New Specification

Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2018

# Mathematics

StemUp Model Solution

Unit M4
**(With calculator)**
Higher Tier



**[GMC41]**

*GMC41*

**THURSDAY 24 MAY, 9.15am - 11.15am**

**TIME**

2 hours.

**INSTRUCTIONS TO CANDIDATES**

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
**You must answer the questions in the spaces provided.**
**Do not write outside the boxed area on each page or on blank pages.**
Complete in black ink only. **Do not write with a gel pen.**
All working should be clearly shown since marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all twenty-three** questions.

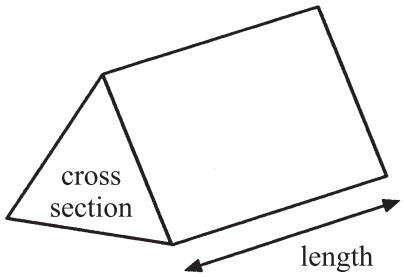
**INFORMATION FOR CANDIDATES**

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

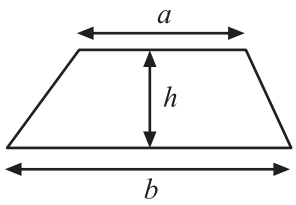
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# Formula Sheet

**Volume of prism** = area of cross section × length

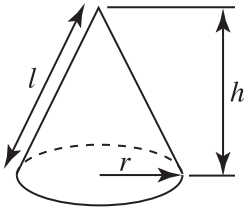



**Area of trapezium** = $\frac{1}{2}(a+b)h$



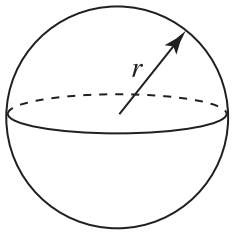

**Volume of cone** = $\frac{1}{3}\pi r^2 h$

**Curved surface area of cone** = $\pi rl$

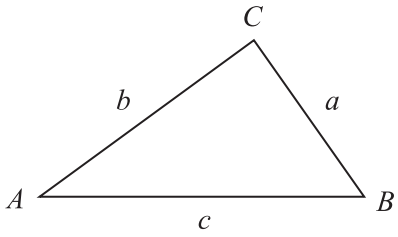



**Volume of sphere** = $\frac{4}{3}\pi r^3$

**Surface area of sphere** = $4\pi r^2$




**In any triangle *ABC***




**Quadratic Equation**

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Sine Rule:** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine Rule:** $a^2 = b^2 + c^2 - 2bc\cos A$

**Area of triangle** = $\frac{1}{2}ab\sin C$

11725

**1** Best Bank offers a 3 year investment account with a fixed compound interest rate of 1.75 % per annum.

Mr Lucas invests £8000 in this account.

What is the value of his investment at the end of the 3 year period?

Multiplier = 1 + 0.0175 = 1.0175

$8000 \times 1.0175^3$

= 8427.392875

= £8427.39 (to the nearest penny)

Answer £ 8427.39 [4]

**[Turn over**

**2** Last year a company spent a total of £2400 on advertising.

This year they spent £2796

What was the percentage increase in their spending on advertising?

Increase = 2796 − 2400 = 396

396 ÷ 2400 × 100

= 16.5%

Answer 16.5 % [3]

**3** Solve the equation $p + 15 = 2(4p - 3)$

$p + 15 = 8p - 6$

$15 + 6 = 8p - p$

$21 = 7p$

$p = 3$

Answer $p =$ 3 [3]

**4** Sue is training to compete in a 10 km walk.

A diagram of her local athletics track is shown below.

The track consists of a rectangle and two semicircles.

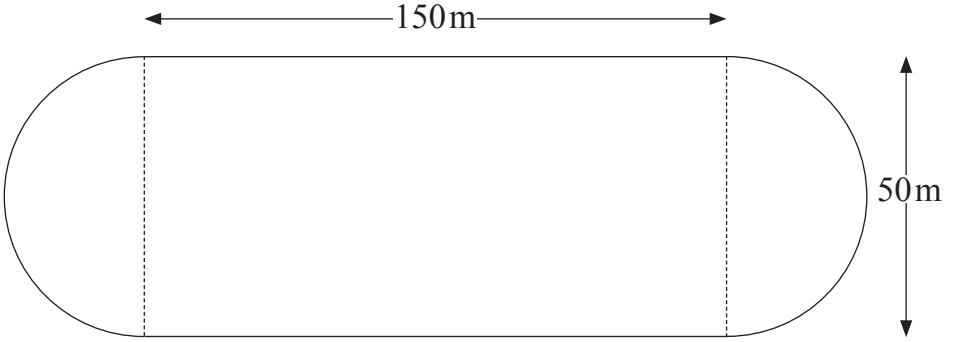



diagram not drawn accurately

How many complete laps are needed to ensure she walks 10 km?

