Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2025

Centre Number

Candidate Number

Mathematics

StemUp Model Solution

Assessment Unit AS 1
assessing
Pure Mathematics

[SMT11]

SMT11

THURSDAY 15 MAY, AFTERNOON

TIME

1 hour 45 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all nine** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an HB pencil.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions. **Answers without working may not gain full credit**.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

15857

1 Solve the simultaneous equations

$$\begin{aligned} 4x + 3y - z &= 18 \\ 5x - 2y + z &= 3 \\ 7x - 4y - 3z &= 5 \end{aligned}$$ [7]

$4x + 3y - z = 18$ (1)

$5x - 2y + z = 3$ (2)

$7x - 4y - 3z = 5$ (3)

(1) + (2): $9x + y = 21$ (4)

3 × (2): $15x - 6y + 3z = 9$

3 × (2) + (3): $22x - 10y = 14$ (5)

10 × (4): $90x + 10y = 210$

(5) + 10 × (4): $112x = 224$

$x = 2$

In (4): $9(2) + y = 21 \Rightarrow y = 3$

In (2): $5(2) - 2(3) + z = 3$

$10 - 6 + z = 3 \Rightarrow z = -1$

Check in (3): $7(2) - 4(3) - 3(-1) = 14 - 12 + 3 = 5$ ✓

$\therefore x = 2, \quad y = 3, \quad z = -1$

[Turn over

2 **Fig. 1** below shows a sketch of the graph of the function $y = f(x)$

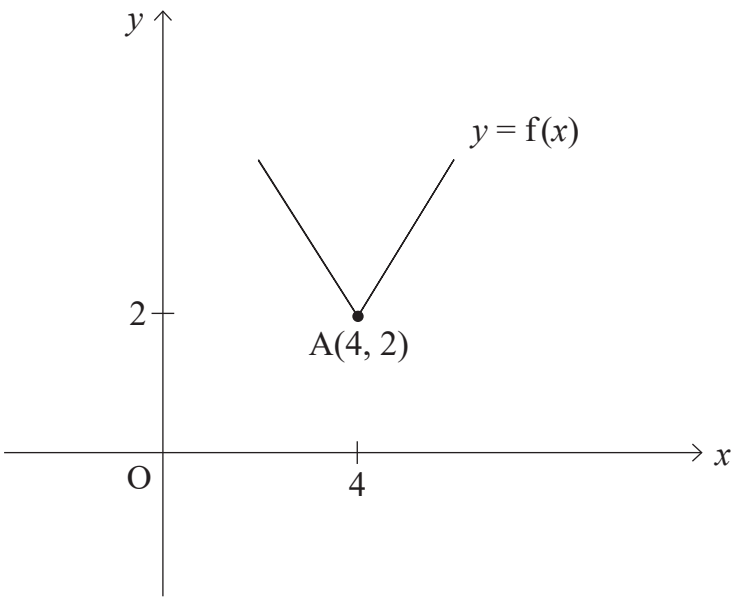


Fig. 1

Point A has coordinates (4, 2).

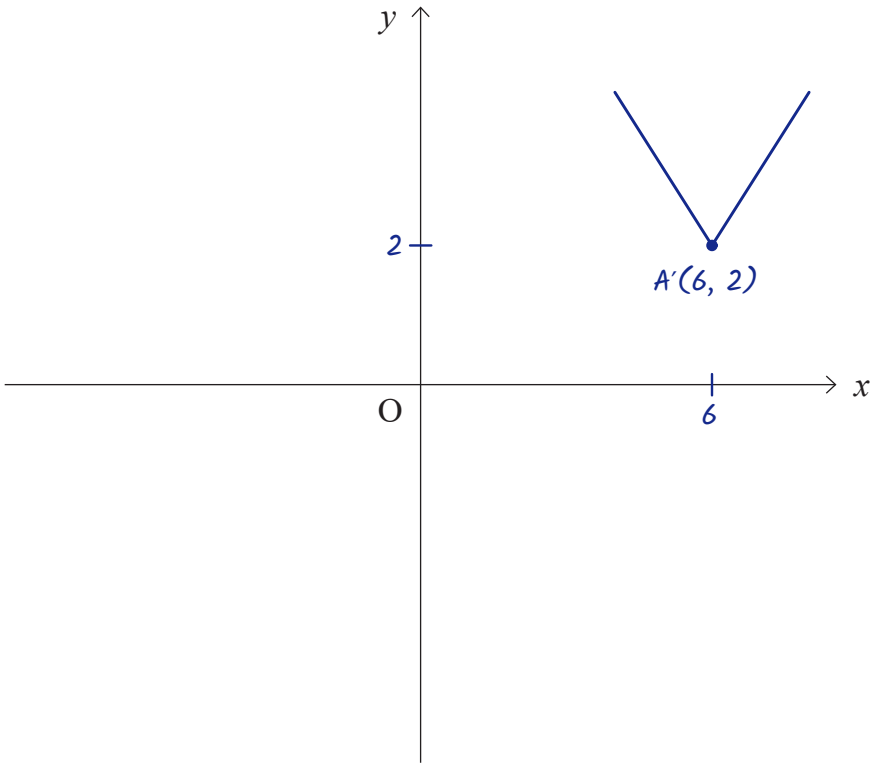
Sketch, on the axes opposite, the graphs of:

