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ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2025

Centre Number

Candidate Number

Mathematics

StemUp Model Solution

Assessment Unit AS 2
assessing
Applied Mathematics

[SMT21]

SMT21

TUESDAY 27 MAY, AFTERNOON

TIME

1 hour 15 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Questions which require drawing or sketching should be completed using an HB pencil.
Candidates must answer **all** questions from sections A and B.
Equal time should be spent on each section. Show clearly the full development of your answers.
Answers without working may not gain full credit.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 70. The total available mark for each section of this paper is 35.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

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Answer all questions.

SECTION A

Mechanics

1 At time $t = 0$ seconds a particle is at rest at the point A.

The particle has a constant acceleration of $(3\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-2}$

(i) Find the velocity of the particle when $t = 5$ [3]

$v = u + at, \quad u = 0$

$v = 0 + 5(3i + 4j)$

$v = (15i + 20j)\ m\ s^{-1}$

(ii) Find the distance travelled by the particle from $t = 0$ to $t = 5$ [5]

$s = ut + \frac{1}{2}at^2$

$s = 0(5) + \frac{1}{2}(3i + 4j)(5^2)$

$s = (25/2)(3i + 4j)$

$s = (37.5i + 50j)\ m$

Motion is in a straight line from rest, so

distance $= |s| = \sqrt{(37.5^2 + 50^2)}$

$= \sqrt{3906.25}$

Distance travelled = 62.5 m

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Point A has a position vector $(-6\mathbf{i} + 2\mathbf{j})$ m relative to a fixed point O.

(iii) Find the displacement of the particle from O when $t = 5$ [2]

Displacement from O = (−6i + 2j) + (37.5i + 50j)
= (31.5i + 52j) m

[Turn over

2 An athlete is training for a race.

During one of her training sessions, she runs along a horizontal path in a straight line.

Fig. 1 below shows the displacement (km) of the athlete at time t (hours) from her starting point O.

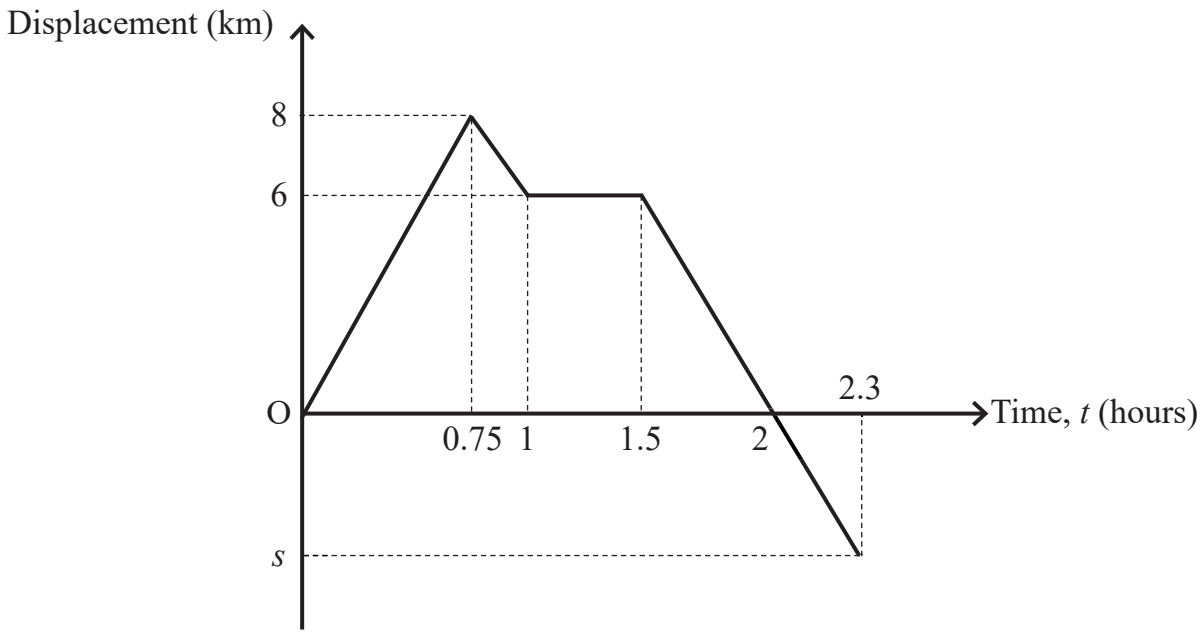


Fig. 1

(i) Find the velocity of the athlete from $t = 0.75$ to $t = 1$ [2]

Velocity = gradient = (6 − 8) ÷ (1 − 0.75)

= −2 ÷ 0.25

Velocity = −8 km h^{-1}

The athlete runs at a constant velocity from $t = 1.5$ to $t = 2.3$

(ii) Find the displacement of the athlete from O at $t = 2.3$ [4]

Constant velocity from $t = 1.5$ to $t = 2.3$, so use $t = 1.5$ to $t = 2$:

velocity $= (0 - 6) \div (2 - 1.5) = -6 \div 0.5 = -12$ km h^{-1}

From $t = 2$ to $t = 2.3$: velocity $= (s - 0) \div (2.3 - 2)$

$-12 = s \div 0.3$

$s = -12 \times 0.3$

$s = -3.6$ km

Displacement from O at $t = 2.3$ is -3.6 km

(i.e. 3.6 km from O on the opposite side to her starting direction)

[Turn over

3 **Fig. 2** below shows a lift of mass 1000 kg.

