Rewarding Learning

General Certificate of Secondary Education

Centre Number

Candidate Number

Mathematics

StemUp Model Solution

Unit M4
(With calculator)
Higher Tier



[GMC41]
Assessment

GMC41

TIME

2 hours.

Assessment Level of Control:
Tick the relevant box (✓)

Controlled Conditions	
Other	

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all twenty-four** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

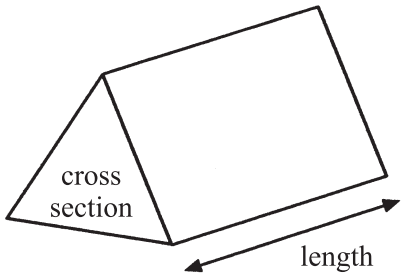
INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

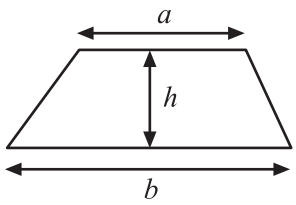
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Formula Sheet

Volume of prism = area of cross section × length

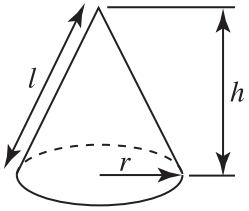


Area of trapezium = $\frac{1}{2}(a+b)h$



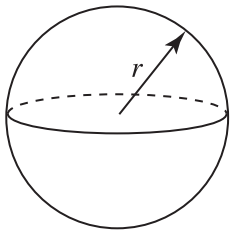
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = πrl

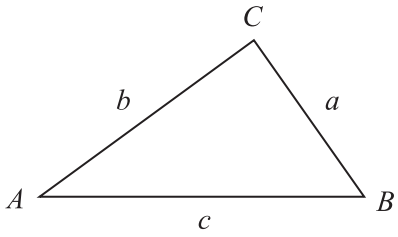


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

12371

1 Solve $2(3x - 1) + 5 = 4(x + 2)$

$2(3x - 1) + 5 = 4(x + 2)$

$6x - 2 + 5 = 4x + 8$

$6x + 3 = 4x + 8$

$6x - 4x = 8 - 3$

$2x = 5$

$x = 2.5$

Answer $x =$ 2.5 [3]

[Turn over

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2 Data on the weights of 16 players on a sports team is recorded in the grouped frequency table.

Weight (W kg)	Frequency	Midpoint x	fx
$60 < W \leq 70$	1	65	65
$70 < W \leq 80$	5	75	375
$80 < W \leq 90$	4	85	340
$90 < W \leq 100$	6	95	570

The manager states that "the estimated mean weight of the team lies within the median class".

Is his statement correct? **You must justify your answer fully.**

Median: 16 players, so median is between the 8th and 9th values

Cumulative: 1, 6, 10, 16, so the 8th and 9th lie in $80 < W \leq 90$

Median class = $80 < W \leq 90$

$$\text{Mean} = \frac{1 \times 65 + 5 \times 75 + 4 \times 85 + 6 \times 95}{16} = \frac{1350}{16} = 84.375 \text{ kg}$$

84.375 lies in the class $80 < W \leq 90$, which is the median class.

Yes, his statement is correct.

[4]

3 Which of these numbers is prime?

Explain your reasoning clearly for each number below.

Number	11	111	1111
Yes/No	Yes	No	No
Reason	Only factors are 1 and 11 (itself), exactly 2 factors	111 = 3 × 37 divisible by 3	1111 = 11 × 101 divisible by 11

[2]

[Turn over

4 A caravan depreciates in value by 15% each year.

Two years ago Malcolm bought a new caravan costing £24 000

(a) What is its value now?

After 1 year: 24 000 × 0.85 = £20 400

After 2 years: 20 400 × 0.85 = £17 340

Answer £ 17 340 [2]

(b) Malcolm states his caravan has depreciated by 30% over the two years.

Is he correct?

Explain your answer clearly.

Overall depreciation = 24 000 − 17 340 = £6660

Percentage depreciation = $\frac{6660}{24\,000} \times 100 = 27.75\%$

No, he is not correct. The caravan has depreciated

by 27.75%, not 30% (30% of £24 000 = £7200, more than £6660).

The 15% each year is taken off a smaller value in year 2.

