Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
January 2020

Mathematics

StemUp Model Solution

Unit M4
(With calculator)
Higher Tier



[GMC41]

GMC41

MONDAY 13 JANUARY, 9.15am–11.15am

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all twenty-four** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

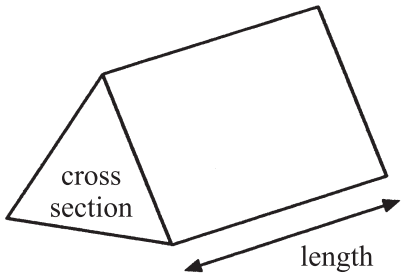
INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

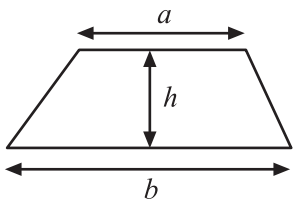
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Formula Sheet

Volume of prism = area of cross section × length

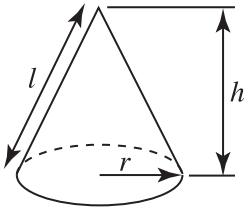


Area of trapezium = $\frac{1}{2}(a+b)h$



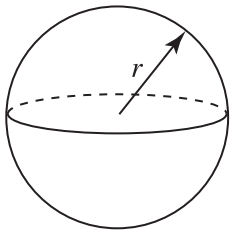
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

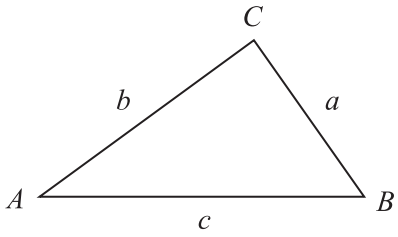


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

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1 BMI (Body Mass Index) is calculated using the formula

$$\text{BMI} = \frac{w}{h^2}$$

where w = weight in kilograms and h = height in metres.

Mark has a BMI of 22
His height is 1.83 m.

Calculate his weight.

$22 = \frac{w}{1.83^2}$

$w = 22 \times 1.83^2$

$w = 73.6758$

$w = 73.7$ kg

Answer 73.7 kg [2]

2 Solve $5(3x - 2) = 7x + 4$

$15x - 10 = 7x + 4$

$15x - 7x = 4 + 10$

$8x = 14$

$x = \frac{14}{8} = 1\frac{3}{4} = 1.75$

Answer $x =$ 1.75 [3]

[Turn over

3 A badge is made from a triangle and three-quarters of a circle as shown.

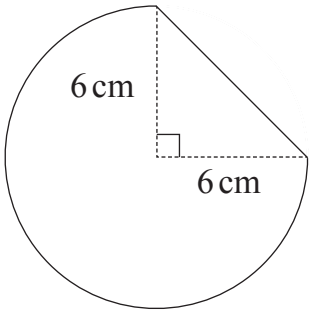


diagram not drawn accurately

Work out the perimeter of the badge.

Circumference of full circle = 2 × π × 6 = 37.69911184

Three-quarters of the circle:

¾ × 37.69911184 = 28.27433388 cm

Straight edge (hypotenuse):

$h^2 = 6^2 + 6^2 = 72$

$h = \sqrt{72} = 8.485281374$ cm

Perimeter = 28.27433388 + 8.485281374

= 36.75961525...

= 36.76 cm

Answer 36.76 cm [6]

4 Write down two numbers, greater than 1, which have a lowest common multiple (LCM) of 48

$3 = 3,\ 16 = 2^4,\ LCM = 2^4 \times 3 = 48$

Answer 3 , 16 [2]