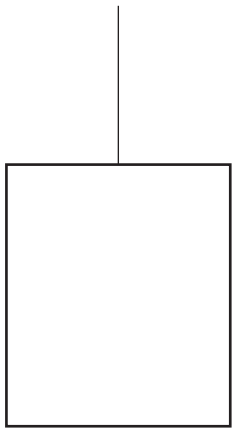


Fig. 2

A vertical cable attached to the top of the lift raises and lowers it.

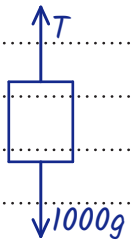
(i) If the empty lift is travelling at constant speed, find the tension in the cable. [2]

Constant speed $\Rightarrow a = 0$

$R(\uparrow)$: $T - 1000g = 1000 \times 0$

$T = 1000 \times 9.8$

Tension = 9800 N



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Safety guidance recommends that the greatest tension the cable can withstand is 18 000 N.

(ii) If the average mass of a passenger is 62 kg, find the maximum number of passengers that the lift can safely carry when it is accelerating upwards at $0.5\,\text{m s}^{-2}$ [6]

Let n = number of passengers.

Total mass = $(1000 + 62n)$ kg

N2L (↑): $T - 1000g - 62ng = (1000 + 62n)(0.5)$

$T = (1000 + 62n)(0.5) + 1000g + 62ng$

Cable must not exceed its limit: $T \leq 18\,000$

$(1000 + 62n)(0.5) + 1000g + 62ng \leq 18\,000$

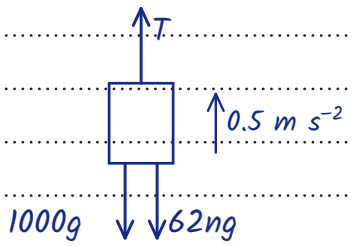
$500 + 31n + 9800 + 607.6n \leq 18\,000$

$638.6n \leq 7700$

$n \leq 12.0576...$

n must be a whole number, so

maximum number of passengers = 12



[Turn over

4 **Fig. 3** below shows a box at rest on a rough plane inclined at 30° to the horizontal.

The coefficient of friction between the box and the plane is $\frac{2}{9}$

When a horizontal force of magnitude 24 N is applied to the box, it is on the point of slipping down the plane.

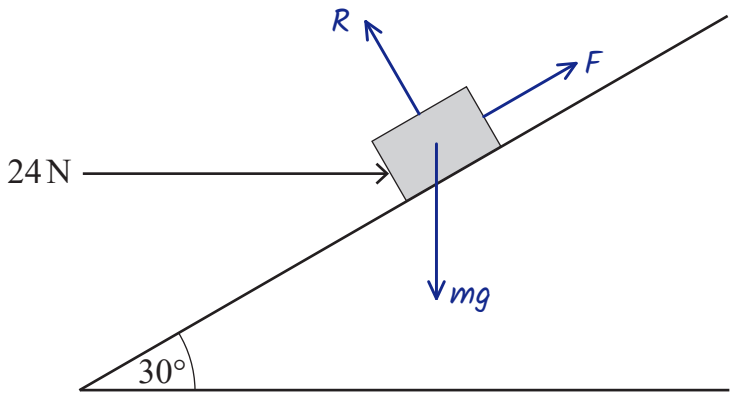


Fig. 3

(i) Complete the diagram above, showing all of the external forces acting on the box. [2]

(ii) Find the mass of the box. [9]

On the point of slipping down, so friction acts up the plane and $F = \mu R$

Resolve perpendicular to the plane:

$R = mg \cos 30° + 24 \sin 30°$

$R = 8.4870...m + 12$

Resolve parallel to the plane:

$F + 24 \cos 30° = mg \sin 30°$

Limiting friction: $F = (2/9)R = (2/9)(mg \cos 30° + 24 \sin 30°)$

$(2/9)(mg \cos 30° + 24 \sin 30°) + 24 \cos 30° = mg \sin 30°$

$1.8860...m + 2.6666... + 20.7846... = 4.9m$

i.e. $(\sqrt{3}/9)mg + 8/3 + 12\sqrt{3} = \frac{1}{2}mg$

$(\frac{1}{2} - \sqrt{3}/9)mg = 12\sqrt{3} + 8/3$

Q4 (ii) continued

3.0139...m = 23.4513...

m = 23.4513... ÷ 3.0139...

m = 7.7808...