(i) $y = f(x - 2)$ [2]

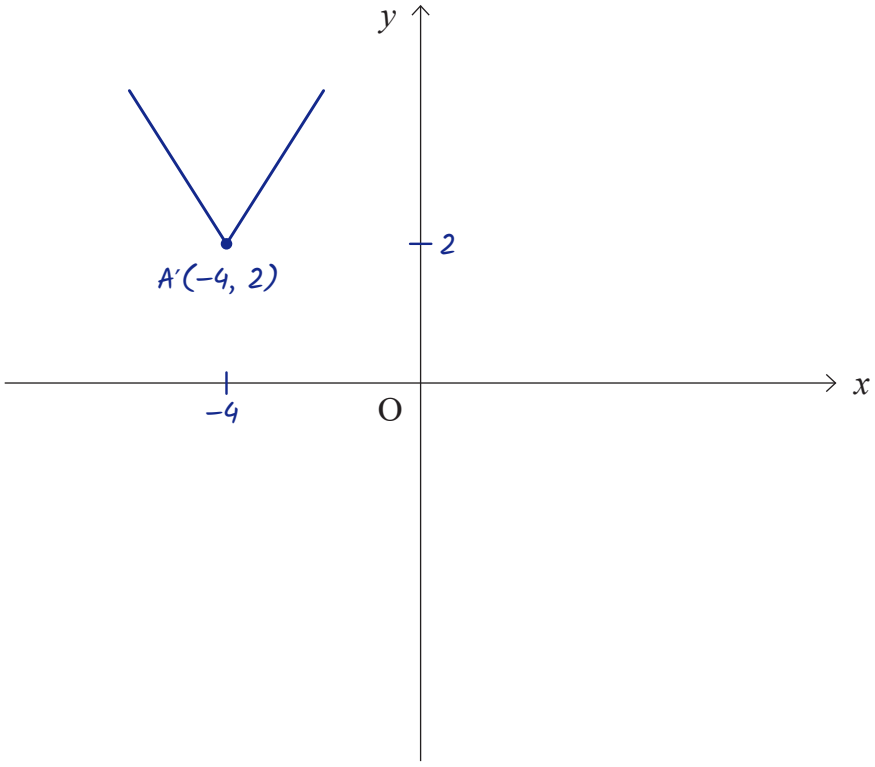
(ii) $y = f(-x)$ [2]

clearly labelling the image of the point A in each case.

(i)



(ii)



[Turn over

3 **(a)** Find the equation of the normal to the curve given by the equation

$$y = 2x^3 + 6x^2 - 5x - 12$$

at the point where $x = -2$

Leave your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found. [7]

When $x = -2$: $y = 2(-2)^3 + 6(-2)^2 - 5(-2) - 12$

$= -16 + 24 + 10 - 12 = 6$

$\frac{dy}{dx} = 6x^2 + 12x - 5$

At $x = -2$: $\frac{dy}{dx} = 6(4) + 12(-2) - 5 = -5$

Gradient of tangent $= -5$

Gradient of normal $= \frac{-1}{-5} = \frac{1}{5}$

Normal through $(-2, 6)$: $y - 6 = \frac{1}{5}(x + 2)$

$5y - 30 = x + 2$

$x - 5y + 32 = 0$

(b) Solve

$$x^{\frac{2}{3}} - 3x^{\frac{1}{3}} = 28$$ [6]