**You must show all your working.**

Two semicircles = one circle, diameter 50 m

Curved parts = π × 50 = 157.0796... m

Straights = 2 × 150 = 300 m

One lap = 300 + 157.0796... = 457.0796... m

10 km = 10 000 m

10 000 ÷ 457.0796... = 21.878...

21 laps is not enough, so she needs 22 laps

Answer 22 laps [4]

**[Turn over**

**5** At a concert 40% of the audience are children.

One third of the **rest** of the audience are men.

There are 120 women in the audience.

Work out the total number of people in the audience.

Rest of audience = 100% − 40% = 60%

Men = ⅓ of 60% = 20%

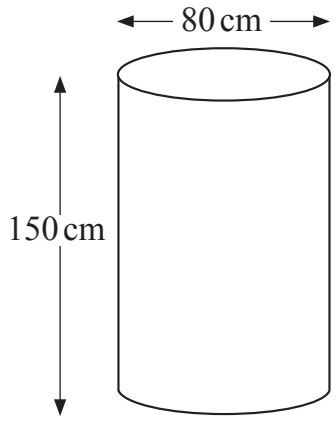
Women = 100% − 40% − 20% = 40%

40% = 120, so 1% = 3

100% = 3 × 100 = 300

Answer 300 [3]

**6** **(a)** A cylindrical tank has a diameter of 80 cm and a height of 150 cm as shown.




Calculate the volume of water the tank can hold when full.

Give your answer correct to the nearest litre.

radius = 80 ÷ 2 = 40 cm
$V = \pi r^2 h = \pi \times 40^2 \times 150$
$= 753\,982.2369\ cm^3$
1 litre = 1000 $cm^3$, so 753 982.2369 ÷ 1000 = 753.98...
= 754 litres (nearest litre)

Answer 754 litres [4]

**(b)** Calculate the curved surface area of the tank.

CSA = $2\pi rh = 2 \times \pi \times 40 \times 150$
= 37 699.11184...

Answer 37 699.11 $cm^2$ [2]

**[Turn over**

7 Peter, John and Matthew are three brothers.

Peter is 10 years old.

John is $x$ years old.

Matthew is a year younger than twice John's age.

The mean of their ages is 7 years.

Work out John's age.

Matthew = $2x - 1$

$$\text{Mean} = \frac{10 + x + 2x - 1}{3} = 7$$

$10 + x + 2x - 1 = 21$

$3x + 9 = 21$

$3x = 12$

$x = 4$

Answer 4 years [4]

**8** Which average (mean, mode or median) would be most suitable for each set of data?

Explain your choice.

**(a)** The data is fairly evenly spread but there is one extreme value at the upper end.

Answer *median* because *the extreme value would distort*

*(pull up) the mean, but the median is not affected by it* [1]

**(b)** One value appears much more frequently than the others and it is not at the upper or lower end of the data.

Answer *mode* because *it is the most common value*

*so it represents the data set best* [1]

**[Turn over**

**9** A school timetable is being arranged.

The day can be arranged in 30-minute classes or 50-minute classes or 60-minute classes.

No matter which of the three choices is made, the total daily teaching time will be the same.

Ignoring the time for break or lunch, what is the daily teaching time?

**You must show all your working.**

$30 = 2 \times 3 \times 5$

$50 = 2 \times 5 \times 5$

$60 = 2 \times 2 \times 3 \times 5$

$LCM = 2 \times 2 \times 3 \times 5 \times 5 = 300$

Check: $300 \div 30 = 10$, $300 \div 50 = 6$, $300 \div 60 = 5$

Daily teaching time = 300 minutes = 5 hours

Answer 300 minutes [4]

**10** Solve the equation

$$\frac{2x-1}{3} + \frac{x+2}{2} + \frac{x}{6} = 8$$

**Show all your working clearly.**

**A solution by trial and improvement will not be accepted.**

Multiply every term by 6:

$2(2x - 1) + 3(x + 2) + x = 48$

$4x - 2 + 3x + 6 + x = 48$

$8x + 4 = 48$

$8x = 44$

$x = 5.5$

Answer $x =$ 5.5 [5]

**11** Factorise $y^2 - 6y + 8$

$-2 \times -4 = 8, \quad -2 + -4 = -6$

Answer $(y - 2)(y - 4)$ [2]

**[Turn over**

**12** A man has mass 74 kg and his son has mass 42 kg, both measured to the nearest kilogram.

What is the maximum difference in mass between the man and his son?

