New Specification

Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
January 2019

Mathematics

StemUp Model Solution

Unit M4
(With calculator)
Higher Tier



[GMC41] *GMC41*

TUESDAY 8 JANUARY, 9.15am–11.15am

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Answer **all nineteen** questions.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
You **may** use a calculator for this paper.

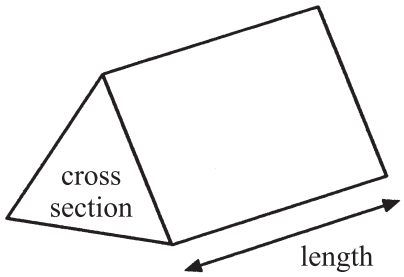
INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You should have a calculator, ruler, compasses and a protractor.
The Formula Sheet is on page 2.

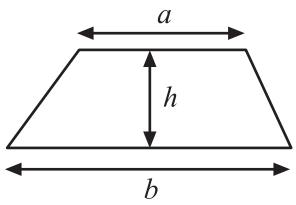
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Formula Sheet

Volume of prism = area of cross section × length

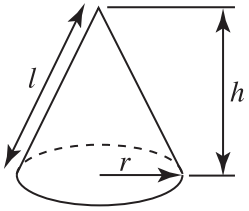


Area of trapezium = $\frac{1}{2}(a+b)h$



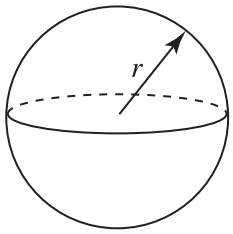
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

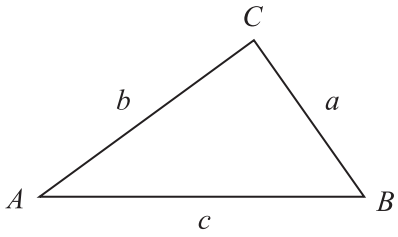


Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



In any triangle *ABC*



Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$

1 A badge is the shape of a quarter circle as shown below.

Calculate the perimeter of the badge.

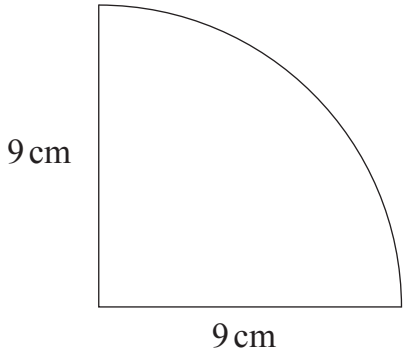


diagram not
drawn accurately

Arc = $\frac{1}{4} \times 2 \times \pi \times 9$

= 56.54866776 ÷ 4 = 14.13716694 cm

Perimeter = 14.13716694 + 9 + 9

= 32.13716694

= 32.14 cm

Answer 32.14 cm [3]

[Turn over

2 The test scores for the 10 boys in a class are

7 8 5 8 7 9 4 5 3 9

The mean test score for the 5 girls in the class is 8

Calculate the mean for this class.

Show your working clearly.

Boys total = 7 + 8 + 5 + 8 + 7 + 9 + 4 + 5 + 3 + 9 = 65

Girls total = 8 × 5 = 40

Class total = 65 + 40 = 105

Mean = 105 ÷ 15 = 7

Answer 7 [3]

3 **(a)** Given that $4500 = 2^a \times 3^2 \times 5^b$

work out the values of a and b.

$4500 = 2 \times 2250$

$= 2 \times 2 \times 1125$

$= 2 \times 2 \times 3 \times 375$

$= 2 \times 2 \times 3 \times 3 \times 125$

$= 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5$

$= 2^2 \times 3^2 \times 5^3$

Answer $a =$ 2 $b =$ 3 [3]

(b) Hence, write down the lowest value by which 4500 needs to be multiplied to make a **cube** number.

$2^2 \times 3^2 \times 5^3$ needs $2^3 \times 3^3 \times 5^3$

so multiply by $2 \times 3 = 6$

Answer 6 [2]

[Turn over

4 The speeds of cars on a road were recorded over a period of time.

The results are recorded in the grouped frequency table.