Let $u = x^{1/3}$, so $x^{2/3} = u^2$

$u^2 - 3u = 28$

$u^2 - 3u - 28 = 0$

$(u - 7)(u + 4) = 0$

$u = 7$ or $u = -4$

$x^{1/3} = 7 \Rightarrow x = 7^3 = 343$

$x^{1/3} = -4 \Rightarrow x = (-4)^3 = -64$

$\therefore x = 343$ or $x = -64$

[Turn over

4 **(a)** The shape in **Fig. 2** below shows a triangle PQR joined to a semicircle with diameter PR.

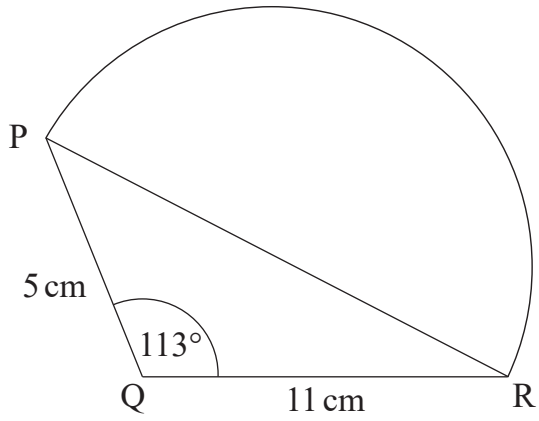


Fig. 2

PQ = 5 cm QR = 11 cm PQ̂R = 113°

Find the area of the shape. [7]

Cosine rule: $PR^2 = PQ^2 + QR^2 - 2(PQ)(QR)\cos PQR$

$PR^2 = 5^2 + 11^2 - 2(5)(11)\cos 113°$

$= 146 + 42.980... = 188.980...$

$PR = 13.747...$ cm

Area of triangle $= \frac{1}{2} \times 5 \times 11 \times \sin 113°$

$= 25.3138...$ cm^2

Radius of semicircle $= 13.747... \div 2 = 6.8735...$ cm

Area of semicircle $= \frac{\pi(6.8735...)^2}{2}$

$= 74.2124...$ cm^2

Total area $= 25.3138... + 74.2124...$

$= 99.526...$ cm^2

Area of shape = 99.5 cm^2 (3 s.f.)

[Turn over

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(b) Solve

$$2\cos 3x = -\sqrt{3} \qquad 0 < x < 180° \qquad [7]$$

$\cos 3x = -\frac{\sqrt{3}}{2}$

$0° < x < 180° \Rightarrow 0° < 3x < 540°$

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$

cos negative in 2nd and 3rd quadrants:

$3x = 180° - 30°,\ 180° + 30°,\ 360° + 150°$

$3x = 150°,\ 210°,\ 510°$

(next value 570° is outside the range)

$x = 50°,\ 70°,\ 170°$

5 **(i)** Prove that

$$\log_a x + \log_a y = \log_a xy \qquad [6]$$

Let $p = \log_a x$ and $q = \log_a y$

Then $a^p = x$ and $a^q = y$

$xy = a^p \times a^q$

$xy = a^{p+q}$

Taking logs to base a: $\log_a xy = p + q$

$\therefore \log_a xy = \log_a x + \log_a y$

(ii) Hence find the exact solution of the equation

$$\log_3 x + \log_3(x + 4) = 2$$ [6]

$\log_3 x + \log_3(x + 4) = 2$

$\log_3 x(x + 4) = 2$ (using part (i))

$x(x + 4) = 3^2 = 9$

$x^2 + 4x - 9 = 0$

$(x + 2)^2 - 4 - 9 = 0$

$(x + 2)^2 = 13$

$x + 2 = \pm\sqrt{13}$

$x = -2 + \sqrt{13}$ or $x = -2 - \sqrt{13}$

$x = -2 - \sqrt{13} < 0$, so $\log_3 x$ is undefined: reject

$x = -2 + \sqrt{13}$

[Turn over

6 **(a)** Expand

$$(1 - 2x)(2 + x)^8$$

in ascending powers of x up to and including the term in x^3 [7]