Mass of the box = 7.78 kg

[Turn over



SECTION B

Statistics

5 Joan has collected a large set of data relating to tourism statistics for Northern Ireland.

Before using her data, Joan is planning to clean it.

(i) List three possible problems which Joan might encounter with the data. [3]

1. Some data values may be missing.
2. There may be outliers in the data.
3. Some data may be recorded in an incorrect format (or contain errors, e.g. typing mistakes).

Joan wishes to use a histogram to display the nationalities of visitors to Northern Ireland.

(ii) Explain why Joan's choice of diagram is not appropriate. [1]

Nationality is qualitative (categorical) data, not numerical, so a histogram (for continuous numerical data) is not appropriate; a bar chart or pie chart should be used instead.

Joan estimates the total number of visitors to the Giant's Causeway during 2023 by finding the number of visitors in July 2023 and multiplying this by 12

(iii) Comment on Joan's method for estimating the total number of visitors to the Giant's Causeway during 2023 [1]

The method is likely to be inaccurate (an overestimate): July is in the summer peak season, so there are probably more visitors in July than in most other months of the year.

[Turn over

6 The lengths of time, in hours, a number of patients waited at the emergency department of a hospital are shown in **Table 1** below.

Table 1

Time, t (hours)	Frequency
$0 < t \leqslant 3$	28
$3 < t \leqslant 6$	72
$6 < t \leqslant 9$	a
$9 < t \leqslant 12$	8
More than 12	0

An estimate of the mean waiting time is 5.1 hours.

(i) Show that $a = 42$ [3]

Midpoints: 1.5, 4.5, 7.5, 10.5

$$\text{mean} = \frac{28 \times 1.5 + 72 \times 4.5 + 7.5a + 8 \times 10.5}{28 + 72 + a + 8} = 5.1$$

$$\frac{7.5a + 450}{a + 108} = 5.1$$

$7.5a + 450 = 5.1a + 550.8$

$2.4a = 100.8$

$a = 100.8 \div 2.4 = 42$ as required

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(ii) Calculate an estimate of the variance of their waiting times for these patients. [3]

$\Sigma f = 28 + 72 + 42 + 8 = 150$

$\Sigma ft^2 = 28 \times 1.5^2 + 72 \times 4.5^2 + 42 \times 7.5^2 + 8 \times 10.5^2$

$= 63 + 1458 + 2362.5 + 882 = 4765.5$

$\text{variance} = \frac{\Sigma ft^2}{\Sigma f} - (\text{mean})^2 = \frac{4765.5}{150} - 5.1^2$

$= 31.77 - 26.01$

$\text{Variance} = 5.76 \text{ hours}^2$

These patients were asked to complete a questionnaire about their waiting times on one of the hospital's social media platforms.

(iii) Give a critique of this method of data collection. [1]

Patients may not remember exactly how long they waited, so the data may be inaccurate; also not all patients have access to (or use) social media, so the responses may not be representative (biased).

[Turn over

7 The Venn diagram in **Fig. 4** below shows the number of students in Year 13 in a school who study Spanish (S), Mathematics (M) and History (H).

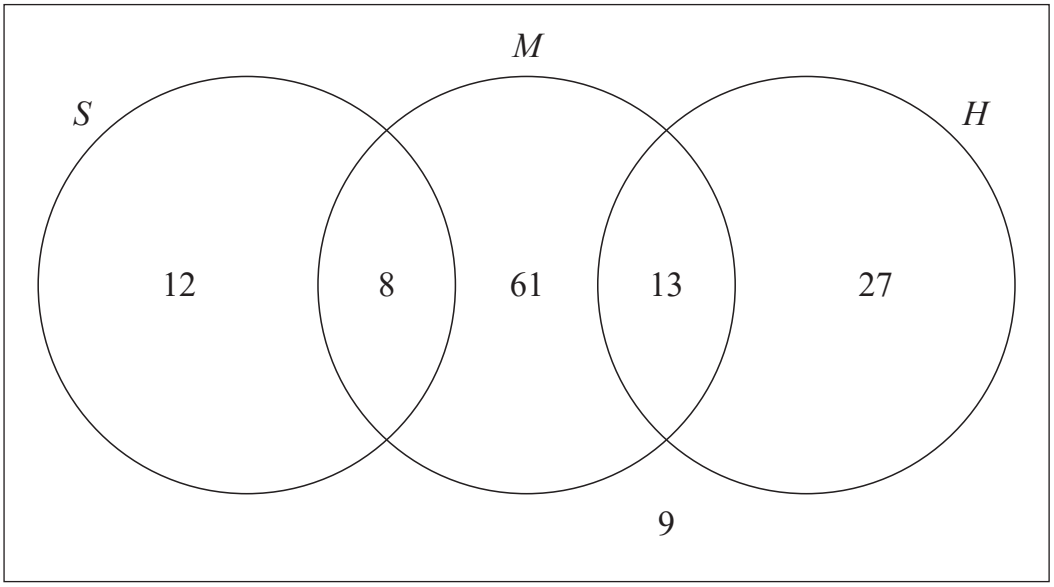


Fig. 4

(i) Explain the meaning of $\overline{S} \cap M$ in this context and state the number of students in this category. [2]