5 Expand and simplify $(p - 6)(p + 2)$

$(p - 6)(p + 2)$

$= p^2 + 2p - 6p - 12$

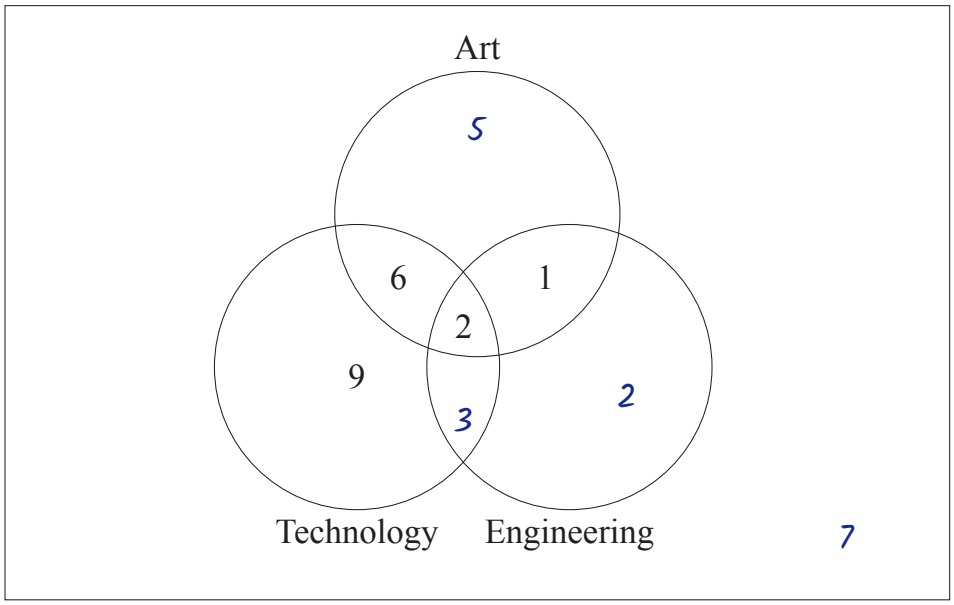
$= p^2 - 4p - 12$

Answer $p^2 - 4p - 12$ [2]

[Turn over

6 There are 35 students in a Form Class.
Some students study Art, some study Technology, some study Engineering and some study a combination of these subjects.
Some study none of these subjects.

The Venn diagram shows some information about the number of students studying the subjects.



5 students study Technology and Engineering.
14 students study Art.
8 students study Engineering.

(a) Use this information to complete the Venn diagram. [4]

(b) Use the Venn diagram to calculate what percentage of Technology students also study Art but not Engineering.

Technology total = 9 + 6 + 2 + 3 = 20

Art but not Engineering = 6, $\frac{6}{20} \times 100 = 30$

Answer 30 % [2]

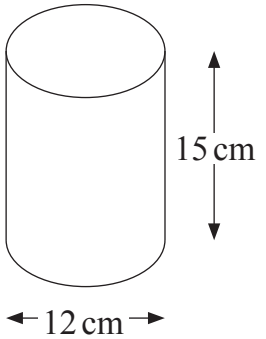
7 Work out the midpoint of the line PQ joining P(4, −6) and Q(8, 2).

x: $\frac{4+8}{2} = 6$ $\quad$ y: $\frac{-6+2}{2} = -2$

Answer (6 , −2) [2]

8 Calculate the volume of a cylinder with diameter 12 cm and height 15 cm.

Include units with your answer.



$r = 12 \div 2 = 6$ cm

$V = \pi r^2 h$

$= \pi \times 6^2 \times 15$

$= 1696.460033...$

Answer 1696.46 cm^3 [3]

[Turn over

9 A restaurant bill, including 15% service charge, was £98.90
How much was the service charge?

115% = £98.90

1% = 98.90 ÷ 115 = £0.86

100% = £86

Service charge = 98.90 − 86 = £12.90

Answer £ 12.90 [3]

