Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2019

Mathematics

StemUp Model Solution

Unit M4
(With calculator)
Higher Tier



[GMC41]

GMC41

TUESDAY 21 MAY, 9.15am–11.15am

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

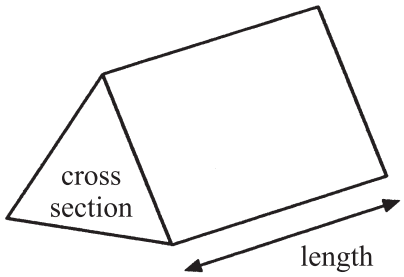
Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all twenty-two** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

INFORMATION FOR CANDIDATES

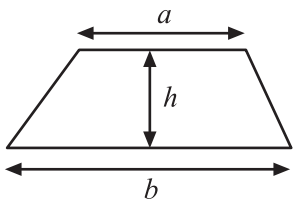
The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

Formula Sheet

Volume of prism = area of cross section × length

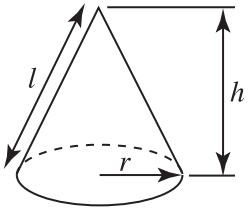


Area of trapezium = $\frac{1}{2}(a+b)h$



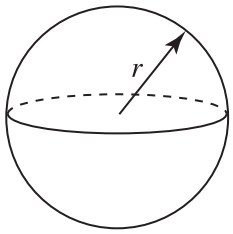
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

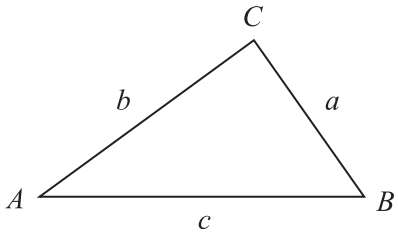


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

1 The waiting times for patients at a surgery are recorded in the table.

Waiting time t (minutes)	Number of patients
$0 < t \leq 5$	7
$5 < t \leq 10$	8
$10 < t \leq 15$	5
$15 < t \leq 20$	5
$20 < t \leq 25$	4
$25 < t \leq 30$	1

Calculate an estimate of the mean waiting time.

$fx = 2.5 \times 7 + 7.5 \times 8 + 12.5 \times 5 + 17.5 \times 5 + 22.5 \times 4 + 27.5 \times 1$

$= 17.5 + 60 + 62.5 + 87.5 + 90 + 27.5 = 345$

$\Sigma f = 7 + 8 + 5 + 5 + 4 + 1 = 30$

$mean = 345 \div 30 = 11.5$

Answer 11.5 minutes [4]

2 Expand and simplify

$$4(2x - 3) - 2(x - 5)$$

$4(2x - 3) - 2(x - 5)$

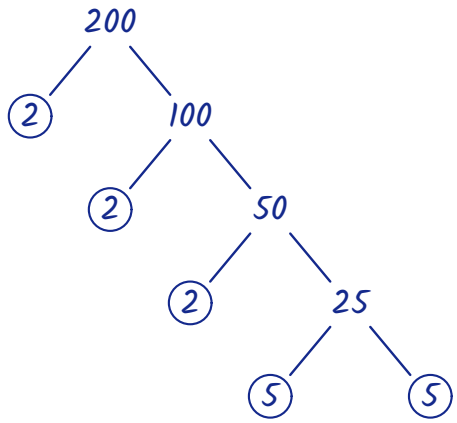
$= 8x - 12 - 2x + 10$

$= 6x - 2$

Answer $6x - 2$ [2]

[Turn over

3 Write 200 as a product of prime factors, using index notation.



$200 = 2 \times 2 \times 2 \times 5 \times 5$

Answer $2^3 \times 5^2$ [3]

4

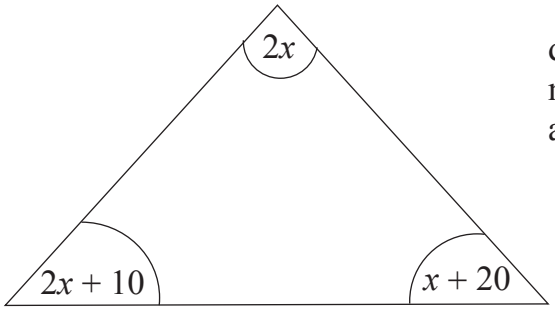


diagram
not drawn
accurately

Form and solve an equation to work out the size of the smallest angle in the triangle above.

