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ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2022

Centre Number

Candidate Number

Mathematics

StemUp Model Solution

Assessment Unit AS 2
assessing
Applied Mathematics

SMT21

[SMT21]
THURSDAY 9 JUNE, MORNING

TIME

1 hour 15 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. Questions which require drawing or sketching should be completed using an HB pencil. **Do not write with a gel pen.**
Candidates must answer **all** questions from sections A and B.
Equal time should be spent on each section. Show clearly the full development of your answers.
Answers without working may not gain full credit.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 70. The total available mark for each section of this paper is 35.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

12949

Answer all questions.

SECTION A

Mechanics

1 Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, act on a body of mass 5 kg.

Given that

$$\mathbf{F}_1 = (3\mathbf{i} - 2\mathbf{j})\text{N} \qquad \mathbf{F}_2 = (7\mathbf{i} + 8\mathbf{j})\text{N} \qquad \mathbf{F}_3 = (-15\mathbf{i} + 9\mathbf{j})\text{N}$$

find the magnitude of the acceleration of the body. [6]

Resultant force $F = F_1 + F_2 + F_3$

$F = (3i - 2j) + (7i + 8j) + (-15i + 9j)$

$F = (-5i + 15j)$ N

Newton's second law: $F = ma$

$-5i + 15j = 5a$

$a = (-i + 3j)$ m s^{-2}

$|a| = \sqrt{((-1)^2 + 3^2)}$

$|a| = \sqrt{10} = 3.1622...$

Magnitude of acceleration = 3.16 m s^{-2}

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[Turn over

2 **Fig. 1** below shows a car of mass 2000 kg towing a trailer of mass 500 kg along a straight horizontal road.

The resistances experienced by the car and trailer are 400 N and 200 N respectively.

The car's engine produces a tractive force of 1800 N.

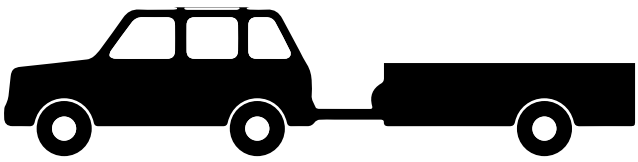


Fig. 1

Find the acceleration of the car and trailer and the tension in the tow bar. [7]

Whole system (car + trailer), total mass 2500 kg:

$R(\rightarrow)$: $1800 - 400 - 200 = 2500a$

$1200 = 2500a$

$a = 0.48\ m\ s^{-2}$

Car alone (tension T in the tow bar acts backwards on the car):

$R(\rightarrow)$: $1800 - 400 - T = 2000a$

$1400 - T = 2000 \times 0.48 = 960$

$T = 440\ N$

Check, trailer: $T - 200 = 500 \times 0.48 \Rightarrow T = 440\ N$

Acceleration = $0.48\ m\ s^{-2}$, tension in tow bar = 440 N

[Turn over

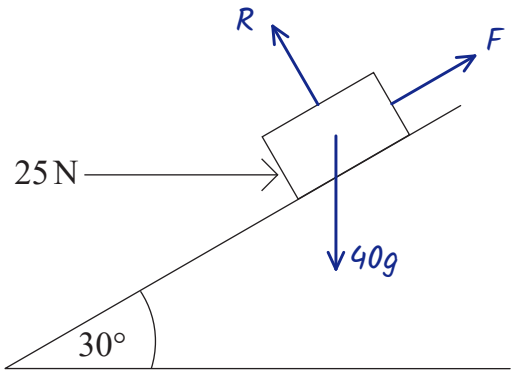
3 A crate of mass 40 kg is held at rest on a rough plane by a horizontal force of magnitude 25 N.

The plane is inclined at an angle of 30° to the horizontal.

All forces act in the same vertical plane.

The crate is on the point of sliding down the plane.

(i) Complete the diagram below showing all the external forces acting on the crate. [2]



(ii) Find the coefficient of friction between the crate and the plane. [8]

Resolve perpendicular to the plane:

$R = 40g \cos 30° + 25 \sin 30°$

$R = 339.48... + 12.5 = 351.98...$ N

Resolve parallel to the plane (friction acts up the plane):

$F + 25 \cos 30° = 40g \sin 30°$

$F = 196 - 21.650... = 174.35...$ N

On the point of sliding, friction is limiting: $F = \mu R$

$\mu(351.98...) = 174.35...$

$\mu = 174.35... \div 351.98... = 0.49533...$

$\mu = 0.495$ (3 s.f.)