10 Simplify $\frac{x^2 - 2x}{3} \times \frac{6}{x^2 + 2x - 8}$

$= \frac{x(x - 2)}{3} \times \frac{6}{(x + 4)(x - 2)}$

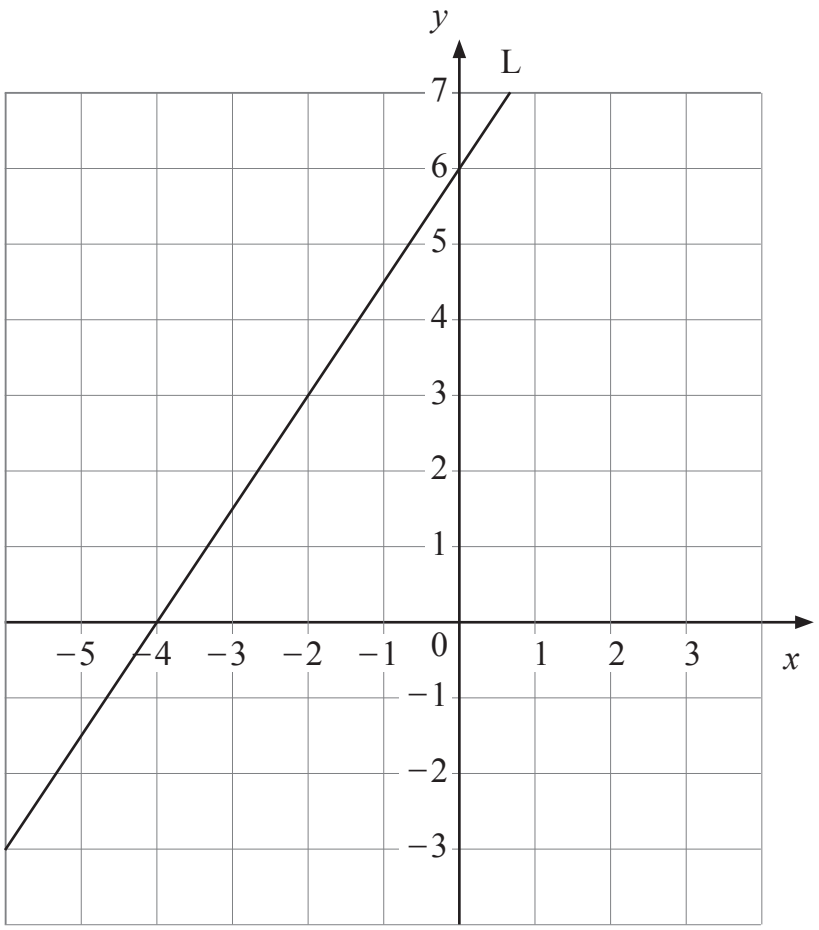
$= \frac{6x}{3(x + 4)}$

$= \frac{2x}{x + 4}$

Answer $\frac{2x}{x + 4}$ [3]

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11



(a) Write down the equation of the line L shown.

gradient $m = 6 \div 4 = 1.5$, y-intercept $c = 6$

$y = mx + c$

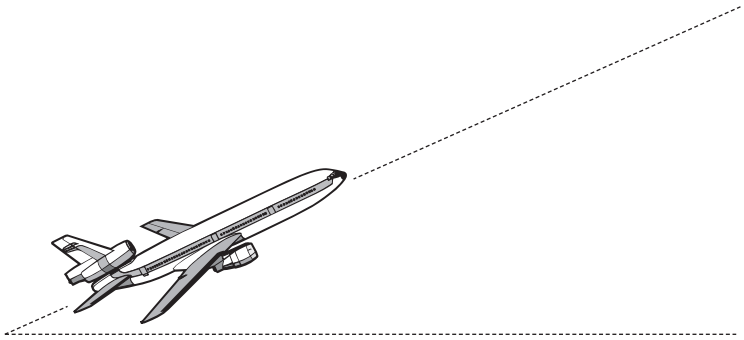
Answer $y = 1.5x + 6$ [3]

(b) Write down the equation of any line parallel to line L.

Answer $y = 1.5x + 1$ [1]

[Turn over

12 After take-off, an aircraft flies for 16 km at an angle of 24° to the horizontal.



Calculate the height it reaches.