[3]

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(Questions continue overleaf)

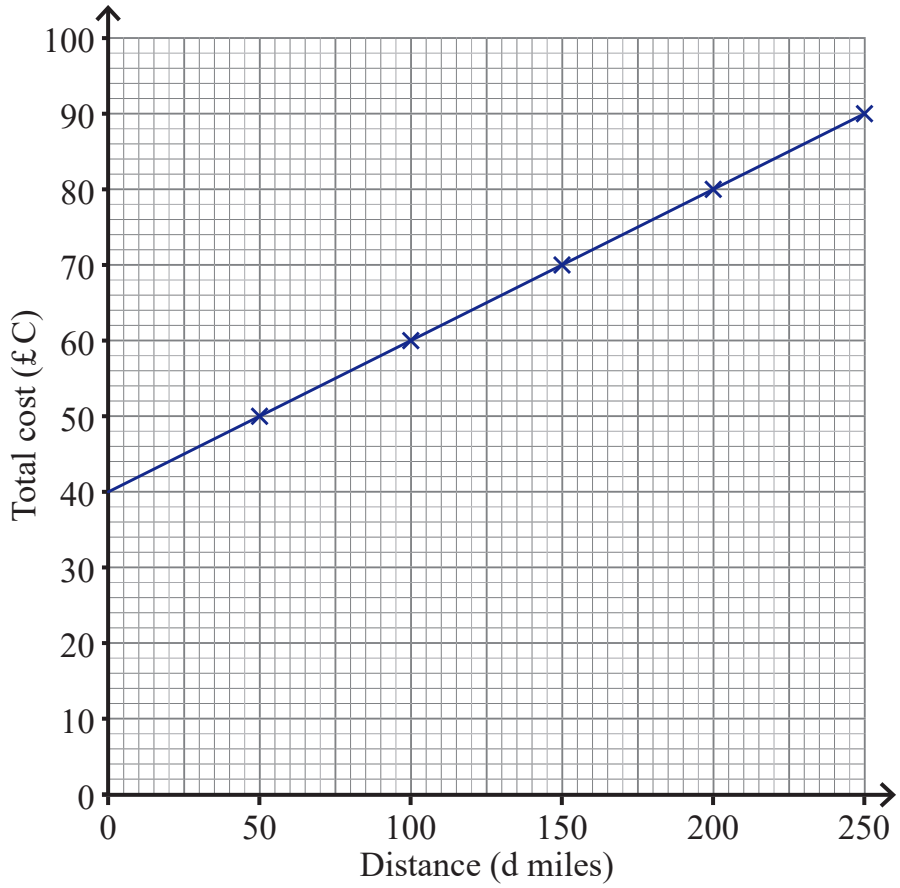
[Turn over

5 Martine wants to hire a van.

The table shows the costs for hiring the van.

Distance (d miles)	50	100	150	200	250
Total cost (£C)	50	60	70	80	90

(a) Draw a straight line graph to illustrate this information.



[2]

(b) Use the graph to find

(i) the initial fixed charge for hiring the van,

Answer £ 40 [1]

Line meets the C axis (d = 0) at 40

(ii) the cost per mile, in pence, for using the van.

Gradient = £10 per 50 miles = 1000p ÷ 50

Answer 20 p [1]

(c) Work out the total cost if the van travels 450 miles.

Cost = £40 + 450 × 20p

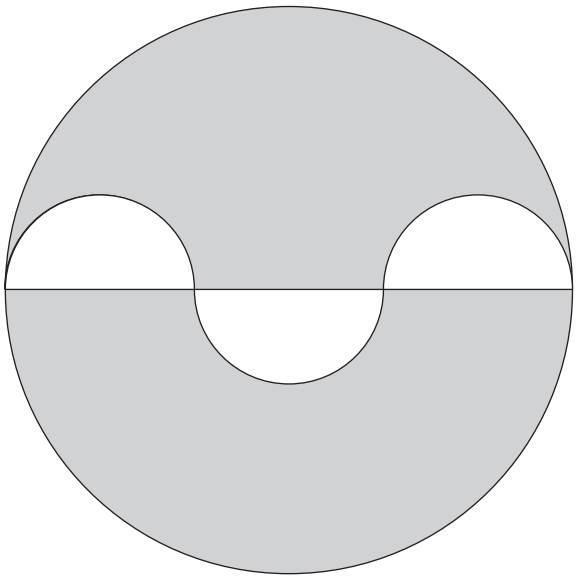
= 40 + 450 × 0.20

= 40 + 90 = £130

Answer £ 130 [2]

[Turn over

6



A large circle has three semicircles of equal diameters placed across its diameter as shown.