Speed (s miles per hour)	**Frequency**	Midpoint x	fx
$20 < s \leqslant 30$	12	25	300
$30 < s \leqslant 40$	16	35	560
$40 < s \leqslant 50$	18	45	810
$50 < s \leqslant 60$	2	55	110
$60 < s \leqslant 70$	2	65	130

(a) How many cars were travelling at more than 40 mph?

18 + 2 + 2 = 22

Answer 22 [1]

(b) Which class interval contains the median speed?

50 cars: median is 25.5th

Answer $30 < s \leq 40$ [1]

(c) Calculate an estimate for the mean speed of the cars on the road.

$\Sigma f = 12 + 16 + 18 + 2 + 2 = 50$

$\Sigma fx = 300 + 560 + 810 + 110 + 130 = 1910$

Mean = 1910 ÷ 50 = 38.2

Answer 38.2 mph [4]

5 A child's height increased from 84 cm to 91 cm.

Calculate the percentage increase.

Increase = 91 − 84 = 7 cm

% increase = $\frac{7}{84} \times 100 = 8.333333...$

= 8.33%

Answer 8.33 % [3]

[Turn over

6 ABCD is a square of side 6 cm.

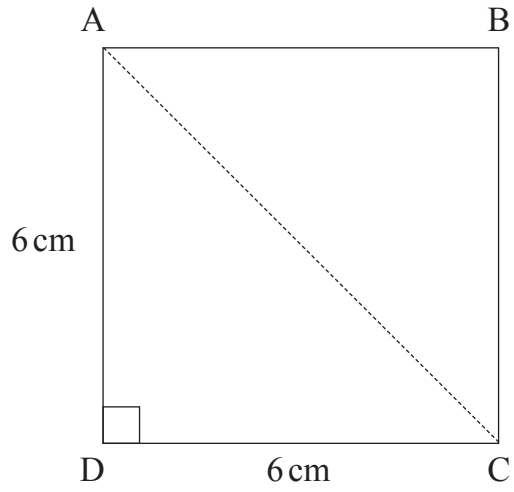


diagram not drawn accurately

How much longer is AC than AD?

You must show all your working.

$AC^2 = 6^2 + 6^2$

$AC^2 = 36 + 36 = 72$

$AC = \sqrt{72} = 8.485281374$ cm

$AC - AD = 8.485281374 - 6$

$= 2.485281374$

$= 2.49$ cm

Answer 2.49 cm [4]

7 Peter, Jack and Colin share a flat. They pay the rent monthly.

Peter pays 30% of the monthly rent.

Jack pays $\frac{3}{8}$ of the monthly rent.

Colin pays £520 of the monthly rent.

Calculate the total monthly rent for the flat.

Jack: $\frac{3}{8} = 3 \div 8 \times 100\% = 37.5\%$

Peter + Jack = 30% + 37.5% = 67.5%

Colin = 100% − 67.5% = 32.5%

32.5% = £520

1% = £520 ÷ 32.5 = £16

100% = £16 × 100 = £1600

Answer £ 1600 [5]

8 Factorise each of the following.

(a) $10cp^2 - 4cp$

Answer $2cp(5p - 2)$ [2]

(b) $y^2 - 1$

Answer $(y - 1)(y + 1)$ [1]

(c) $k^2 - 2k - 3$

$(-3) \times (+1) = -3, (-3) + (+1) = -2$

Answer $(k - 3)(k + 1)$ [2]

(d) $(x - 2)^2 + 5(x - 2)$

$= (x - 2)[(x - 2) + 5]$

Answer $(x - 2)(x + 3)$ [2]

9 Solve $\frac{x+3}{2} = \frac{5x}{6}$

$6 \times \frac{x+3}{2} = 6 \times \frac{5x}{6}$

$3(x+3) = 5x$

$3x + 9 = 5x$

$9 = 2x$

$x = 4.5$

Answer $x =$ 4.5 [4]