Equation 2x + 10 + 2x + x + 20 = 180 [1]

5x + 30 = 180
5x = 150
x = 30
angles: 2x = 60°, 2x + 10 = 70°, x + 20 = 50°
smallest angle = 50°

Answer smallest angle = 50 ° [3]

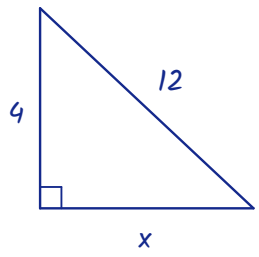
[Turn over

5 The longest side in a right-angled triangle is 12 cm.

One of the shorter sides is 4 cm.

Calculate the perimeter of the triangle.

Give your answer correct to 1 decimal place.



$4^2 + x^2 = 12^2$

$x^2 = 144 - 16 = 128$

$x = \sqrt{128} = 11.3137...$

Perimeter $= 4 + 12 + 11.3137...$

$= 27.3137...$

$= 27.3$ cm (1 d.p.)

Answer 27.3 cm [5]

6 **(a)** The price of a TV is increased by 20%.

In a sale this price is decreased by 20%.

By choosing any starting price for the TV, show that the final sale price is lower than the starting price.

Starting price £100
20% increase: 120% of £100 = 1.2 × 100 = £120
20% decrease: 80% of £120 = 0.8 × 120 = £96
£96 < £100, so the final sale price is lower than the starting price.

[3]

(b) Calculate the overall percentage decrease.

decrease = 100 − 96 = £4
4/100 × 100 = 4%

Answer 4 % [2]

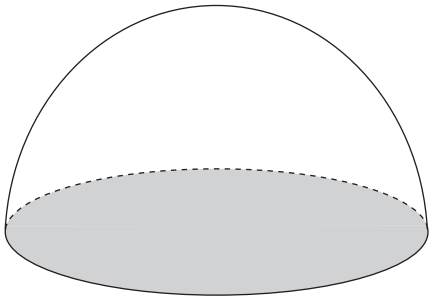
(c) Would the outcome be the same if the 20% decrease was applied first, followed by the 20% increase? Justify your answer.

Yes, it is the same.
£100 × 0.8 = £80, then £80 × 1.2 = £96
0.8 × 1.2 = 1.2 × 0.8 = 0.96 (multiplication can be done in either order), so it is still a 4% decrease to £96.

[2]

[Turn over

7 The **solid** hemisphere has a diameter of 12 cm.



Mary says the total surface area is 226 cm^2 to the nearest cm^2

Martha says the total surface area is 339 cm^2 to the nearest cm^2

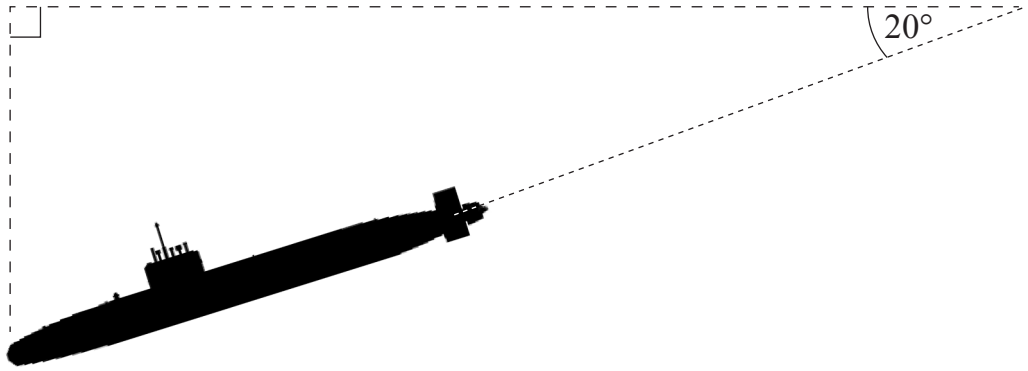
Explain with reasoning who is correct.