The radius of each of the small semicircles is 2 cm.

Work out the area shaded.

Diameter of large circle = 3 × 4 = 12 cm, so radius = 6 cm

Area of large circle = $\pi \times 6^2 = 113.0973355$

Area of each semicircle = $\frac{1}{2} \times \pi \times 2^2 = 6.283185307$

Shaded area = $113.0973355 - 3 \times 6.283185307$

$= 94.24777961...$

$= 94.25 \text{ cm}^2$

Answer 94.25 cm^2 [5]

7 Coffee is sold in 250 gram packets and costs £4.20 a packet.

Tea is sold in 450 gram packets and costs £3.60 a packet.

Helen runs a café and buys the same number of grams of coffee and tea.

What is the least amount of money she could have spent?

250 = 2 × 5 × 5 × 5

450 = 2 × 3 × 3 × 5 × 5

LCM = 2 × 3 × 3 × 5 × 5 × 5 = 2250 g

Coffee: 2250 ÷ 250 = 9 packets

Tea: 2250 ÷ 450 = 5 packets

Cost = 9 × 4.20 + 5 × 3.60

= 37.80 + 18.00

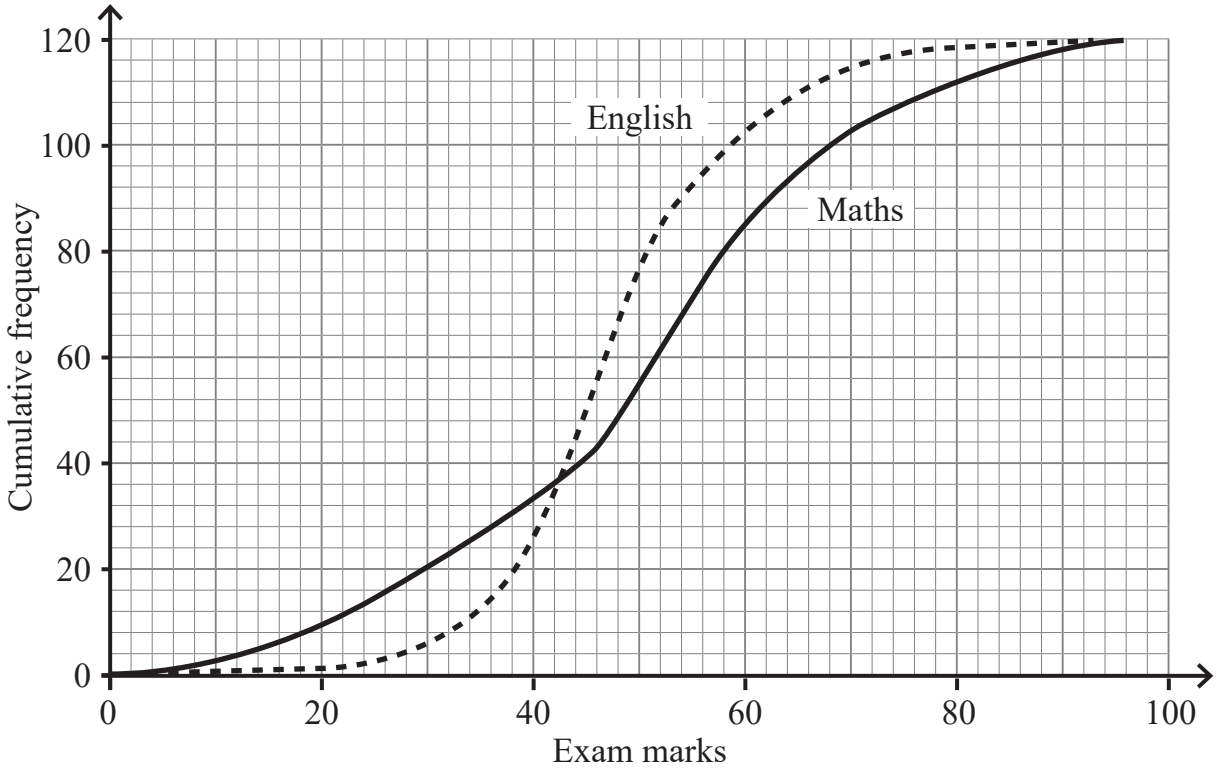
= £55.80

Answer £ 55.80 [5]

[Turn over

8 120 students sat a Maths exam and an English exam.