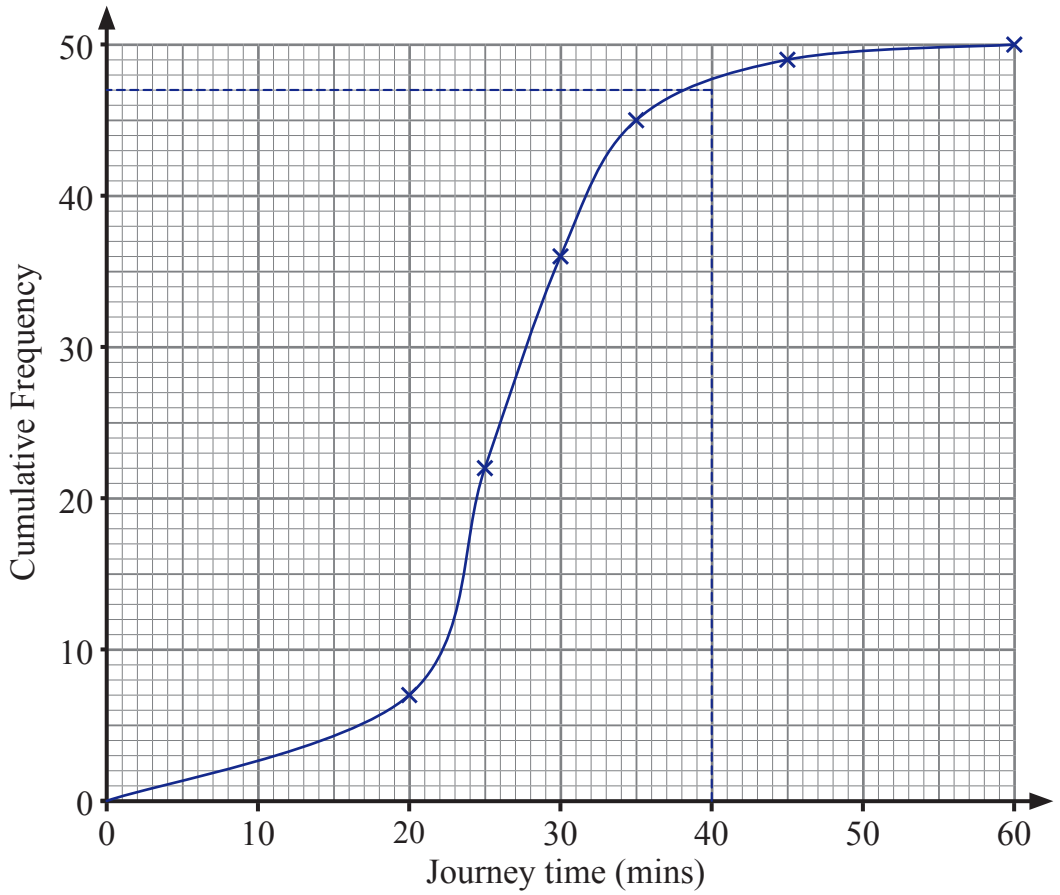
[Turn over

10 The cumulative frequency table gives data about the length of time it takes for 50 workers to travel to work one morning.

Journey time (t minutes)	**Cumulative Frequency**
$t \leqslant 20$	7
$t \leqslant 25$	22
$t \leqslant 30$	36
$t \leqslant 35$	45
$t \leqslant 45$	49
$t \leqslant 60$	50

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(a) On the graph paper below, draw a cumulative frequency graph to illustrate the data.



[3]

(b) Use the graph to estimate the percentage of workers whose journey time was longer than 40 minutes.

$50 - 47 = 3$ workers

$\frac{3}{50} \times 100 = 6$

Answer 6 % [2]

[Turn over

11 PQR is a right-angled triangle.

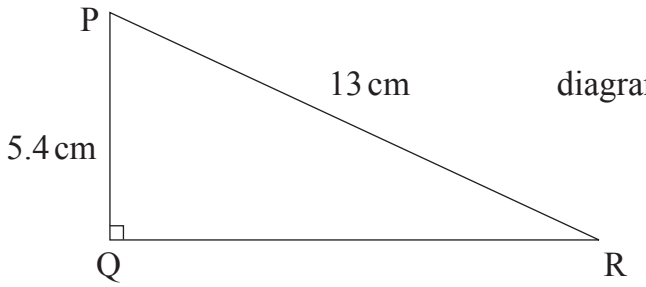


diagram not drawn accurately

By how many degrees is angle P bigger than angle R?

Give your answer to the nearest degree.

Show all your working clearly.