$(2 + x)^8 = 2^8 + {}^8C_1\, 2^7x + {}^8C_2\, 2^6x^2 + {}^8C_3\, 2^5x^3 + \ldots$

$= 256 + 8(128)x + 28(64)x^2 + 56(32)x^3 + \ldots$

$= 256 + 1024x + 1792x^2 + 1792x^3 + \ldots$

$(1 - 2x)(2 + x)^8 = (1 - 2x)(256 + 1024x + 1792x^2 + 1792x^3 + \ldots)$

$= 256 + 1024x + 1792x^2 + 1792x^3$

$- 512x - 2048x^2 - 3584x^3 + \ldots$

$= 256 + 512x - 256x^2 - 1792x^3 + \ldots$

(b) Show that

$$\tan x \equiv \frac{\sin x}{1 - \dfrac{\sin^2 x}{1 + \cos x}}$$ [7]

Denominator: $1 - \frac{\sin^2 x}{1 + \cos x} = \frac{1 + \cos x - \sin^2 x}{1 + \cos x}$

$= \frac{1 - \sin^2 x + \cos x}{1 + \cos x} = \frac{\cos^2 x + \cos x}{1 + \cos x}$ (since $1 - \sin^2 x = \cos^2 x$)

$= \frac{\cos x(\cos x + 1)}{1 + \cos x}$

$= \cos x$

So the right-hand side = sin x ÷ (denominator)

$RHS = \frac{\sin x}{\cos x} = \tan x$

$\therefore$ tan x ≡ RHS as required

[Turn over

7 **Fig. 3** below shows a solid prism.

Its cross-section is formed by joining an isosceles triangle to a rectangle.

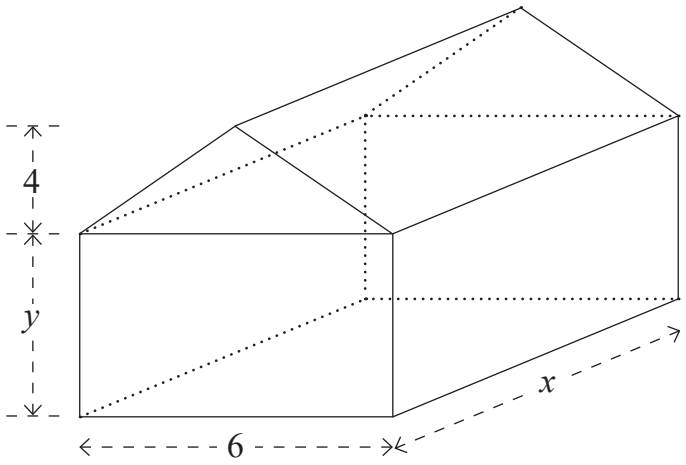


Fig. 3

The rectangle has width 6 metres and height y metres.
The isosceles triangle has perpendicular height 4 metres.

The prism has length x metres and volume $120\,\text{m}^3$

(i) Show that

$$y = \frac{20}{x} - 2$$ [4]

Area of cross-section = $6y + \frac{1}{2} \times 6 \times 4 = 6y + 12$

$V = x(6y + 12) = 6xy + 12x$

$6xy + 12x = 120$

$xy + 2x = 20$

$xy = 20 - 2x$

$y = \frac{20}{x} - 2$

(ii) Hence show that the total surface area S can be expressed as

$$S = 12x + \frac{240}{x} + 40$$ [7]

Sloping edge of triangle = $\sqrt{(3^2 + 4^2)} = 5$

S = 2 ends + base + 2 sides + 2 sloping faces

$S = 2(6y + 12) + 6x + 2xy + 2(5x)$

$S = 16x + 2xy + 12y + 24$

$S = 16x + 2x\left(\frac{20}{x} - 2\right) + 12\left(\frac{20}{x} - 2\right) + 24$

$S = 16x + 40 - 4x + \frac{240}{x} - 24 + 24$

$\therefore \; S = 12x + \frac{240}{x} + 40$

[Turn over

(iii) Hence find the value of x for which S is a minimum. [7]