The cumulative frequency curves show the distribution of results.



With reference to three different statistical measures compare the two sets of results.

The median is higher in Maths (about 53) than in English (about 47). [1]

There is a similar range of marks in both subjects (about 0 to 95). [1]

The interquartile range is greater in Maths (about 26) than English (about 9). [1]

9 Calculate the perimeter of the isosceles triangle.

You must show all your working.

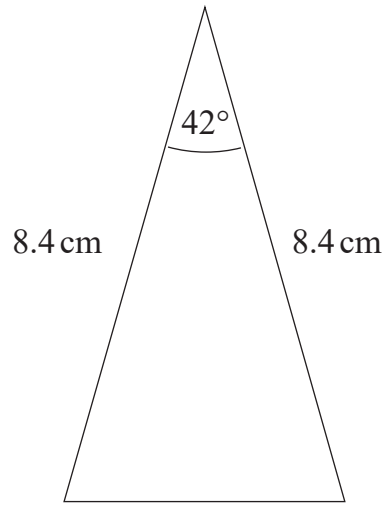


diagram not drawn accurately

Split into two right-angled triangles: 42° ÷ 2 = 21°

$\sin 21° = \frac{x}{8.4}$ (x = half the base)

$x = 8.4 \times \sin 21°$

$x = 3.010290776$

base = 2 × 3.010290776 = 6.020581552

Perimeter = 8.4 + 8.4 + 6.020581552

= 22.82058155...

= 22.82 cm

Answer 22.82 cm [4]

[Turn over

10 A guitar is on sale in a shop.

The sale price is £120.96 after a 28% reduction.

What was the price of the guitar before the sale?

100% − 28% = 72%

72% = £120.96

120.96 ÷ 72 × 100 = £168

Answer £ 168 [2]

11 An elephant puts a total force of 33 000 N on its four feet.

Each foot has an area of 1500 cm^2

What is the pressure the elephant exerts on the ground?

Total area = 4 × 1500 = 6000 cm^2

$P = \frac{F}{A} = \frac{33\,000}{6000} = 5.5$

Answer 5.5 N/cm^2 [2]

12 Find the area of the shaded sector of this circle, centre O and radius 3 cm.

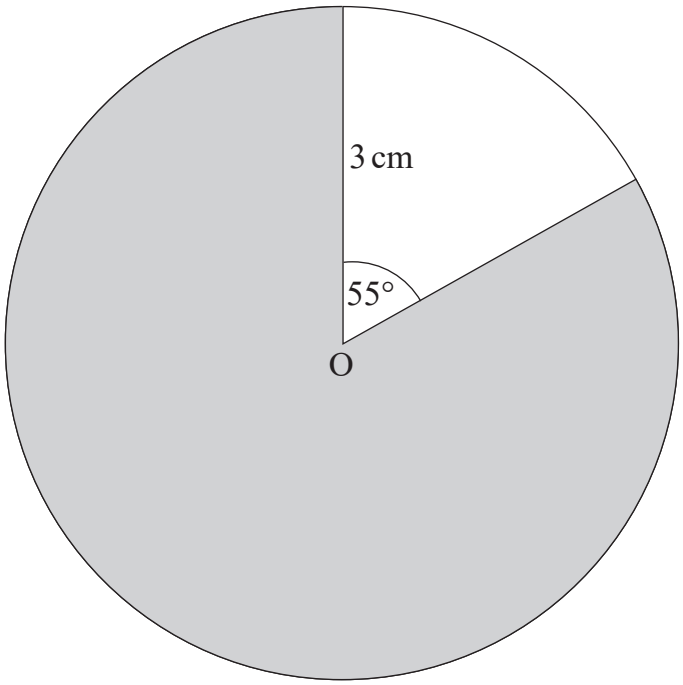


diagram
not drawn
accurately

Shaded angle = 360° − 55° = 305°

$A = \frac{305}{360} \times \pi \times 3^2$

= 23.95464398...

= 23.95 cm^2

Answer 23.95 cm^2 [2]

[Turn over

13 Solve the equation $\frac{2(3x+2)}{5} - \frac{(3x-4)}{3} = \frac{2}{15}$

Show all your working out clearly.

A solution by trial and improvement will not be accepted.

Multiply every term by 15:

$6(3x + 2) - 5(3x - 4) = 2$

$18x + 12 - 15x + 20 = 2$

$3x + 32 = 2$

$3x = 2 - 32$

$3x = -30$

$x = -10$

Answer $x =$ -10 [4]

14 Martin runs 200 metres, correct to the nearest metre.

It takes him 26.4 seconds, correct to the nearest tenth of a second.