$\cos P = \frac{5.4}{13}$

$P = \cos^{-1}(5.4 \div 13) = 65.45646012°$

$R = 90 - 65.45646012 = 24.54353988°$

$P - R = 65.45646012 - 24.54353988$

$= 40.91292024 = 41°$

Answer 41 ° [5]

12 Find the equation of the line which goes through $(0, -4)$ and is perpendicular to the line $2x + 3y = 9$

$3y = -2x + 9$

$y = -\frac{2}{3}x + 3$ so gradient $= -\frac{2}{3}$

Perpendicular gradient $= \frac{3}{2}$ $\left(-\frac{2}{3} \times \frac{3}{2} = -1\right)$

Passes through $(0, -4)$ so $c = -4$

Answer $y = \frac{3}{2}x - 4$ [3]

13 Simplify fully

$$\frac{3x^2 - 6xy}{4x^2 - 8xy - 3xy + 6y^2}$$

Numerator: $3x^2 - 6xy = 3x(x - 2y)$

Denominator: $4x(x - 2y) - 3y(x - 2y) = (4x - 3y)(x - 2y)$

$$\frac{3x(x - 2y)}{(4x - 3y)(x - 2y)} = \frac{3x}{4x - 3y}$$

Answer $\frac{3x}{4x - 3y}$ [2]

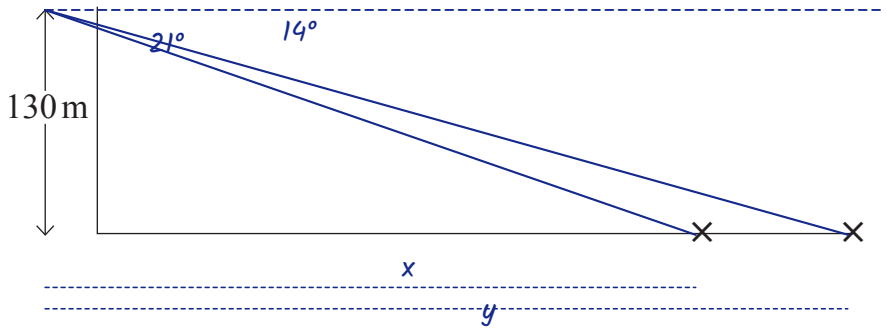
[Turn over

14 A cliff is 130 m high.

From the top of the cliff, the angles of depression of two boats, both due east and level with each other, are 14° and 21° respectively.

Calculate the distance between the boats.

Show all your working clearly.



Angles at top of cliff: 90 − 21 = 69°, 90 − 14 = 76°

Near boat: tan 69° = x ÷ 130

x = 130 × tan 69° = 338.6621... m

Far boat: tan 76° = y ÷ 130

y = 130 × tan 76° = 521.4021... m

Distance = 521.4021... − 338.6621...

= 182.74 = 183 m

Answer 183 m [5]

15 A train travels 736 km (correct to the nearest km).

The journey takes 4.5 hours (correct to the nearest 0.1 hour).

Work out the minimum possible average speed and the maximum possible average speed in km/h.