[Turn over

4 **(a)** A ball is thrown vertically upwards with a speed of $20\,m\,s^{-1}$

Three seconds later a second ball is thrown vertically upwards from the same point with a speed of $22\,m\,s^{-1}$

(i) Find how long it takes from when the first ball is thrown until the two balls collide. [7]

Take upwards as positive, $a = -9.8\ m\ s^{-2}$

Let T s be the time after the first ball is thrown.

First ball: $u = 20$, $t = T$

$s = ut + \frac{1}{2}at^2$: $S = 20T - 4.9T^2$

Second ball: $u = 22$, $t = T - 3$

$S = 22(T - 3) - 4.9(T - 3)^2$

The balls collide when they are at the same height:

$20T - 4.9T^2 = 22(T - 3) - 4.9(T - 3)^2$

$20T - 4.9T^2 = 22T - 66 - 4.9T^2 + 29.4T - 44.1$

$20T = 51.4T - 110.1$

$31.4T = 110.1$

$T = 3.5063...$

Time until the balls collide = 3.51 s

(ii) Find the direction in which the first ball is moving when the two balls collide. [2]

First ball at $T = 3.5063...$: $v = u + at$

$v = 20 - 9.8 \times 3.5063...$

$v = -14.36... = -14.4 \text{ m s}^{-1}$

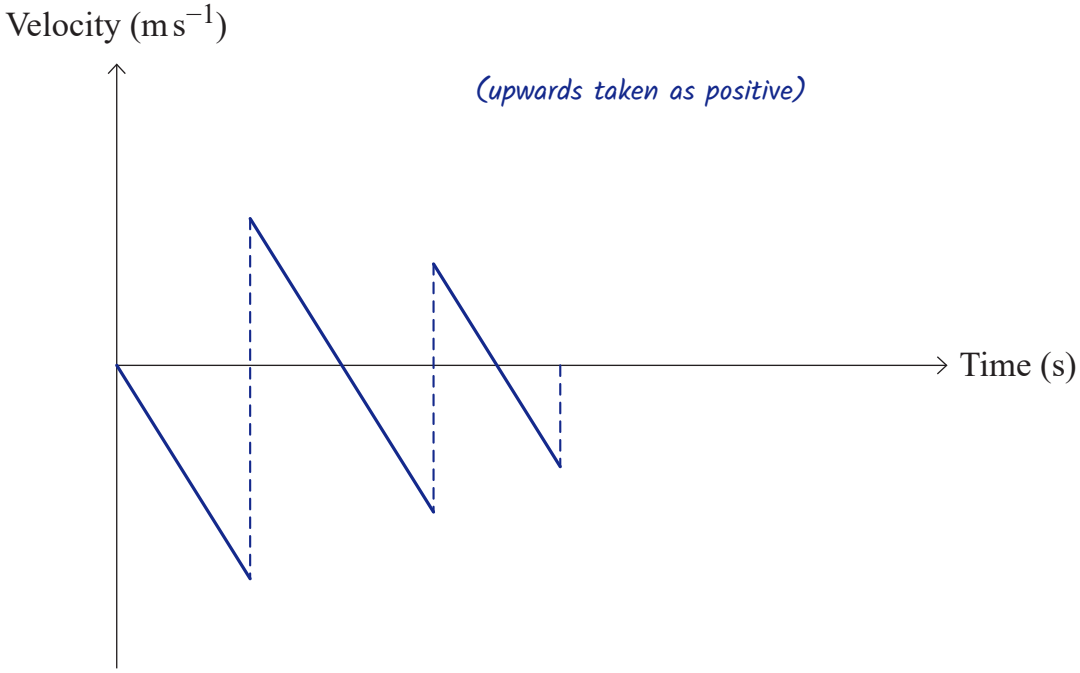
$v < 0$, so the first ball is moving downwards (at 14.4 m s^{-1}) when the two balls collide.

[Turn over

(b) A ball bearing is dropped from a height onto a horizontal metal plate.

Each time the ball bearing strikes the metal plate, it rebounds with its direction of motion reversed and its speed reduced.

On the axes below, sketch the velocity–time graph of the ball bearing until it strikes the plate for the third time. [3]



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SECTION B

Statistics

5 Kevin is doing a Geography project about weather.

He has organised his data in a spreadsheet using one row for each day's results.

Table 1 below shows part of Kevin's spreadsheet.