$\sin 24° = \frac{h}{16}$

$h = 16 \times \sin 24°$

$h = 6.507746...$

$h = 6.51$ km

Answer 6.51 km [3]

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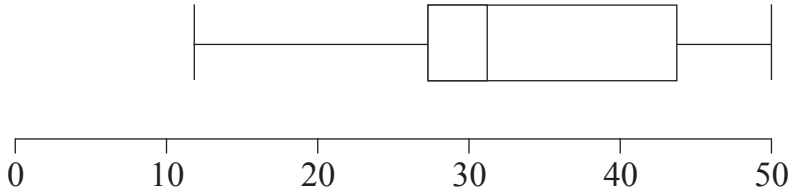
13 The test marks for 35 students are displayed in two different ways.

Stem and leaf

1	2 2 2 3 4
2	0 4 5 8 8 8 8
3	0 0 0 1 1 2 3 4 5 7 8 9 9
4	2 4 5 5 6 7 8 9 9
5	0

KEY: 4 | 6 = 46

Box plot



(a) Give one advantage of the information being displayed in the stem and leaf diagram.

All the raw data is still visible (you can see every test mark). [1]

(b) Give one advantage of the information being displayed in the box plot.

The median and quartiles (summary statistics) are clear to see. [1]

[Turn over

14 **(a)** Write 84 and 154 as a product of their prime factors.

$84 = 2 \times 42 = 2 \times 2 \times 21 = 2 \times 2 \times 3 \times 7$

$154 = 2 \times 77 = 2 \times 7 \times 11$

Answer 84 = $2^2 \times 3 \times 7$ 154 = $2 \times 7 \times 11$ [2]

(b) Hence, or otherwise, find the highest common factor (HCF) of 84 and 154

Common prime factors: 2 and 7

HCF = $2 \times 7 = 14$

Answer 14 [2]

15 Rory has x golf balls.

Graham has one less than Rory.

Darren has one less than Graham.

After a round of golf, Rory still has x golf balls but Graham has lost half of his golf balls and Darren has lost two-thirds of his golf balls.

They **now** have a total of 74 golf balls between them.

Set up a linear equation and solve it to find how many golf balls each player now has.

A solution by trial and improvement will not be accepted.

Rory = x, Graham = x − 1, Darren = x − 2

$x + \frac{1}{2}(x - 1) + \frac{1}{3}(x - 2) = 74$

× 6: $6x + 3(x - 1) + 2(x - 2) = 444$

$6x + 3x - 3 + 2x - 4 = 444$

$11x - 7 = 444$

$11x = 451$

$x = 41$

Rory: 41, Graham: ½ × 40 = 20, Darren: ⅓ × 39 = 13

Answer Rory 41 Graham 20 Darren 13 [5]

[Turn over

16 The diagram shows an L-shape with a total area of 47 cm^2

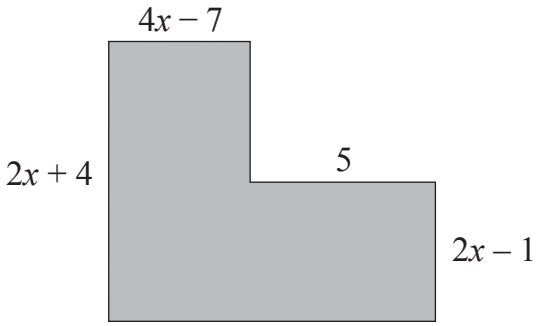


diagram not drawn accurately

All corners are right angles.

All lengths are in cm.

(a) Show that $2x^2 + 3x - 20 = 0$

Split into two rectangles:

Area = $(2x + 4)(4x - 7) + 5(2x - 1) = 47$

$8x^2 - 14x + 16x - 28 + 10x - 5 = 47$

$8x^2 + 12x - 33 = 47$

$8x^2 + 12x - 80 = 0$

$\div 4$: $2x^2 + 3x - 20 = 0$

[4]

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(b) Solve this equation to find the value of x.