Distance: $735.5 \leq d < 736.5$ km

Time: $4.45 \leq t < 4.55$ hours

Minimum speed = $735.5 \div 4.55 = 161.6483516$ km/h

Maximum speed = $736.5 \div 4.45 = 165.505618$ km/h

Answer minimum average speed is 161.65 km/h

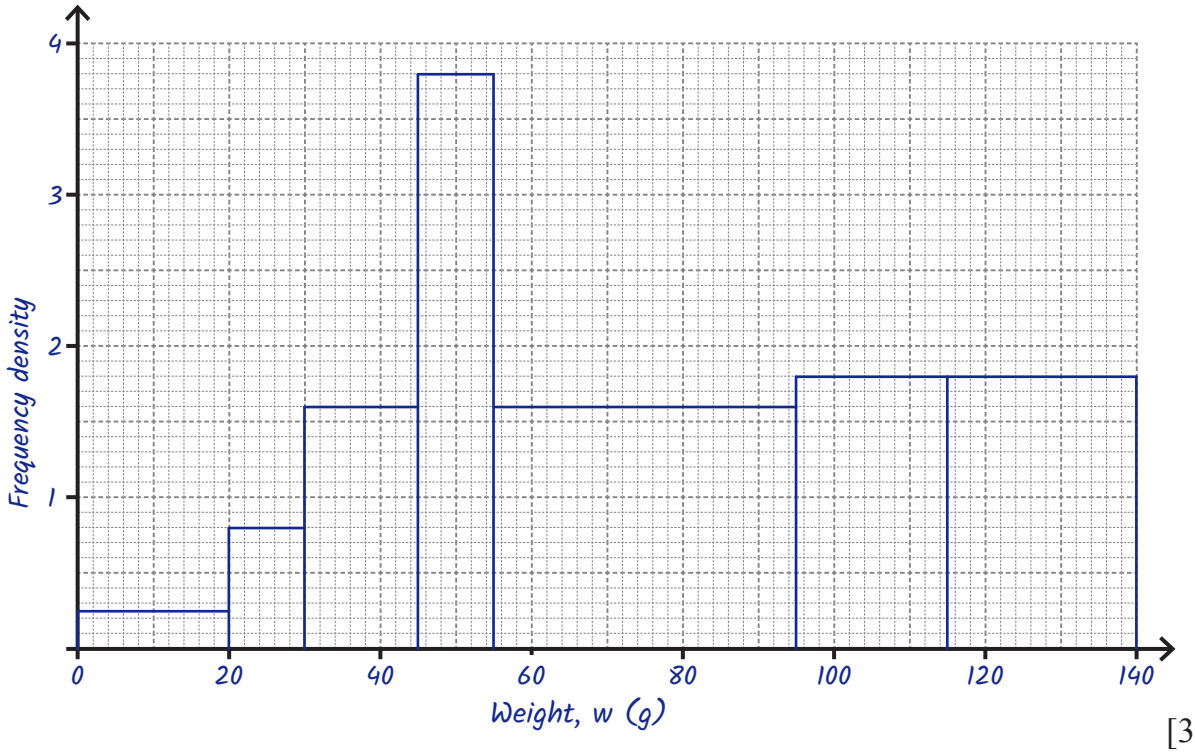
maximum average speed is 165.51 km/h [4]

[Turn over

16 The weights, in grams, of a collection of waste bags from a hospital are shown.

Weight, w (g)	Frequency	f.d.
$0 < w \leqslant 20$	5	5 ÷ 20 = 0.25
$20 < w \leqslant 30$	8	8 ÷ 10 = 0.8
$30 < w \leqslant 45$	24	24 ÷ 15 = 1.6
$45 < w \leqslant 55$	38	38 ÷ 10 = 3.8
$55 < w \leqslant 95$	64	64 ÷ 40 = 1.6
$95 < w \leqslant 115$	36	36 ÷ 20 = 1.8
$115 < w \leqslant 140$	45	45 ÷ 25 = 1.8

(a) Draw a histogram on the axes provided to illustrate this data.



[3]

(b) Estimate the number of bags which weighed more than 60 g.

$55 < w \leq 95$: $\frac{35}{40} \times 64 = 56$

More than 60 g = 56 + 36 + 45 = 137

Answer 137 [2]

A stratified sample of 70 bags was selected from those with weights less than or equal to 95 g.

(c) Estimate how many of this sample weighed less than 40 g.

