 180 |

**(a)** Complete the cumulative frequency column in the table. [1]

**(b)** Plot a cumulative frequency graph on the given axes.




[3]

**[Turn over**


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**(c)** Use your graph to estimate

**(i)** the median,

median at 180 ÷ 2 = 90th pupil

Answer 131 minutes [1]

**(ii)** the inter-quartile range.

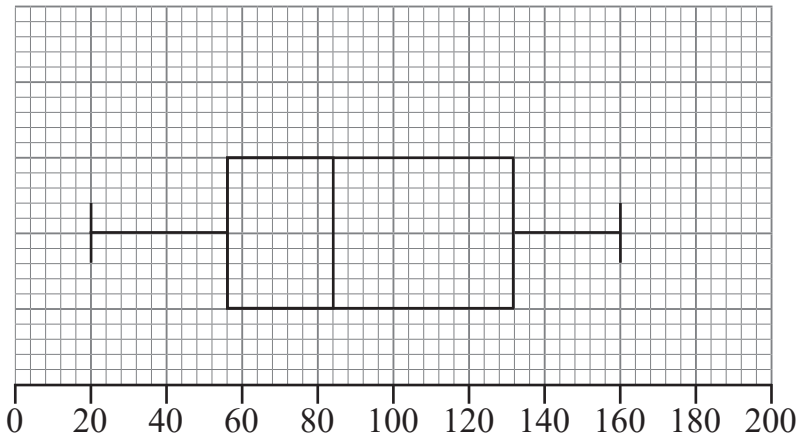
LQ (45th) = 94, UQ (135th) = 176

IQR = 176 − 94 = 82

Answer 82 minutes [2]

The staff at the same school were also asked to record the time they spent on their phones on the same Saturday.

Their results are recorded on the box-plot diagram.




**(d)** Compare the results for the pupils and staff.

1. On average pupils spent more time: median 131 min v staff 84 min

2. Pupils' times more spread out: IQR 82 min v staff 132 − 56 = 76 min

3. Pupils' maximum 240 min, much more than staff maximum 160 min [3]

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| For Examiner's use only | |
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**Examiner Number**

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Rewarding Learning

General Certificate of Secondary Education
Summer 2022

# GCSE Mathematics

# HIGHER TIER
# ADDITIONAL SUPPORT MATERIALS
# (For use in Summer 2022)



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**HIGHER TIER ADDITIONAL SUPPORT MATERIALS (Summer 2022)**

**Numbers**

Lowest common multiple (LCM): The lowest common multiple is the lowest multiple shared by 2 or more numbers.

**Trial and Improvement**

This is a method of trying different values in an equation until you get a suitable solution (e.g to 1 decimal place).

**Metric units**

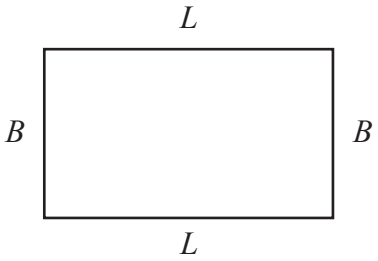
$1 \text{ ml} = 1 \text{ cm}^3$

**Compound Measures**

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

**Perimeter, Area and Volume**




The perimeter of a rectangle is the distance around the outside of the rectangle. It is found by adding the lengths of the 4 sides of the rectangle.

$P = 2L + 2B$ where $P$ is perimeter, $L$ is length and $B$ is breadth.

The area of a rectangle is found by multiplying the length of the rectangle by the breadth.

$A = L \times B$ where $L$ is length and $B$ is breadth.

The volume of a cuboid is found by multiplying the length by the breadth by the height of the cuboid.

$V = L \times B \times H$ where $V$ is volume, $L$ is length, $B$ is breadth and $H$ is height.

The area of a circle is $A = \pi r^2$ where $r$ is the radius of the circle.

The circumference (perimeter) of a circle is $C = 2\pi r$ where $r$ is the radius of the circle. An alternative formula is $C = \pi d$ where $d$ is the diameter of the circle.

**Mid point of a line**

If $(x_1, y_1)$ and $(x_2, y_2)$ are the end points of a line, then the coordinates of the midpoint $M$ of the line are

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Gradient of a line**

If $(x_1, y_1)$ and $(x_2, y_2)$ are two points on a line, then the gradient $m$ of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**Lines**

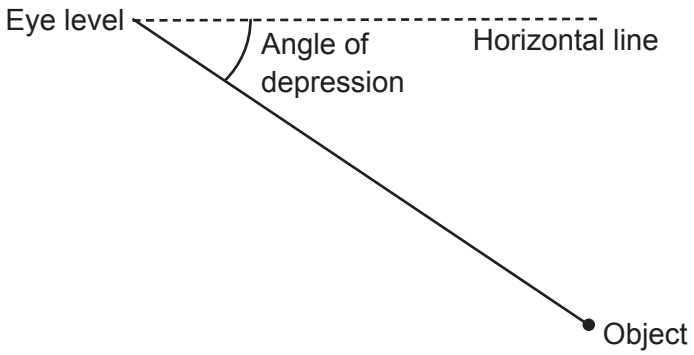
Parallel lines have the same gradient.

If a straight line has gradient $m$, then a line which is perpendicular to this line has a gradient $-\frac{1}{m}$

**Geometry and Angles**

There are 180° on a straight line.
There are 180° inside a triangle.
An isosceles triangle is a triangle with 2 equal sides and 2 equal angles.
The sum of all the angles inside a polygon is given by $180(n - 2)$ where $n$ is the number of sides in the polygon.

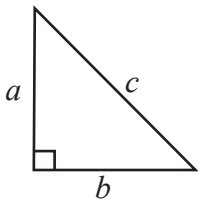
**Angle of depression**
If a person stands and looks down at an object, the **angle of depression** is the angle between the horizontal line of sight and the object.



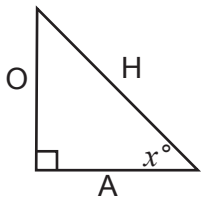

**Pythagoras' Theorem**

If *a, b* and *c* are the sides of a right angled triangle shown below, then

$$a^2 + b^2 = c^2$$



**Trigonometric ratios in right angled triangles**




$\sin x° = \frac{O}{H}$ $\cos x° = \frac{A}{H}$ $\tan x° = \frac{O}{A}$

**Tangent/Radius property**

The tangent to a circle is perpendicular to the radius at the point of contact with the circle.

**Alternate Segment Theorem**
In a circle, the angle between a chord and a tangent through one of the end points of the chord is equal to the angle in the alternate segment.

**Range**
The range of a set of data is the difference between the largest value and the smallest value in the data set.

**Mean**

The mean of a set of data is the sum of all the data values divided by the number of data values.

**Estimate for the mean of a grouped frequency distribution**

Estimated mean = sum of (mid interval values multiplied by their frequency) divided by the sum of all the frequencies.

**Pie Chart**

In a pie chart, the total angle that corresponds to the entire data set is 360°

**Probability**

The sum of the probabilities of all outcomes equals 1

**Frequency density in histograms**

$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$


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