Calculate the upper bound of Martin's average speed.

Upper bound of distance = 200.5 m

Lower bound of time = 26.35 s

Upper bound of speed = UB distance ÷ LB time

$$\text{Speed} = \frac{200.5}{26.35} = 7.609108159...$$

Answer 7.609108159 m/s [3]

[Turn over

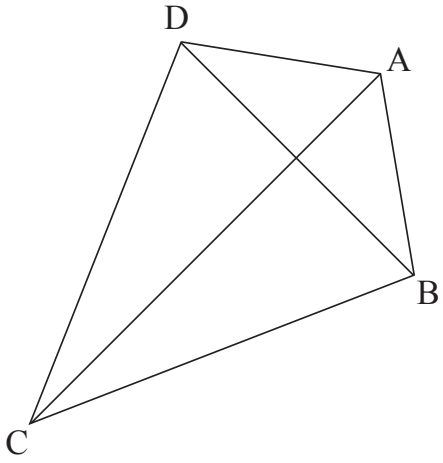


15 The lines AC and BD are diagonals of a kite.

The line AC has equation $y = 3x + 2$

The diagonals meet at (1, 5).

Find the equation of the line BD.



Gradient of AC = 3

Diagonals of a kite are perpendicular, so

gradient of BD = $-\frac{1}{3}$ $\left(3 \times -\frac{1}{3} = -1\right)$

$y = -\frac{1}{3}x + c$ through (1, 5)

$5 = -\frac{1}{3} \times 1 + c$

$c = 5 + \frac{1}{3} = 5\frac{1}{3}$

Answer $y = -\frac{1}{3}x + 5\frac{1}{3}$ [4]

16 In a right-angled triangle the two shorter sides have lengths of $(3x + 3)$ cm and $(2x - 1)$ cm.

The hypotenuse has length $(5x - 2)$ cm.

Form an equation and solve it to find x.

Show all your working.
A solution by trial and improvement will not be accepted.

$(3x + 3)^2 + (2x - 1)^2 = (5x - 2)^2$

$9x^2 + 18x + 9 + 4x^2 - 4x + 1 = 25x^2 - 20x + 4$

$13x^2 + 14x + 10 = 25x^2 - 20x + 4$

$0 = 12x^2 - 34x - 6$

$(12x + 2)(x - 3) = 0$

$x = -\frac{1}{6}$ or $x = 3$

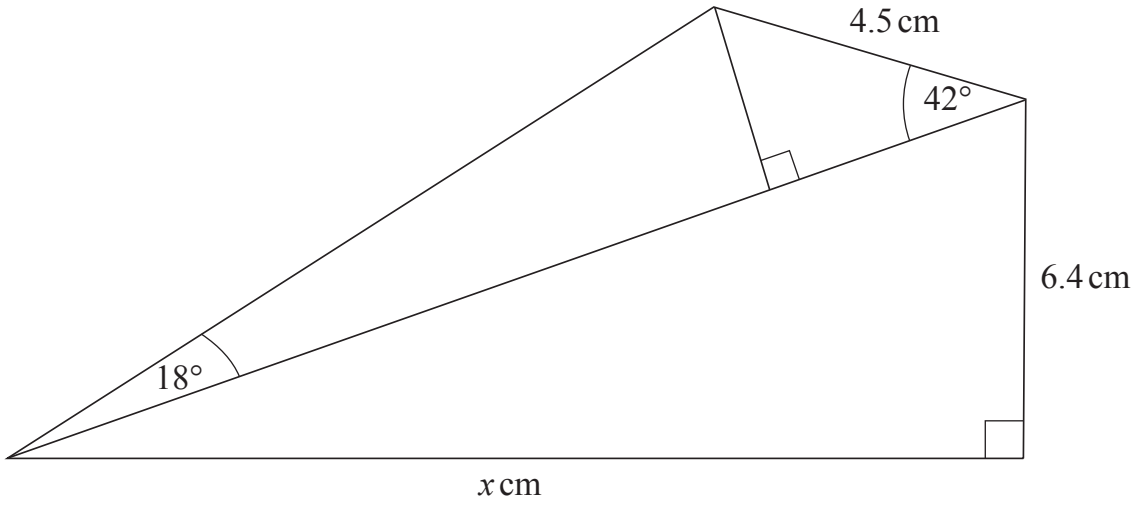
$x = -1/6$ gives negative lengths (e.g. $2x - 1 < 0$),

so $x = 3$

Answer $x =$ 3 [4]