Bags ≤ 95 g: 5 + 8 + 24 + 38 + 64 = 139

Less than 40 g: $5 + 8 + \frac{10}{15} \times 24 = 5 + 8 + 16 = 29$

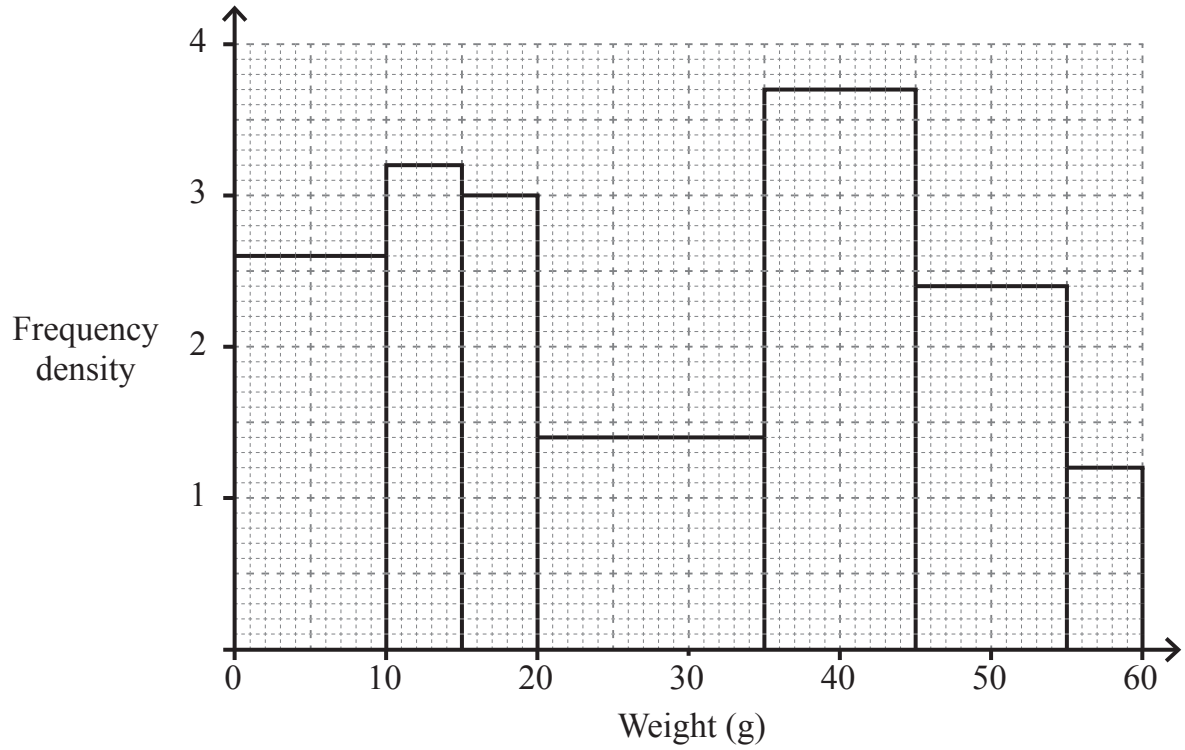
Sample: $\frac{29}{139} \times 70 = 14.60431655$

≈ 15 bags

Answer 15 [3]

[Turn over

The weights of a second collection of bags are shown in the histogram below.



(d) Estimate the mean weight for this collection.

Class (g)	f = f.d. × width	Midpoint	f × mid
0–10	2.6 × 10 = 26	5	130
10–15	3.2 × 5 = 16	12.5	200
15–20	3.0 × 5 = 15	17.5	262.5
20–35	1.4 × 15 = 21	27.5	577.5
35–45	3.7 × 10 = 37	40	1480
45–55	2.4 × 10 = 24	50	1200
55–60	1.2 × 5 = 6	57.5	345
Total	145		4195

Mean = 4195 ÷ 145 = 28.93103448

= 28.93 g

Answer 28.93 g [4]

(e) Estimate the median weight for this collection.

Median = 145 ÷ 2 = 72.5th value

Cumulative frequencies: 26, 42, 57, 78, ...