$\overline{S} \cap M$ is the set of Year 13 students who study Mathematics but do not study Spanish.

Number of students = 61 + 13 = 74

One Year 13 student is selected at random from this school.

(ii) Find the probability that this student:

(a) does not study Spanish or History; [2]

Total = 12 + 8 + 61 + 13 + 27 + 9 = 130

P(not S and not H) = $\frac{61 + 9}{130}$ = $\frac{70}{130}$ = $\frac{7}{13}$

(b) studies just one of the three subjects. [1]

P(just one subject) = $\frac{12 + 61 + 27}{130}$ = $\frac{100}{130}$ = $\frac{10}{13}$

(iii) Show why studying Spanish and studying History are not independent of each other. [2]

P(S) = $\frac{20}{130}$ = $\frac{2}{13}$, P(H) = $\frac{40}{130}$ = $\frac{4}{13}$

P(S) × P(H) = $\frac{2}{13}$ × $\frac{4}{13}$ = $\frac{8}{169}$ ≠ 0

but P(S ∩ H) = 0 (no student studies both), so P(S ∩ H) ≠ P(S) × P(H)

∴ studying Spanish and studying History are not independent.

[Turn over

8 David is researching a possible connection between the air quality, x, in 20 major cities and the rate of respiratory disease, y, among the population living there.

He calculates the following summary statistics:

$\Sigma x = 214.5$ $\Sigma y = 107.6$ $\Sigma x^2 = 2367.8$ $\Sigma y^2 = 665.3$ $\Sigma xy = 1081.2$

(i) Calculate the product-moment correlation coefficient between the air quality and the rate of respiratory disease for these 20 cities. [4]

$S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n} = 1081.2 - \frac{214.5 \times 107.6}{20} = -72.81$

$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 2367.8 - \frac{214.5^2}{20} = 67.2875$

$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n} = 665.3 - \frac{107.6^2}{20} = 86.412$

$r = \frac{S_{xy}}{\sqrt{(S_{xx} S_{yy})}} = \frac{-72.81}{\sqrt{(67.2875 \times 86.412)}}$

$= -0.95485...$

$r = -0.955$

(ii) Give an interpretation of the result in part **(i)**. [1]

Strong negative (linear) correlation: cities with higher values of air quality tend to have lower rates of respiratory disease.

In a different part of his research, for the same 20 cities, David obtains an equation for the line of regression of the average life expectancy, z years, on the air quality, x units.

The equation of his regression line is

$$z = 76.8 + 0.703x$$

(iii) Estimate the average life expectancy for the population of a city in which the air quality is 10.5 units. [1]

$z = 76.8 + 0.703 \times 10.5 = 84.1815$

Average life expectancy ≈ 84.2 years

(iv) Explain what is represented by the gradient of this regression line. [2]

The gradient 0.703 means that the average life expectancy increases by 0.703 years for every increase of 1 unit in the air quality.

[Turn over

9 Elaine runs a website selling second-hand furniture.

Over a period of time, Elaine estimates that 15% of all transactions on her website are fraudulent.

She takes a random sample of 50 transactions during one month and plans to use the binomial distribution to model the number of fraudulent transactions.

(i) State three conditions which must be satisfied for Elaine to use the binomial distribution as a model for the number of fraudulent transactions. [3]

1. There are only two outcomes for each transaction: fraudulent or not fraudulent.
2. Transactions are independent of each other.
3. The probability that a transaction is fraudulent is the same (0.15) for every transaction, with a fixed number of trials (50).

(ii) Using this model, calculate the probability that:

(a) exactly 8 transactions are fraudulent; [2]

X = number of fraudulent transactions, $X \sim B(50, 0.15)$

$P(X = 8) = {}^{50}C_8 (0.15)^8 (0.85)^{42}$

$= 0.14934...$

$= 0.149$

(b) at least 10 of the transactions are fraudulent. [3]

$P(X \geq 10) = 1 - P(X \leq 9)$

$= 1 - 0.79109...$

$= 0.20890...$

$= 0.209$

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THIS IS THE END OF THE QUESTION PAPER

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Question Number	**Marks**
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Total Marks	

Examiner Number

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