$2x^2 + 3x - 20 = 0$

$(2x - 5)(x + 4) = 0$

$x = 2.5$ or $x = -4$

x cannot be negative (length), so $x = 2.5$

Answer $x =$ 2.5 [2]

17 Factorise fully $8x^2 - 50y^2$

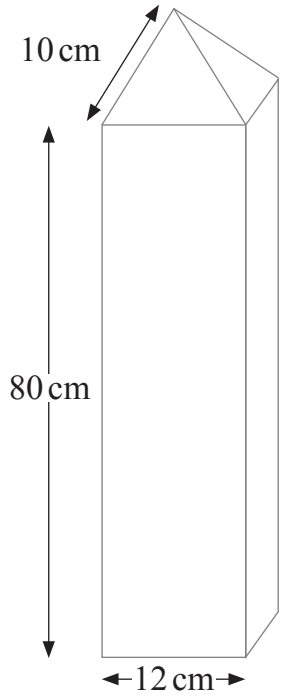
$8x^2 - 50y^2 = 2(4x^2 - 25y^2)$

$= 2(2x - 5y)(2x + 5y)$

Answer 2(2x − 5y)(2x + 5y) [3]

[Turn over

18 The diagram shows a wooden post consisting of a cuboid with a square-based pyramid on top. Each slanted edge is 10 cm.



Calculate the total surface area of the wooden post.

Height of each triangular face: $h^2 = 10^2 - 6^2 = 64$, $h = 8$ cm

Four rectangles: $4 \times (12 \times 80) = 3840$

Square base: $12 \times 12 = 144$

Four triangles: $4 \times (\frac{1}{2} \times 12 \times 8) = 192$

Total $= 3840 + 144 + 192 = 4176$ cm^2

Answer 4176 cm² [5]

19 The head cook at Dundee Secondary School wants to take a stratified sample to complete a survey on school meals.

A sample of 50 pupils is required.

The table below shows the number of pupils in each year group.

Year	8	9	10	11	12
Number of pupils	182	124	150	108	97

(a) How many pupils should be selected from each year group using a stratified sample?

Total = 182 + 124 + 150 + 108 + 97 = 661

Y8: $\frac{182}{661} \times 50 = 13.77$

Y9: $\frac{124}{661} \times 50 = 9.38$

Y10: $\frac{150}{661} \times 50 = 11.35$

Y11: $\frac{108}{661} \times 50 = 8.17$

Y12: $\frac{97}{661} \times 50 = 7.34$

Rounding gives 49, so one more from Year 9 (next largest decimal) to make 50

Answer Year 8 14

Year 9 10

Year 10 11

Year 11 8

Year 12 7 [4]

(b) Write down one advantage and one disadvantage of using a stratified sample.

Advantage every year group is represented in proportion to its size

Disadvantage it is time consuming to work out and select [2]

[Turn over

20

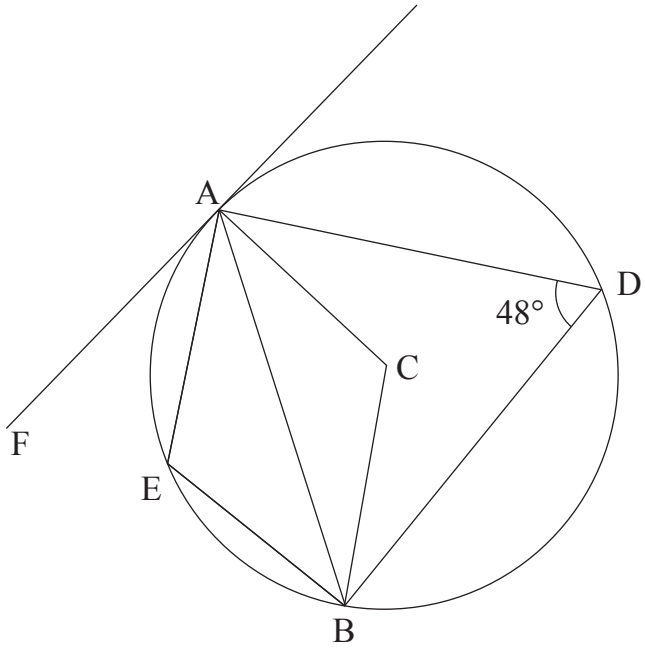


diagram not drawn accurately

A, B, D and E are points on the circumference of a circle, centre C.