so the median is in the class $20 < w \leq 35$

72.5 − 57 = 15.5 into this class of 21

$$\text{Median} = 20 + \frac{15.5}{21} \times 15 = 31.07142857$$

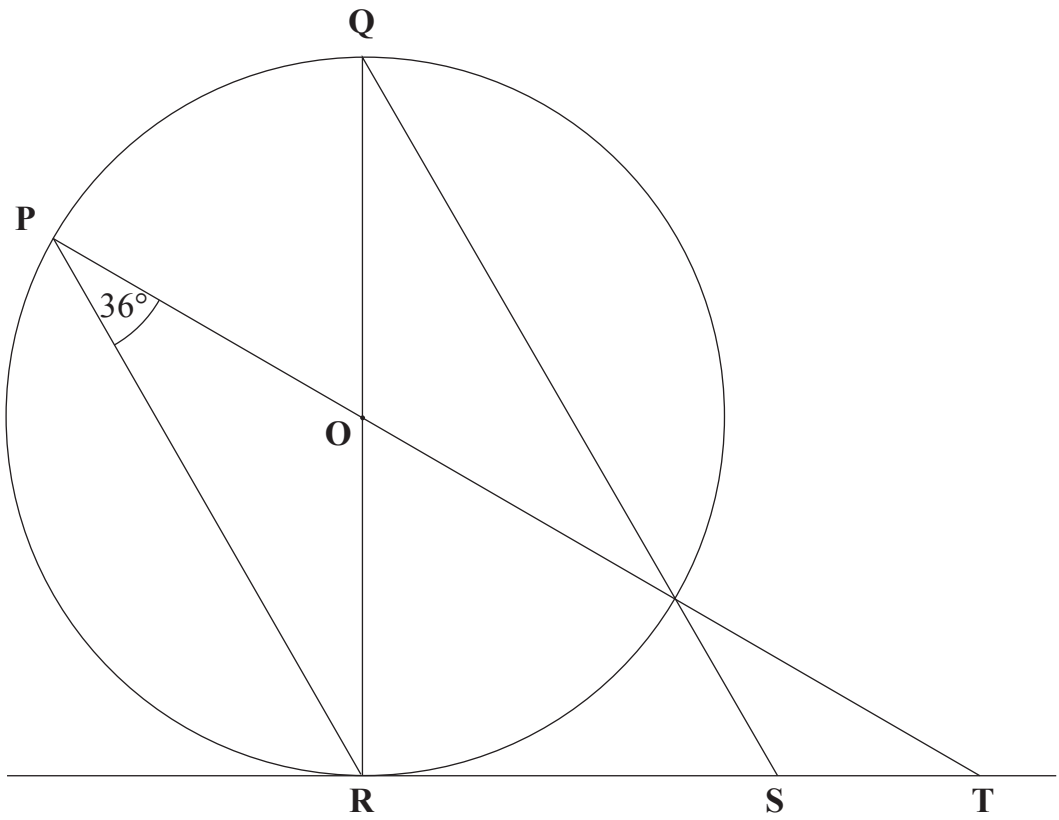
Answer 31.1 g [3]

[Turn over

17 (a) In the diagram shown, O is the centre of the circle.

P, Q and R are points on the circumference of the circle.

RST is a tangent to the circle.



Calculate the size of

(i) angle ROT,

OP = OR so ∠ORP = 36°; 36 + 36 = 72°

Answer 72 ° [1]

(ii) angle OTR,

∠ORT = 90°; 180 − 90 − 72 = 18°

Answer 18 ° [1]

(iii) angle QSR.

∠RQS = ∠RPT = 36° (same segment)

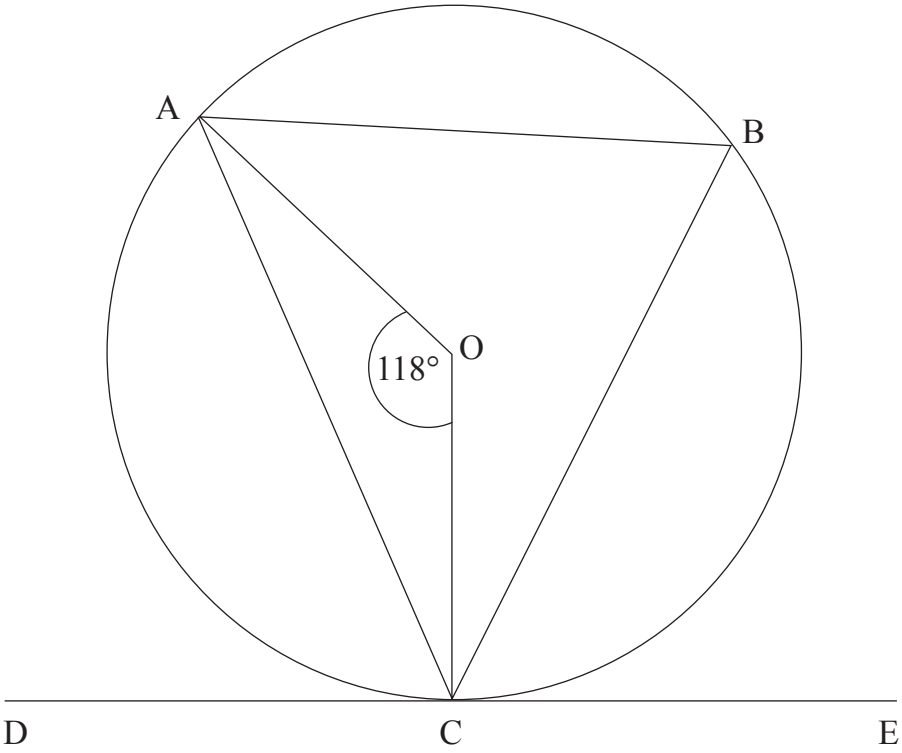
180 − 90 − 36 = 54°

Answer 54 ° [1]

(b) O is the centre of the circle through A, B and C.