*Man upper bound = 74.5 kg, son lower bound = 41.5 kg*

*74.5 − 41.5 = 33*

Answer *33* kg [2]

**13**

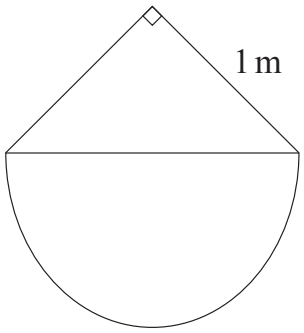



diagram not drawn accurately

The composite shape consists of a right-angled isosceles triangle and a semicircle.

**(a)** Show that the area of the composite shape is approximately 1.285 $m^2$

*Hypotenuse$^2$ = $1^2$ + $1^2$ = 2*

*d = √2 = 1.4142...*

*r = 1.4142... ÷ 2 = 0.7071...*

*Triangle = ½ × 1 × 1 = 0.5*

*Semicircle = ½ × π × $0.7071...^2$*

*= 0.7853...*

*Total area = 0.5 + 0.7853... = 1.2853... ≈ 1.285 $m^2$*

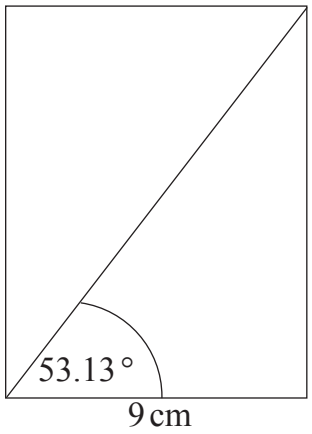
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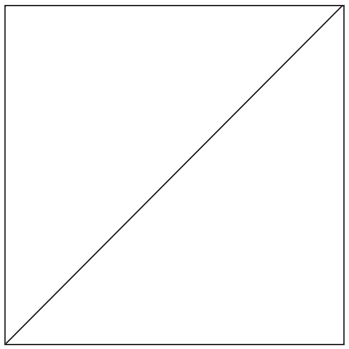
**(b)** Find the force applied to the area of the composite shape when the pressure is 5 $N/m^2$

*Force = pressure × area = 5 × 1.285 = 6.425*

Answer *6.43* N [2]

**14** A rectangle and a square have the same length of diagonal.



diagrams not drawn accurately

Calculate the length of the side of the square.

Give your answer correct to 1 decimal place.

$\cos 53.13° = \frac{9}{d}$

$d = 9 \div \cos 53.13°$

$= 14.99996... = 15$ cm (diagonal)

Square, side s: $s^2 + s^2 = 15^2$

$2s^2 = 225$

$s^2 = 112.5$

$s = \sqrt{112.5} = 10.6066...$

$= 10.6$ cm (1 d.p.)

Answer 10.6 cm [6]

**[Turn over**

**15** Factorise fully

$4q^2 - r^2$

$(2q)^2 - r^2$

Answer $(2q + r)(2q - r)$ [2]

**16** The lines STR and BCR are tangents to the circle shown.

Angle RTC = 47 ° and angle ADC = 94 °

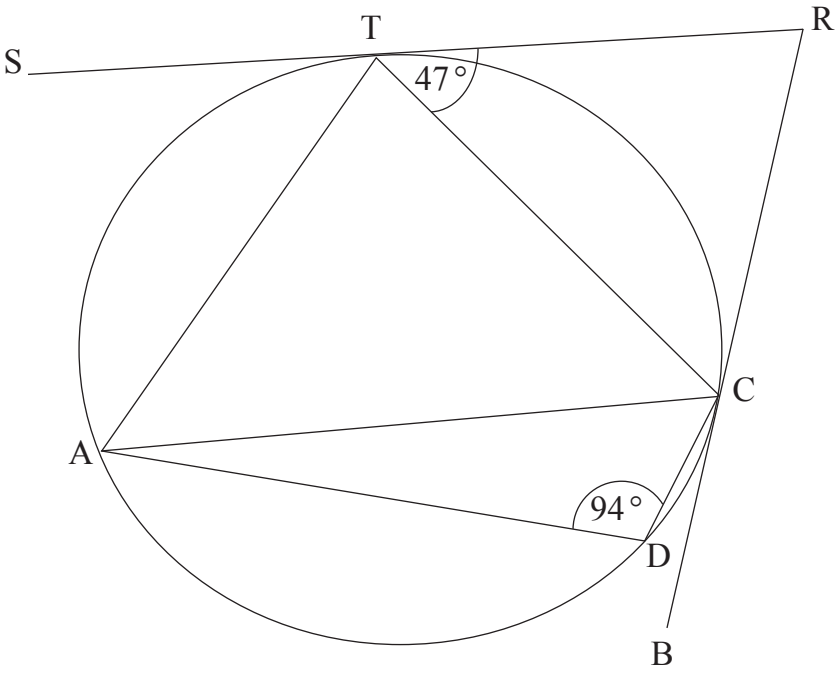



diagram not drawn accurately

John proved that the lines AC and SR are parallel.
He used the following proof but didn't give his reasons.