[Turn over

17 Find the value of x in the diagram below.



Top triangle (hypotenuse 4.5 cm):

$\sin 42° = \frac{a}{4.5}$ $a = 4.5 \sin 42° = 3.011087729$

$\cos 42° = \frac{b}{4.5}$ $b = 4.5 \cos 42° = 3.344151715$

$\tan 18° = \frac{3.011087729}{c}$ $c = 3.011087729 \div \tan 18° = 9.267175131$

Hypotenuse of bottom triangle = b + c

$= 3.344151715 + 9.267175131 = 12.61132685$

$x^2 = 12.61132685^2 - 6.4^2 = 118.0855630$

$x = \sqrt{118.0855630} = 10.86671822...$

$x = 10.9$ cm

Answer $x =$ 10.9 [5]

18 Rob records the number of miles he drives each day for one week.

His results are given in the table below:

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Distance (miles)	16	26	24	20	18	14	88

Which average is the most appropriate to use to represent this data?

Give a reason for your answer.

Answer Median because it is not affected by the extreme value of 88 on Sunday, which would make the mean too high (and there is no mode). [2]

[Turn over

19 A, B, C and D are points on the circumference of a circle with centre M.

BD is the diameter of the circle.

Angle BAC = 38°

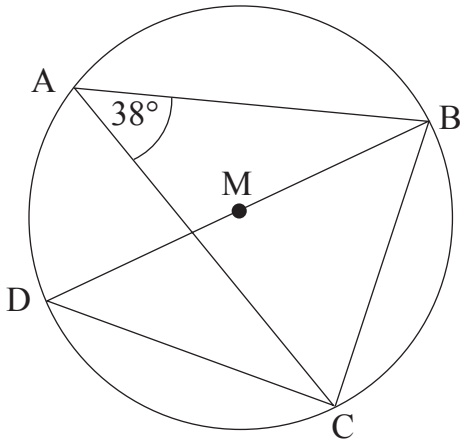


diagram not drawn accurately

(a) Find the size of angle BDC, giving a reason for your answer.

Answer 38 ° because angles in the same segment are equal [2]

(b) Find the size of angle BCD, giving a reason for your answer.

Answer 90 ° because the angle in a semicircle is 90° [2]

(c) Find the size of angle BMC, giving a reason for your answer.

Answer 76 ° because the angle at the centre is twice the angle at the circumference (2 × 38° = 76°) [2]

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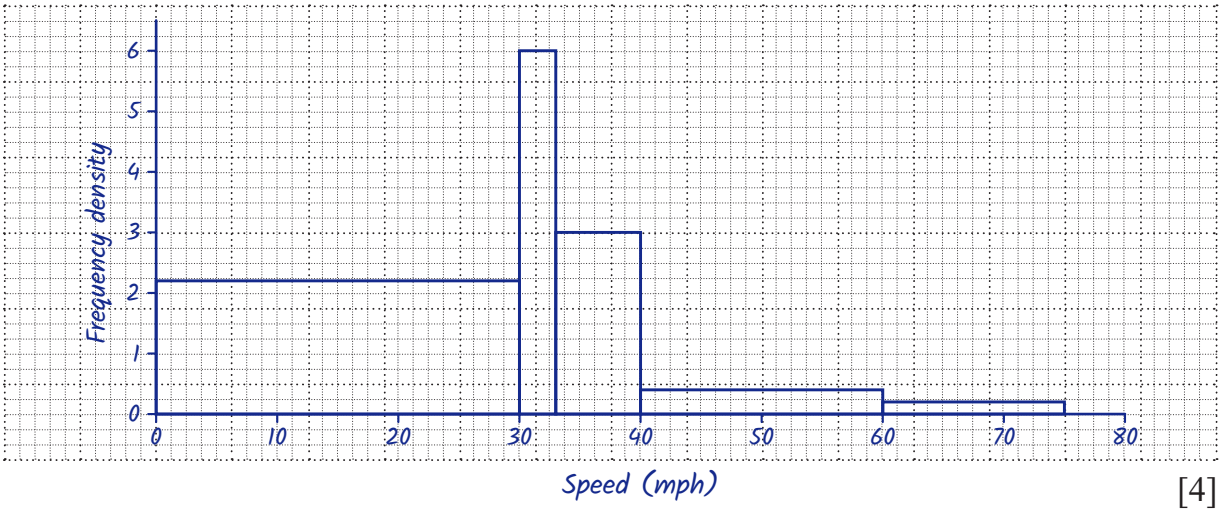
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(Questions continue overleaf)