radius = 12 ÷ 2 = 6 cm
SA of whole sphere = 4 × π × 6^2 = 452.389...
curved SA of hemisphere = 452.389... ÷ 2 = 226.194...
base (circle) = π × 6^2 = 113.097...
total SA = 226.194... + 113.097... = 339.29... = 339 cm^2
Mary has only found the curved surface. The solid hemisphere also has a flat circular base, so the total is 339 cm^2.

Answer Martha is correct [4]

8 A submarine makes a diving angle of 20° below the horizontal as shown.
It travels at a constant speed of 12 m/s.

Work out how deep the front end of the submarine is after one minute.



distance in 1 minute = 12 × 60 = 720 m
sin 20° = d ÷ 720
d = 720 × sin 20°
d = 246.2545...
depth = 246.25 m

Answer 246.25 m [4]

9 Solve the equation $x^2 - x - 12 = 0$

$x^2 - x - 12 = 0$
$(x - 4)(x + 3) = 0$
$x = 4$ or $x = -3$

Answer $x = 4,\ x = -3$ [3]

[Turn over

10 Solve

$$\frac{a-1}{4} + \frac{a+1}{8} = \frac{3}{2}$$

Give your answer as a mixed number.

$8 \times \frac{a-1}{4} + 8 \times \frac{a+1}{8} = 8 \times \frac{3}{2}$

$2(a-1) + (a+1) = 12$
$2a - 2 + a + 1 = 12$
$3a - 1 = 12$
$3a = 13$

$a = \frac{13}{3} = 4\frac{1}{3}$

Answer $a =$ $4\frac{1}{3}$ [4]

11 Write down the equation of a line parallel to the line with equation $y = 3x + 5$

parallel lines have the same gradient, m = 3

Answer $y = 3x + 1$ [2]

12 After a 7.5% pay rise Mr Jones' salary was £29 455

What was his salary before the pay rise?

107.5% = £29 455
100% = 29 455 ÷ 1.075
= £27 400

Answer £ 27 400 [3]