Using the properties of tangents and circle theorems complete John's argument.

1. Angle RCT = 47 ° because *tangents from R are equal, so triangle RTC is isosceles*

2. Angle RTC = Angle TAC because *alternate segment theorem*

3. Angle ATC = 86 ° because *opposite angles of a cyclic quadrilateral add to 180°*

4. Angle STA = 47 ° because *angles on a straight line add to 180° (180 – 47 – 86)*

5. So the lines AC and SR are parallel because *STA = TAC = 47°, alternate angles*

[5]

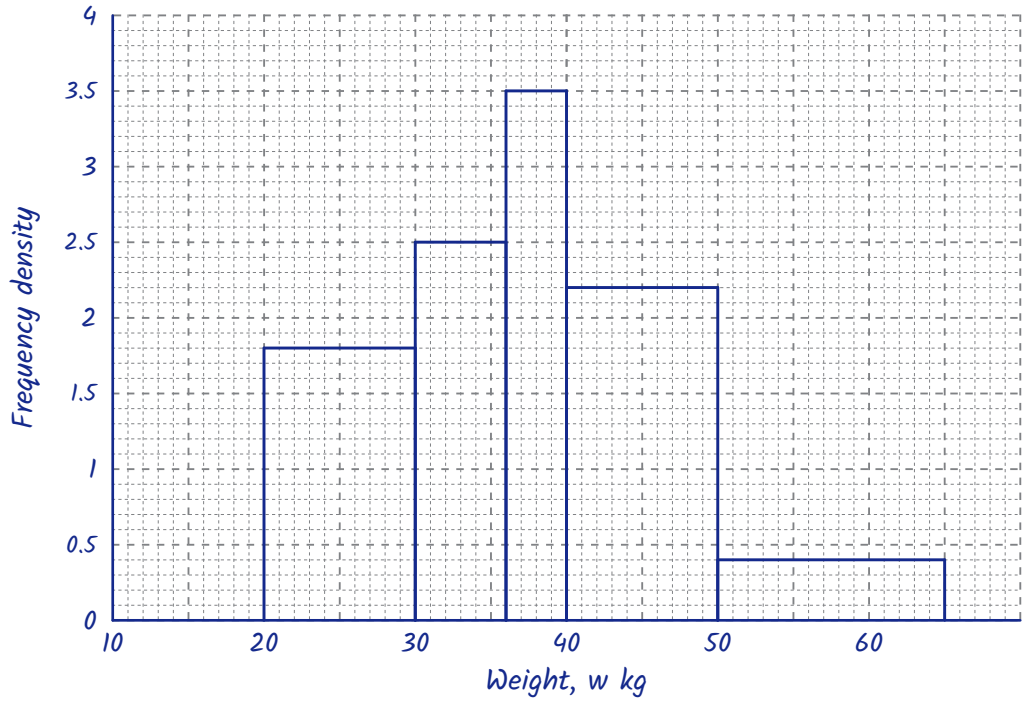
**[Turn over**

**17** The table gives information about the weights of 75 children.

| Weight, $w$ kg | Number of Children |
|---|---|
| $20 \leqslant w < 30$ | 18 |
| $30 \leqslant w < 36$ | 15 |
| $36 \leqslant w < 40$ | 14 |
| $40 \leqslant w < 50$ | 22 |
| $50 \leqslant w < 65$ | 6 |

$fd = f \div width$
$18 \div 10 = 1.8$
$15 \div 6 = 2.5$
$14 \div 4 = 3.5$
$22 \div 10 = 2.2$
$6 \div 15 = 0.4$

**(a)** Illustrate the data by drawing a histogram on the graph paper below.




[3]

**(b)** A stratified sample of 30 children was taken from those whose weight was less than 40 kg.

Estimate how many of the sample were taken from the interval 30–36

Under 40 kg: $18 + 15 + 14 = 47$, $\frac{30}{47} \times 15 = 9.574... \approx 10$

Answer 10 [2]

**18** **(a)** Give a reason why a stratified sample is usually better than a random sample.

*each group (stratum) is represented in proportion to its size* [1]

**(b)** Give a reason why someone might choose to take a random sample rather than a stratified sample.

*it is quicker and simpler, as the strata do not need to be set up first* [1]

**19** The length of a rectangular field is 20 m longer than its width.
The area of the field is 406.25 m$^2$
By setting up and solving a quadratic equation find the length of the field in metres.