20 A speed camera in a 30 mph speed zone records the following results over a two-hour period:

Speed(s) in mph	Number of cars	
$0 < s \leq 30$	66	$fd = f \div width$
$30 < s \leq 33$	18	$66 \div 30 = 2.2$
$33 < s \leq 40$	21	$18 \div 3 = 6$
$40 < s \leq 60$	8	$21 \div 7 = 3$
$60 < s \leq 75$	3	$8 \div 20 = 0.4$
		$3 \div 15 = 0.2$

(a) Illustrate this data by drawing a histogram on the grid below.



[4]

(b) A stratified sample of 50 cars was taken from the data on the previous page.

Estimate how many cars sampled had a speed between 33 and 40 mph.

Total = 66 + 18 + 21 + 8 + 3 = 116

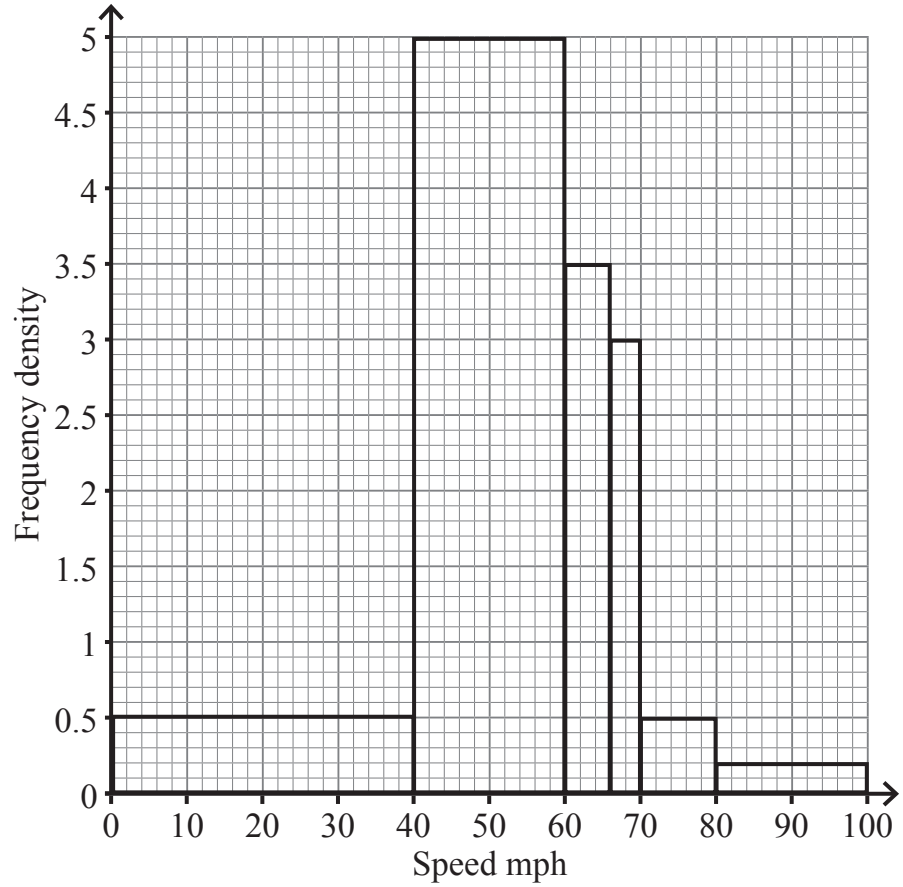
$\frac{21}{116} \times 50 = 9.05...$

Answer 9 [2]

[Turn over



(c) A speed camera in a 60 mph speed zone records the speeds of cars and the results are shown in the histogram below.



Calculate an estimate for the mean speed over the two-hour period for this data.