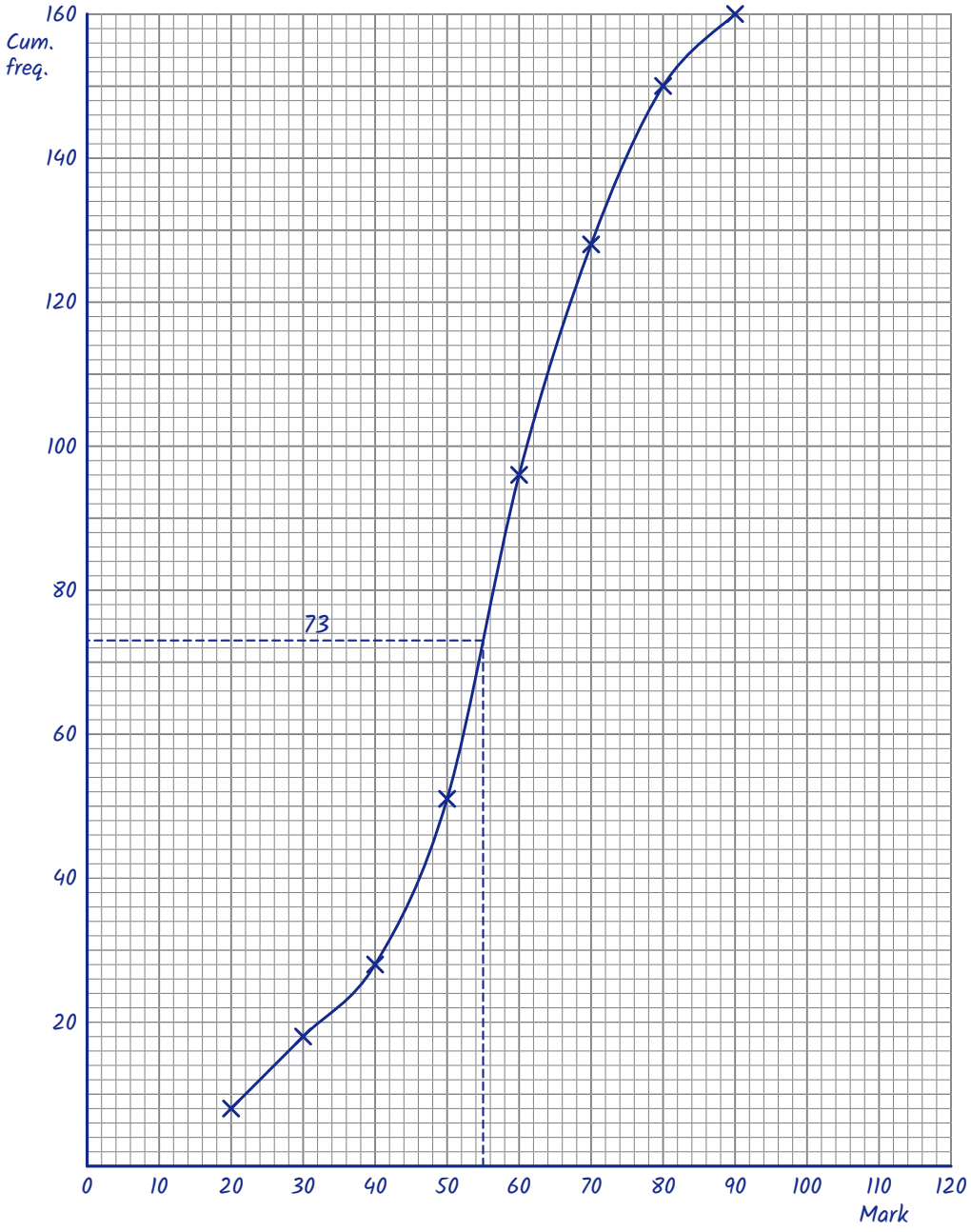
[Turn over

13 160 pupils in Year 8 sat a Science examination at the end of the year.

Their results are given in the cumulative frequency table below.

Examination Mark, x	**Cumulative Frequency**
$x \leq 20$	8
$x \leq 30$	18
$x \leq 40$	28
$x \leq 50$	51
$x \leq 60$	96
$x \leq 70$	128
$x \leq 80$	150
$x \leq 90$	160

(a) On the graph paper below draw a cumulative frequency graph for the data given.



[3]

(b) The pass mark for this examination was 55

Use your graph to estimate the number of pupils who passed the examination.

reading at 55: 73 pupils, 160 − 73 = 87

Answer 87 [2]

[Turn over

14 Stephen wants to survey 50 pupils in his school.

The number of pupils in each year group is given in the table below.

Year 8	Year 9	Year 10	Year 11	Year 12
126	161	154	145	170

For a stratified sample, how many pupils should Stephen include from Year 8?

Show your working out.

Total = 126 + 161 + 154 + 145 + 170 = 756

Year 8: $\frac{126}{756} \times 50 = 8.333... \approx 8$

Answer 8 [2]

15 $a = 3.2$ and $b = 5.8$ are both correct to 1 decimal place.

Find

(a) the minimum possible value of $b - a$,

$\min b - a = 5.75 - 3.25 = 2.5$

Answer 2.5 [1]

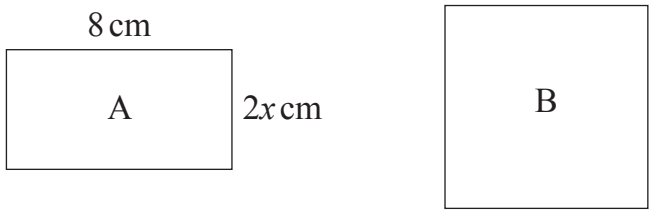
(b) the maximum possible value of $\frac{b}{a}$

$\max \frac{b}{a} = \frac{\max b}{\min a} = \frac{5.85}{3.15} = 1.857142857...$

Answer 1.857142857... [2]

[Turn over

16 A is a rectangle of length 8 cm and width $2x$ cm, and B is a square.



The perimeters of the rectangle and the square are equal.