Table 1

	A	B	C	D	E	F
1	**Mean temperature (°C)**	**Rainfall (mm)**	**Mean windspeed (kn)**	**Sunshine (hours)**	**Mean pressure (hPa)**	**Mean relative humidity (%)**
2	12.2	0	6	6.6	1023	89
3	11.3	4.2	21	5.1	1008	AB
4	10.3	0.7	12	5.9	1000	95
5	8.5	0	6	8.2	1021	86
6	7.4	0.1	4	92	1025	81
7	9.6	6.1	9	3.7	1010	97
8	8.8	0	8	7.7	1012	88
9		2	13	2.1	1013	89
10	11.2	5.7	12	0.9	1022	95

(i) Identify three reasons why Kevin needs to clean his data. [3]

1. Cell A9 is blank: the mean temperature value is missing.
2. Cell D6 gives 92 hours of sunshine, which is impossible (more than 24 hours in a day), so it must be an error.
3. Cell F3 gives the humidity as "AB", which is not a valid number, so the value is unclear.

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Fig. 2 below shows the box plot Kevin drew for the rainfall, x mm, for all of the data in his spreadsheet.

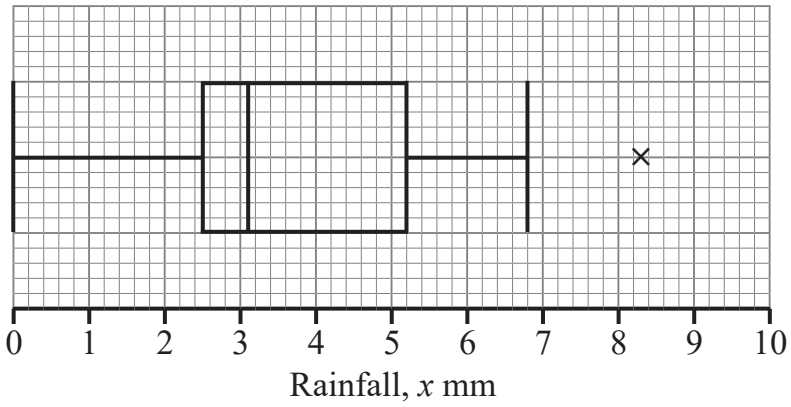


Fig. 2

Kevin claimed that one value was an outlier and marked it as × on the box plot.

An outlier is defined as a value which is outside the range

$$Q_1 - 1.5(Q_3 - Q_1) \leqslant x \leqslant Q_3 + 1.5(Q_3 - Q_1)$$

(ii) Show that Kevin's claim is not correct. [3]

From the box plot: $Q_1 = 2.5$, $Q_3 = 5.2$, marked value = 8.3

$Q_3 - Q_1 = 5.2 - 2.5 = 2.7$

Upper limit: $Q_3 + 1.5(Q_3 - Q_1) = 5.2 + 1.5(2.7) = 9.25$

Lower limit: $Q_1 - 1.5(Q_3 - Q_1) = 2.5 - 4.05 = -1.55$

$-1.55 \leqslant 8.3 \leqslant 9.25$ ($8.3 < 9.25$), so 8.3 lies inside the range and is not an outlier. Hence Kevin's claim is not correct.

[Turn over

6 An airline operates flights between Belfast and Edinburgh. Each passenger is allowed to check in one piece of hold luggage.

For one flight the mass, x kg, of the hold luggage of each of the 220 passengers is recorded.

The results are summarised below.

$\Sigma x = 2214.44$ $\Sigma x^2 = 31\,454.76$ $n = 220$

(i) Calculate the mean and variance for the masses of the hold luggage of the 220 passengers on this flight. [5]

Mean $= \frac{\Sigma x}{n} = \frac{2214.44}{220}$

Mean $= 10.065\ldots = 10.1$ kg

Variance $\sigma^2 = \frac{\Sigma x^2}{n} - (\text{mean})^2$

$\sigma^2 = \frac{31454.76}{220} - (10.065\ldots)^2$

$\sigma^2 = 142.976\ldots - 101.316\ldots$

Variance $= 41.660\ldots = 41.7$ kg^2

In addition, each passenger is allowed to carry one piece of hand luggage on board.

The mass, y kg, of the hand luggage of each of the same 220 passengers is also recorded.

Some further summary statistics are as follows:

$S_{xy} = 196.205$ $S_{xx} = 9165.012$ $S_{yy} = 2804.465$

(ii) Calculate the product–moment correlation coefficient between the masses of hand luggage and hold luggage for this flight. [2]

$r = \frac{S_{xy}}{\sqrt{(S_{xx} \cdot S_{yy})}}$

$r = \frac{196.205}{\sqrt{(9165.012 \times 2804.465)}}$

$r = 0.038721... = 0.0387$

(iii) Comment on the value you obtained in part **(ii)**. [2]

$r = 0.0387$ is very close to zero, so there is (virtually) no linear correlation between the masses of hold luggage and hand luggage for this flight.

[Turn over

7 It is known that one in six seeds in a large batch will not germinate when planted.

(i) Write down the probability that a seed from the batch will germinate. [1]

$P(\text{seed germinates}) = 1 - \frac{1}{6} = \frac{5}{6}$

Twelve seeds are sown.