$f = fd \times \text{width}$, $x = \text{midpoint}$

Speed	f	x	fx
$0 < s \leq 40$	$0.5 \times 40 = 20$	20	400
$40 < s \leq 60$	$5 \times 20 = 100$	50	5000
$60 < s \leq 66$	$3.5 \times 6 = 21$	63	1323
$66 < s \leq 70$	$3 \times 4 = 12$	68	816
$70 < s \leq 80$	$0.5 \times 10 = 5$	75	375
$80 < s \leq 100$	$0.2 \times 20 = 4$	90	360

$\Sigma f = 20 + 100 + 21 + 12 + 5 + 4 = 162$ $\quad \Sigma fx = 8274$

$$\text{Mean} = \frac{8274}{162} = 51.07407\ldots = 51.1 \text{ mph}$$

Answer 51.1 mph [4]

(d) Anyone driving at a speed that is greater than 10% above the speed limit is given a fine.

Compare the percentage of fines given in the two speed zones.

Fined if speed > 10% above the limit:

30 mph zone (over 33 mph): $21 + 8 + 3 = 32$, $\frac{32}{116} \times 100 = 27.6\%$

60 mph zone (over 66 mph): $12 + 5 + 4 = 21$, $\frac{21}{162} \times 100 = 13.0\%$

So over twice the percentage of drivers were fined in the 30 mph zone (27.6%) as in the 60 mph zone (13.0%).

[2]

21 Factorise $15x^2 + 2xy - 8y^2$

$15 \times (-8) = -120$: $12 \times (-10) = -120$, $12 - 10 = 2$

$15x^2 + 12xy - 10xy - 8y^2$

$= 3x(5x + 4y) - 2y(5x + 4y)$

$= (3x - 2y)(5x + 4y)$

Answer $(3x - 2y)(5x + 4y)$ [2]

[Turn over

22 Solve the equation

$$\frac{3}{2x+5} = \frac{1}{11} + \frac{2}{4x-1}$$

A solution by trial and improvement will not be accepted.

Multiply every term by $11(2x + 5)(4x - 1)$:

$33(4x - 1) = (2x + 5)(4x - 1) + 22(2x + 5)$

$132x - 33 = 8x^2 - 2x + 20x - 5 + 44x + 110$

$132x - 33 = 8x^2 + 62x + 105$

$0 = 8x^2 - 70x + 138$

$0 = 2(4x^2 - 35x + 69)$

$0 = 2(4x - 23)(x - 3)$

$4x - 23 = 0$ or $x - 3 = 0$

$x = 5.75$ or $x = 3$

Answer $x =$ 5.75 or 3 [6]



23 Simplify $\dfrac{8x^2 - 18}{6x^2 + 7x - 3}$

$= \dfrac{2(4x^2 - 9)}{(2x + 3)(3x - 1)}$

$= \dfrac{2(2x + 3)(2x - 3)}{(2x + 3)(3x - 1)}$

$= \dfrac{2(2x - 3)}{3x - 1}$

Answer $\dfrac{2(2x - 3)}{3x - 1}$ [3]

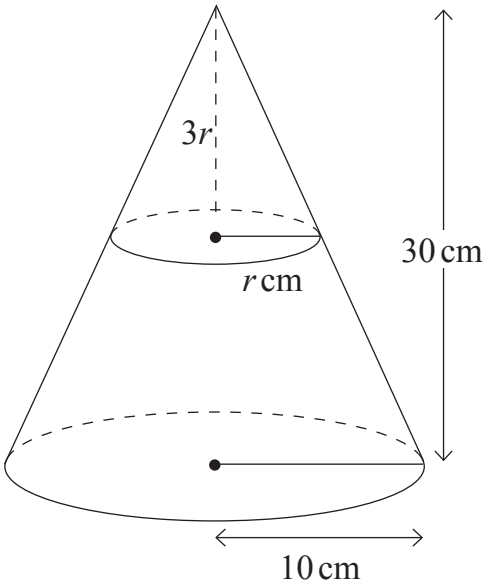
[Turn over

24 A cone of radius r cm and height $3r$ cm is removed from a cone of radius 10 cm and height 30 cm to give a frustum.

The volume of the frustum is 2855 cm^3

Calculate the value of r.

Show all your working.



Large cone: $V = \frac{1}{3} \times \pi \times 10^2 \times 30 = 3141.592654$

Small cone: $V = \frac{1}{3} \times \pi \times r^2 \times 3r = \pi r^3$ $(= 3.141592654r^3)$

Frustum = large cone − small cone

$3141.592654 - 3.141592654r^3 = 2855$

$3.141592654r^3 = 286.5926536$

$r^3 = 91.22527495$

$r = \sqrt[3]{91.22527495} = 4.50165...$

$r = 4.5$

Answer 4.5 [6]

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Examiner Number

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