(a) Write down an expression in terms of x for the length of the side of the square B.

perimeter of A = 8 + 8 + 2x + 2x = 16 + 4x
side of square = (16 + 4x) ÷ 4 = 4 + x

Answer 4 + x [2]

The area of the square is 4 cm^2 more than the area of the rectangle.

(b) (i) Write down an equation satisfied by x and show that it simplifies to

$$x^2 - 8x + 12 = 0$$

area of A = 8 × 2x = 16x, area of B = $(4 + x)^2$
$(4 + x)^2 = 16x + 4$
$16 + 8x + x^2 = 16x + 4$
$x^2 + 8x - 16x + 16 - 4 = 0$
$x^2 - 8x + 12 = 0$

[3]

(ii) Solve this equation, giving the two possible values of x.

$x^2 - 8x + 12 = 0$

$(x - 2)(x - 6) = 0$

$x = 2$ or $x = 6$

Answer $x = 2,\ x = 6$ [2]

[Turn over



17 The line l_1 passes through the points $(-1, -4)$ and $(2, 8)$.

The line l_2 is perpendicular to l_1 and passes through the point $(1, 1)$.

Find the equation of the line l_2 in the form $y = mx + c$.

gradient of $l_1 = \frac{8-(-4)}{2-(-1)} = \frac{12}{3} = 4$

gradient of $l_2 = -\frac{1}{4}$ (since $4 \times -\frac{1}{4} = -1$)

$y = -\frac{1}{4}x + c$

through $(1, 1)$: $1 = -\frac{1}{4}(1) + c$

$c = 1 + \frac{1}{4} = \frac{5}{4}$

$y = -\frac{1}{4}x + \frac{5}{4}$

Answer $y = -\frac{1}{4}x + \frac{5}{4}$ [5]

18 The diagram shows a sector AOB of a circle, with radius 13 cm and centre O.

The point C lies on OB and angle ACO is 90°

OC = 5 cm.

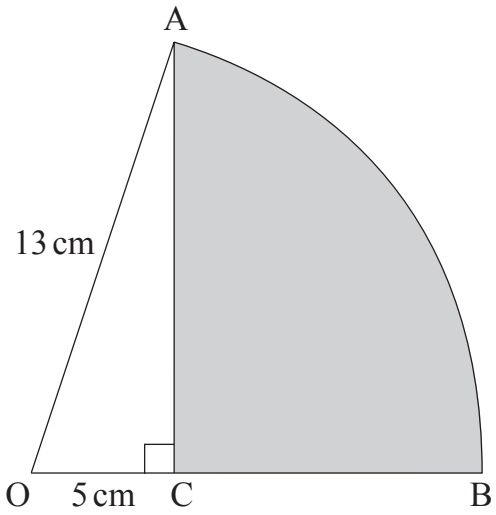


diagram not drawn accurately

Find the area of the shaded section ABC.

$\cos AOC = \frac{5}{13}$

angle $AOC = \cos^{-1}(5 \div 13) = 67.38°$

sector $AOB = \frac{67.38}{360} \times \pi \times 13^2 = 99.37\ldots \text{ cm}^2$

$AC^2 = 13^2 - 5^2 = 169 - 25 = 144$

$AC = 12 \text{ cm}$

triangle $OAC = \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$

shaded area $ABC = 99.37\ldots - 30$

$= 69.37\ldots = 69.4 \text{ cm}^2$

Answer 69.4 cm^2 [8]