$S = 12x + 240x^{-1} + 40$

$\frac{dS}{dx} = 12 - 240x^{-2} = 12 - \frac{240}{x^2}$

For a minimum $\frac{dS}{dx} = 0$: $12 - \frac{240}{x^2} = 0$

$12x^2 = 240$

$x^2 = 20$

$x = \sqrt{20} = 2\sqrt{5}$ $(x > 0$ as it is a length$)$

$\frac{d^2S}{dx^2} = 480x^{-3} = \frac{480}{x^3}$

At $x = 2\sqrt{5}$: $\frac{d^2S}{dx^2} = \frac{480}{(2\sqrt{5})^3} = 5.366... > 0$

$\Rightarrow$ minimum

S is a minimum when $x = 2\sqrt{5} = 4.47$ (3 s.f.)

[Turn over

8 **Fig. 4** below shows a sketch of the graphs of a curve and a straight line.

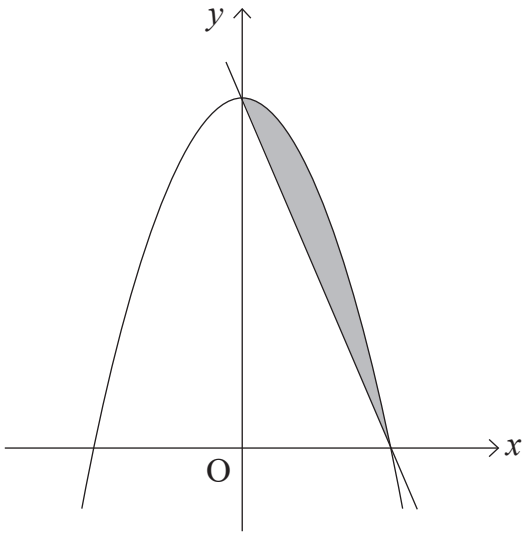


Fig. 4

The curve is given by the equation $y = a^2 - x^2$ where a is a positive constant.

The curve and the straight line meet at the x and y axes.

Find, in terms of a, the area of the shaded region which is enclosed between the curve and the line. [10]

Curve meets y-axis: $x = 0 \Rightarrow y = a^2$, i.e. $(0, a^2)$

Curve meets x-axis: $y = 0 \Rightarrow x = \pm a$; line meets at $(a, 0)$

Area under curve $= \int_0^a (a^2 - x^2)\,dx$

$= \left[a^2x - \frac{x^3}{3}\right]_0^a$

$= \left(a^3 - \frac{a^3}{3}\right) - (0) = \frac{2a^3}{3}$

Q8 continued

Line joins $(0, a^2)$ and $(a, 0)$: triangle with the axes

Area of triangle $= \frac{1}{2} \times a \times a^2 = \frac{a^3}{2}$

Shaded area $= \frac{2a^3}{3} - \frac{a^3}{2}$

$= \frac{4a^3}{6} - \frac{3a^3}{6}$

$= \frac{a^3}{6}$ units^2

[Turn over

9 A straight line is given by the equation $x + y = p$ where p is a constant $[p \neq 0]$.

The straight line is a tangent to the curve given by the equation

$$y^2 = 2p - 3xy$$

Find the value of p. [8]

$x + y = p \Rightarrow y = p - x$

Substitute into $y^2 = 2p - 3xy$:

$(p - x)^2 = 2p - 3x(p - x)$

$p^2 - 2px + x^2 = 2p - 3px + 3x^2$

$0 = 2x^2 - px + 2p - p^2$

$2x^2 - px + (2p - p^2) = 0$

Tangent $\Rightarrow$ equal roots $\Rightarrow b^2 - 4ac = 0$

$(-p)^2 - 4(2)(2p - p^2) = 0$

$p^2 - 16p + 8p^2 = 0$

$9p^2 - 16p = 0$

$p(9p - 16) = 0$

$p = 0$ (rejected, since $p \neq 0$) or $p = \frac{16}{9}$

$\therefore p = \frac{16}{9}$

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Question Number	**Marks**
1	
2	
3	
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Total Marks	

Examiner Number

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