**A method involving trial and improvement will receive no marks.**

Let width = $x$ m, length = $(x + 20)$ m

$x(x + 20) = 406.25$

$x^2 + 20x - 406.25 = 0$

$$x = \frac{-20 \pm \sqrt{(20^2 - 4 \times 1 \times (-406.25))}}{2 \times 1}$$

$$x = \frac{-20 \pm \sqrt{2025}}{2} = \frac{-20 \pm 45}{2}$$

$x = 12.5$ or $x = -32.5$ (width cannot be negative)

Width = 12.5 m

Length = 12.5 + 20 = 32.5 m

Answer 32.5 m [6]

**[Turn over**





**20** **(a)** Simplify $\dfrac{x^2 - 4}{3x^2 - 5x - 2}$

$\dfrac{x^2 - 4}{3x^2 - 5x - 2} = \dfrac{(x + 2)(x - 2)}{(3x + 1)(x - 2)} = \dfrac{x + 2}{3x + 1}$

Answer $\dfrac{x + 2}{3x + 1}$ [3]

**(b)** Given $(x + 1)(x - 1) \equiv (x + a)^2 - 6x - b$
find the values of $a$ and $b$.

$x^2 - 1 \equiv x^2 + 2ax + a^2 - 6x - b$

x terms: $0 = 2a - 6$ so $2a = 6$, $a = 3$

numbers: $-1 = a^2 - b$

$-1 = 9 - b$ so $b = 10$

Answer $a =$ 3 $b =$ 10 [4]

**21** Solve the equation

$$\frac{3}{3x+5} - \frac{5}{2x+3} = 2$$

Multiply by $(3x + 5)(2x + 3)$:

$3(2x + 3) - 5(3x + 5) = 2(3x + 5)(2x + 3)$

$6x + 9 - 15x - 25 = 2(6x^2 + 9x + 10x + 15)$

$-9x - 16 = 12x^2 + 38x + 30$

$0 = 12x^2 + 47x + 46$

$(x + 2)(12x + 23) = 0$

$x + 2 = 0$ or $12x + 23 = 0$

$x = -2$ or $x = -\frac{23}{12}$

Answer $x = -2$ or $x = -\frac{23}{12}$ [7]

**[Turn over**

**22** Find the equation of the line L which is perpendicular to the line $3x - 2y + 6 = 0$ and passes through the point $(3, -5)$

$3x - 2y + 6 = 0$

$2y = 3x + 6$

$y = 1.5x + 3$, gradient $= 1.5 = 3/2$

Perpendicular gradient $= -\frac{2}{3}$ (since $3/2 \times -2/3 = -1$)

$y = -\frac{2}{3}x + c$

$-5 = -\frac{2}{3}(3) + c$

$-5 = -2 + c$

$c = -3$

Answer $y = -\frac{2}{3}x - 3$ [4]

**23** A student prepared a frequency table for an experiment involving measuring weights, $w$, in grams.

| $w$ (grams) | Frequency |
|---|---|
| $0 \leqslant w < 10$ | 5 |
| $10 \leqslant w < a$ | 9 |
| $a \leqslant w < 25$ | 27 |
| $25 \leqslant w < 30$ | 14 |
| $30 \leqslant w < 40$ | 17 |

The frequency density for the third group in the table was twice the frequency density for the second group.

**(a)** Find the value of $a$.

fd: $\frac{27}{25-a} = 2 \times \frac{9}{a-10}$

$27(a-10) = 18(25-a)$

$27a - 270 = 450 - 18a$

$45a = 720$

$a = 16$

Answer $a =$ 16 [3]

**(b)** Using this value of $a$ calculate an estimate for the interquartile range of his data.

$n = 72$. Cumulative frequency: 5, 14, 41, 55, 72

LQ = ¼ × 72 = 18th, UQ = ¾ × 72 = 54th

$LQ = 16 + \frac{18-14}{27} \times 9 = 17.333...$

$UQ = 25 + \frac{54-41}{14} \times 5 = 29.642...$

$IQR = 29.642... - 17.333... = 12.309... = 12.31$ g

Answer 12.31 g [5]

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