DCE is a tangent to the circle at C.

Angle AOC = 118°



(i) Find the size of angle ACD.
Give a reason for each step.

Answer Angle ACD = 59 °

Reasons:
∠ABC = 118 ÷ 2 = 59° (angle at the centre is twice the angle at the circumference)
∠ACD = ∠ABC = 59° (angle between tangent and chord equals the angle in the alternate segment)

[3]

[Turn over

(ii) Angle BCE = x.

Express the angles BAC and BCA in terms of x.

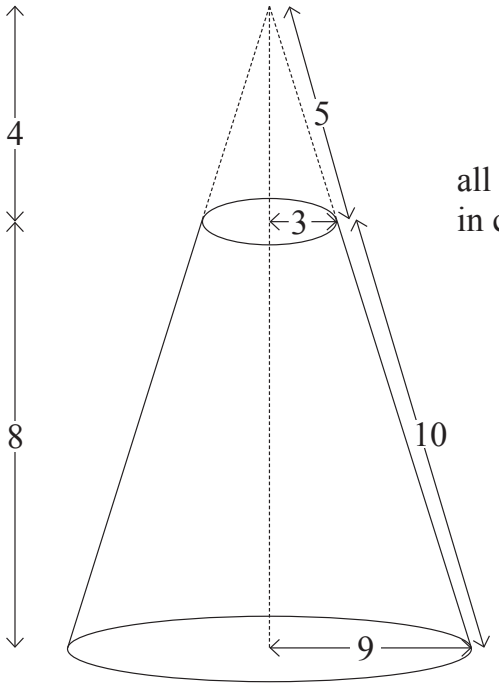
$\angle BAC = \angle BCE = x$ (alternate segment theorem)

$\angle ABC = 59°$, so $\angle BCA = 180 - (x + 59) = 121 - x$

Answer BAC = x ° [1]

BCA = $121 - x$ ° [2]

18 The diagram represents the frustum of a solid cone formed when a small cone is removed from the top of a large cone.



all measurements in cm

Work out the total surface area of the frustum.

You must show all your working.

Slant height of large cone = 10 + 5 = 15 cm

Curved area = $\pi \times 9 \times 15 - \pi \times 3 \times 5$

$= 135\pi - 15\pi = 120\pi = 376.9911184$ cm^2

Top + base = $\pi \times 3^2 + \pi \times 9^2 = 9\pi + 81\pi = 90\pi$

$= 282.7433388$ cm^2

Total = 376.9911184 + 282.7433388

$= 659.7344572 = 659.73$ cm^2

Answer: 659.73 cm^2 [4]

[Turn over

19 A is a fraction whose denominator is 3 more than its numerator.

A new fraction B is produced.

4 is added to the numerator of A to give the numerator of B.

4 is added to the denominator of A to give the denominator of B.

Fraction B is now larger than fraction A by $\frac{1}{8}$

By setting up and solving a suitable equation find the original fraction A.

A method of trial and improvement will not be accepted.

Let $A = \frac{x}{x+3}$ then $B = \frac{x+4}{x+7}$

$\frac{x+4}{x+7} - \frac{x}{x+3} = \frac{1}{8}$

$8[(x+4)(x+3) - x(x+7)] = (x+7)(x+3)$

$8[x^2 + 7x + 12 - x^2 - 7x] = x^2 + 10x + 21$

$96 = x^2 + 10x + 21$

$x^2 + 10x - 75 = 0$

$(x+15)(x-5) = 0$

$x = 5$ or $x = -15$

$x = 5$: $A = \frac{5}{8}$, $B = \frac{9}{12} = \frac{3}{4}$, $\frac{3}{4} - \frac{5}{8} = \frac{1}{8}$ ✓

($x = -15$ gives $A = \frac{-15}{-12}$, also works)

Answer $\frac{5}{8}$ or $\frac{-15}{-12}$ [8]

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