[Turn over

19 Solve the equation

$$\frac{4}{x+3} - \frac{3}{x+4} = 1$$

$\frac{4(x+4) - 3(x+3)}{(x+3)(x+4)} = 1$

$\frac{4x + 16 - 3x - 9}{x^2 + 7x + 12} = 1$

$\frac{x + 7}{x^2 + 7x + 12} = 1$

$x + 7 = x^2 + 7x + 12$

$x^2 + 6x + 5 = 0$

$(x + 1)(x + 5) = 0$

$x = -1$ or $x = -5$

Answer $x = -1,\ x = -5$ [6]

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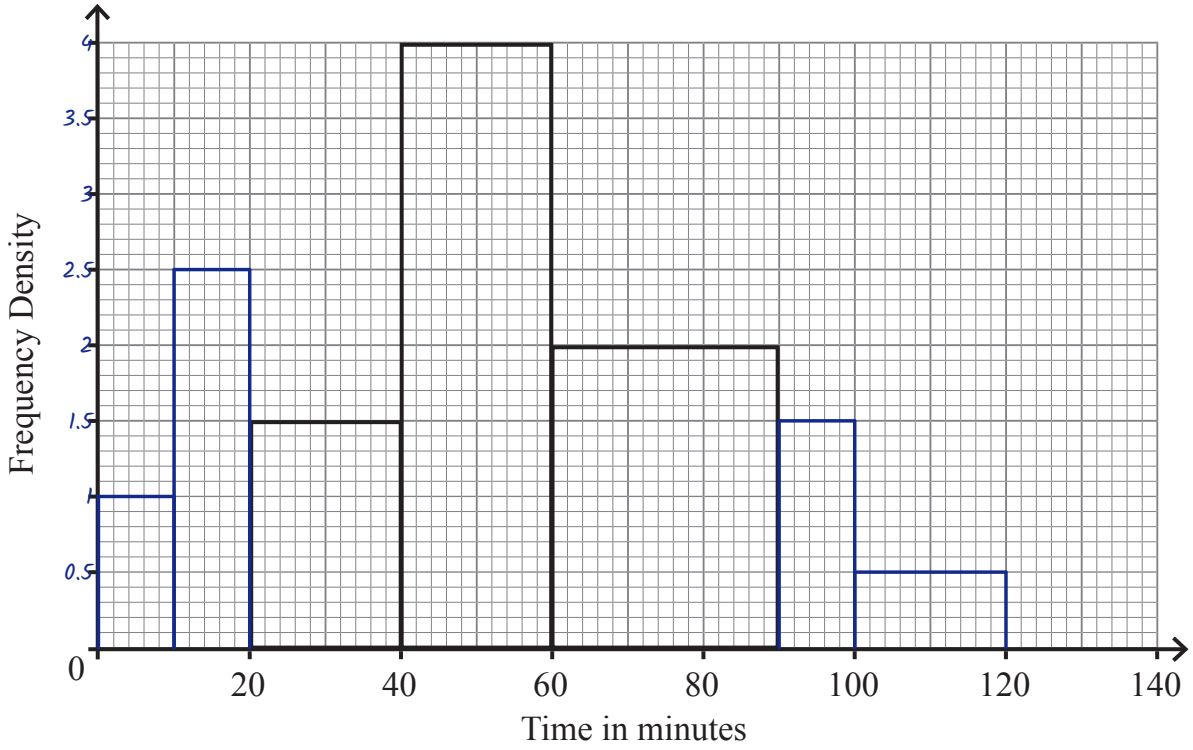
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20 The table and histogram show information about the length of time 230 pupils spent on social media on a week night.

No pupil spent more than 120 minutes on social media on a week night.