(ii) Calculate the probability that exactly nine of these seeds germinate. [3]

X = number of seeds that germinate, $X \sim B(12, \frac{5}{6})$

$P(X = 9) = {}^{12}C_9 \left(\frac{5}{6}\right)^9 \left(\frac{1}{6}\right)^3$

$= 220 \times 0.19380... \times 0.0046296...$

$= 0.19740...$

$P(X = 9) = 0.197$

(iii) Calculate the probability that at most ten of these seeds germinate. [4]

$P(X \leq 10) = 1 - [P(X = 11) + P(X = 12)]$

$P(X = 11) = {}^{12}C_{11} \left(\frac{5}{6}\right)^{11}\left(\frac{1}{6}\right) = 0.26918...$

$P(X = 12) = \left(\frac{5}{6}\right)^{12} = 0.11216...$

$P(X \leq 10) = 1 - (0.26918... + 0.11216...)$

$= 1 - 0.38134... = 0.61865...$

$P(X \leq 10) = 0.619$

(iv) Find the most likely number of these twelve seeds that will germinate. [3]

Probabilities near the mean $np = 10$:

$P(X = 12) = 0.112$, $P(X = 11) = 0.269$

$P(X = 10) = {}^{12}C_{10} \left(\frac{5}{6}\right)^{10}\left(\frac{1}{6}\right)^{2} = 0.296$

$P(X = 9) = 0.197$, $P(X = 8) = 0.0888$

$P(X = 10)$ is the largest probability, so the most likely number of seeds to germinate is 10.

[Turn over

8 **Fig. 3** below shows a Venn diagram with two events, A and B, and probabilities associated with these events.

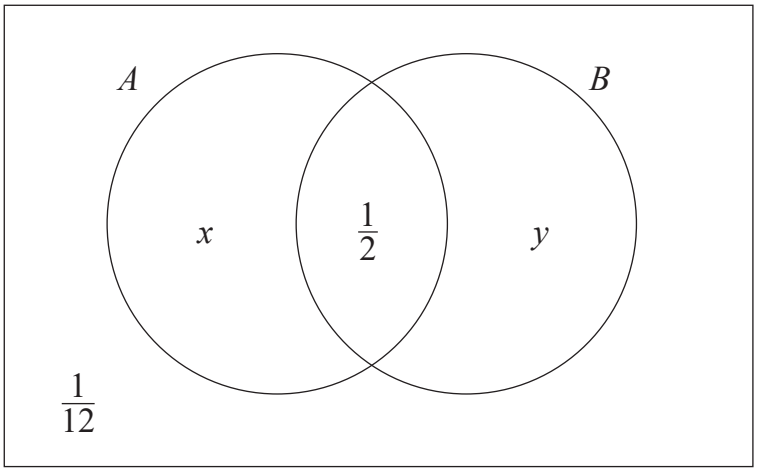


Fig. 3

(i) Show that $x + y = \frac{5}{12}$ [1]

Total probability = 1: $x + \frac{1}{2} + y + \frac{1}{12} = 1$

$x + y = 1 - \frac{7}{12} = \frac{5}{12}$ as required

(ii) Given that the events A and B are independent and that $P(A) > P(B)$, find the values of x and y. [8]

$P(A) = x + \frac{1}{2}$, $P(B) = y + \frac{1}{2}$, $P(A \cap B) = \frac{1}{2}$

Independent $\Rightarrow$ $P(A \cap B) = P(A) \times P(B)$

Q8 (ii) continued

$(x + \frac{1}{2})(y + \frac{1}{2}) = \frac{1}{2}$

$xy + \frac{1}{2}(x + y) + \frac{1}{4} = \frac{1}{2}$

$xy + \frac{1}{2}(\frac{5}{12}) - \frac{1}{4} = 0 \quad \Rightarrow \quad xy = \frac{1}{24}$

Substitute $y = \frac{5}{12} - x$: $\quad x(\frac{5}{12} - x) = \frac{1}{24}$

$\frac{5}{12}x - x^2 = \frac{1}{24}$

$\times 24$: $\quad 10x - 24x^2 = 1$

$24x^2 - 10x + 1 = 0$

$(6x - 1)(4x - 1) = 0$

$x = \frac{1}{6} \quad$ or $\quad x = \frac{1}{4}$

$\Rightarrow \quad y = \frac{1}{4} \quad$ or $\quad y = \frac{1}{6}$

$P(A) > P(B) \Rightarrow x > y$, so $x = \frac{1}{4}$, $y = \frac{1}{6}$

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