$\angle ADB = 48°$

AF is a tangent to the circle.

Find the size of the following angles, giving a reason for each answer.

(a) $\angle AEB =$ 132 ° because opposite angles in a cyclic quadrilateral add up to 180° (180 − 48 = 132) [2]

(b) $\angle ACB =$ 96 ° because the angle at the centre is twice the angle at the circumference (2 × 48 = 96) [2]

(c) $\angle BAF =$ 48 ° because of the alternate segment theorem (angle between tangent and chord = angle in alternate segment, ∠ADB) [2]

21 Solve the equation

$$\frac{4}{2x-3} = \frac{2}{3x+1} + 2$$

giving your answers correct to 2 decimal places.

$\frac{4}{2x-3} = \frac{2+2(3x+1)}{3x+1} = \frac{6x+4}{3x+1}$

$4(3x+1) = (2x-3)(6x+4)$

$12x + 4 = 12x^2 + 8x - 18x - 12$

$12x + 4 = 12x^2 - 10x - 12$

$0 = 12x^2 - 22x - 16$

$6x^2 - 11x - 8 = 0$

$x = \frac{11 \pm \sqrt{((-11)^2 - 4 \times 6 \times (-8))}}{2 \times 6} = \frac{11 \pm \sqrt{313}}{12}$

$x = 2.391164...$ or $x = -0.557830...$

$x = 2.39$ or $x = -0.56$

Answer $x =$ 2.39 or −0.56 [7]

[Turn over

22 The line joining $(a, 7)$ to $(-1, 10)$ is perpendicular to the line joining $(a, 7)$ to $(9, -6)$.

Find the possible values of a.

A solution by trial and improvement will not be accepted.

Gradient 1 $= \frac{10 - 7}{-1 - a} = \frac{3}{-1 - a}$ Gradient 2 $= \frac{-6 - 7}{9 - a} = \frac{-13}{9 - a}$

Perpendicular: $\frac{3}{-1 - a} \times \frac{-13}{9 - a} = -1$

$-39 = -(-1 - a)(9 - a)$

$39 = (-1 - a)(9 - a)$

$39 = -9 + a - 9a + a^2$

$39 = a^2 - 8a - 9$

$a^2 - 8a - 48 = 0$

$(a - 12)(a + 4) = 0$

$a = 12$ or $a = -4$

Answer $a =$ 12 or $a =$ −4 [6]

23 Given $m = \frac{\sqrt{s}}{t}$ and

s = 5.14 rounded to 2 decimal places
t = 9.384 rounded to 3 decimal places

find the upper bound of m.

s: lower bound 5.135, upper bound 5.145

t: lower bound 9.3835, upper bound 9.3845

Upper bound of m uses upper s and lower t:

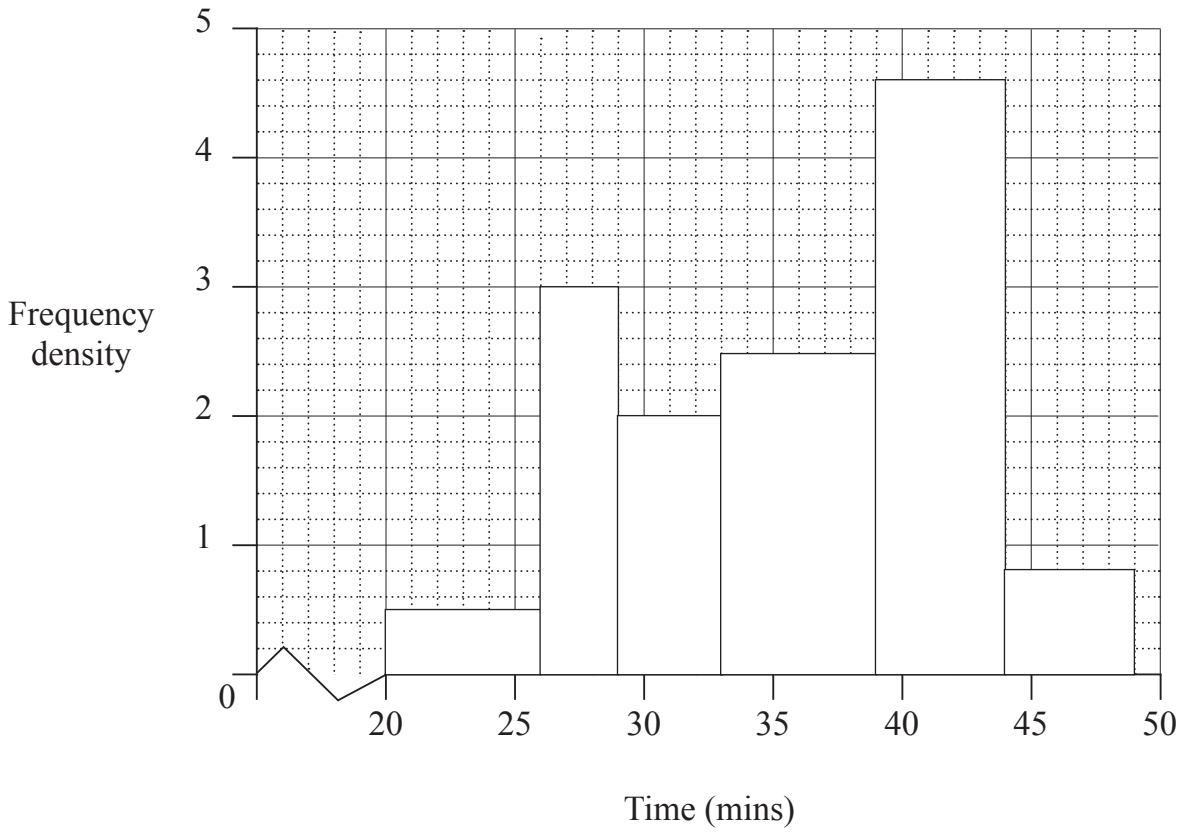
$m = \frac{\sqrt{5.145}}{9.3835} = 0.2417284856...$

Answer 0.2417284856 [3]

[Turn over

24 62 pupils completed a couch to 5km programme, which ended with everyone completing a 5 km run.

Their times were recorded and the histogram below illustrates the data collected.



(a) Complete the table below.

Time (t minutes)	Frequency
$20 \leqslant t < 26$	3
$26 \leqslant t < 29$	9
$29 \leqslant t < 33$	8
$33 \leqslant t < 39$	15
$39 \leqslant t < 44$	23
$44 \leqslant t < 49$	4

$f = fd \times \text{width}$

$0.5 \times 6 = 3$

$3 \times 3 = 9$

$2 \times 4 = 8$

$2.5 \times 6 = 15$

$4.6 \times 5 = 23$

$0.8 \times 5 = 4$

[4]

(b) Find an estimate for the mean time taken by the 62 pupils to complete the 5 km.

Midpoints: 23, 27.5, 31, 36, 41.5, 46.5

$\Sigma fx = 3\times23 + 9\times27.5 + 8\times31 + 15\times36 + 23\times41.5 + 4\times46.5$

$= 69 + 247.5 + 248 + 540 + 954.5 + 186 = 2245$

$$\text{Mean} = \frac{2245}{62} = 36.20967\ldots$$

$= 36.2$ minutes

Answer 36.2 minutes [4]

THIS IS THE END OF THE QUESTION PAPER

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For Examiner's use only	
Question Number	**Marks**
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