Length of time in minutes, *m*	**Frequency**
$0 < m \leq 10$	10
$10 < m \leq 20$	25
$20 < m \leq 40$	30
$40 < m \leq 60$	80
$60 < m \leq 90$	60
$90 < m \leq 100$	15
$100 < m \leq 120$	10

From the histogram, $20 < m \leq 40$ bar has frequency density 1.5:
frequency = $1.5 \times 20 = 30$
$90 < m \leq 100$: $230 - (10 + 25 + 30 + 80 + 60 + 10) = 230 - 215 = 15$
Frequency densities: 0 to 10: $10 \div 10 = 1$, 10 to 20: $25 \div 10 = 2.5$
90 to 100: $15 \div 10 = 1.5$, 100 to 120: $10 \div 20 = 0.5$



(a) Complete the table and the histogram. [6]

(b) Use the histogram to estimate the median time spent on social media.

median = 230 ÷ 2 = 115th value

up to 40 minutes: 10 + 25 + 30 = 65

need 115 − 65 = 50 more from the 40 to 60 class (80 pupils)

$$\text{median} = 40 + \frac{50}{80} \times 20 = 40 + 12.5 = 52.5$$

Answer 52.5 minutes [2]

[Turn over

21

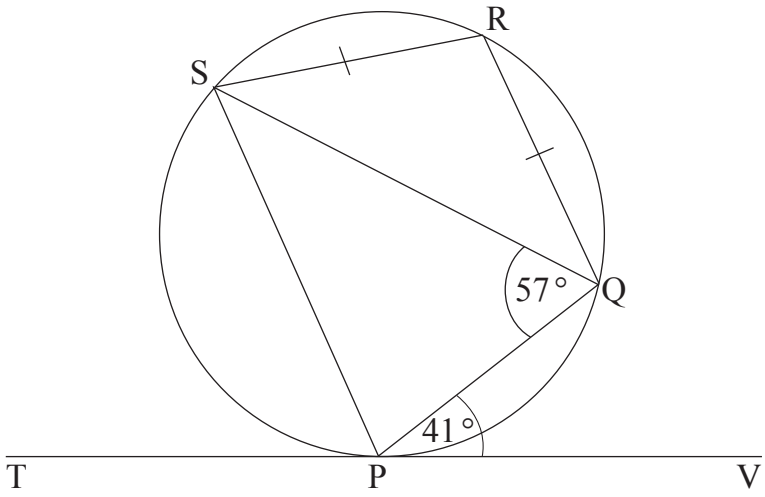


diagram
not drawn
accurately

TV is a tangent to the circle at P.
SR = RQ
Angle QPV = 41° and angle SQP = 57°
Show that SP is parallel to RQ.
You must give reasons to justify any angles that you calculate.

Angle PSQ = 41° (alternate segment theorem)
Angle SPQ = 180 – 57 – 41 = 82° (angles in a triangle add to 180°)
Angle SRQ = 180 – 82 = 98° (opposite angles in a cyclic quadrilateral add to 180°)
Triangle SRQ is isosceles as SR = RQ, so
Angle SQR = (180 – 98) ÷ 2 = 41° (base angles of isosceles triangle)
Angle PSQ = Angle SQR = 41°, and these are alternate angles,
so SP is parallel to RQ.

[5]

22 **(a)** Factorise $2a^2 + 7ab - 4b^2$

$2a^2 + 7ab - 4b^2$
$= 2a^2 + 8ab - ab - 4b^2$
$= 2a(a + 4b) - b(a + 4b)$
$= (2a - b)(a + 4b)$

Answer $(2a - b)(a + 4b)$ [2]

(b) Simplify the following

$$\left(\frac{x+1}{2x-1} + \frac{3x-4}{x-4}\right) \times \frac{2x-1}{x}$$

$= \dfrac{(x + 1)(x - 4) + (3x - 4)(2x - 1)}{(2x - 1)(x - 4)} \times \dfrac{2x - 1}{x}$

$= \dfrac{x^2 - 3x - 4 + 6x^2 - 11x + 4}{(2x - 1)(x - 4)} \times \dfrac{2x - 1}{x}$

$= \dfrac{7x^2 - 14x}{(2x - 1)(x - 4)} \times \dfrac{2x - 1}{x}$

$= \dfrac{7x(x - 2)}{x - 4} \times \dfrac{1}{x}$

$= \dfrac{7(x - 2)}{x - 4} = \dfrac{7x - 14}{x - 4}$

Answer $\dfrac{7(x - 2)}{x - 4}$ [4]

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