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ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2022

Centre Number

Candidate Number

# Mathematics

StemUp Model Solution

Assessment Unit AS 1
*assessing*
Pure Mathematics

*SMT11*

## [SMT11]
## THURSDAY 19 MAY, AFTERNOON

**TIME**

1 hour 45 minutes.

**INSTRUCTIONS TO CANDIDATES**

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all nine** questions in the spaces provided.
**Do not write outside the boxed area on each page or on blank pages.**
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an HB pencil.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions. Answers without working may not gain full credit.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

**INFORMATION FOR CANDIDATES**

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

12955.05 **R**

**1** Solve the simultaneous equations

$$\begin{aligned} x + 3y &= 10 \\ 2x - y + 4z &= 2 \\ 3x + 2y - 3z &= 19 \end{aligned}$$ [6]

(2) × 3: $6x - 3y + 12z = 6$

(3) × 4: $12x + 8y - 12z = 76$

Add: $18x + 5y = 82$ (4)

(1) × 18: $18x + 54y = 180$

(4) − (1) × 18: $-49y = -98 \Rightarrow y = 2$

$x = 10 - 3(2) = 4$

In (2): $8 - 2 + 4z = 2 \Rightarrow 4z = -4 \Rightarrow z = -1$

$x = 4, \quad y = 2, \quad z = -1$

**2** **(a)** **(i)** Write $2x^2 + 4x - 5$ in the form $2[(x+p)^2 - q]$ [2]

$2x^2 + 4x - 5 = 2[x^2 + 2x - \frac{5}{2}]$

$= 2[(x+1)^2 - 1 - \frac{5}{2}]$

$= 2[(x+1)^2 - \frac{7}{2}]$

$p = 1, \quad q = \frac{7}{2}$

**(ii)** Hence state the minimum value of $2x^2 + 4x - 5$ and the value of $x$ at which it occurs. [2]

Minimum value $= 2(0 - 7/2) = -7$

occurring when $x + 1 = 0$, i.e. $x = -1$

**(b)** In a controlled experiment, the number of bacteria $N$ after $t$ hours is given by

$$N = 500e^{2t}$$

**(i)** Find the initial value of $N$. [1]

$t = 0$: $N = 500e^0 = 500$

**(ii)** Find, in whole minutes, the time for $N$ to increase by 48% [4]

48% increase: $N = 1.48 \times 500 = 740$

$500e^{2t} = 740$

$e^{2t} = 1.48$

$2t = \ln 1.48$

$t = \frac{\ln 1.48}{2} = 0.19601...$ hours

$= 0.19601... \times 60 = 11.76...$ minutes

$t = 12$ minutes

**[Turn over**

**3** **(a)** Solve $\sin(2x - 60°) = -0.5$ $0° \leqslant x \leqslant 360°$ [5]

Let $y = 2x - 60°$, so $-60° \leq y \leq 660°$

$\sin y = -0.5 \Rightarrow$ principal value $y = -30°$

$y = -30°, 180° + 30° = 210°, 360° - 30° = 330°,$

$360° + 210° = 570°$

$2x - 60° = -30°, 210°, 330°, 570°$

$2x = 30°, 270°, 390°, 630°$

$x = 15°, 135°, 195°, 315°$

**(b)** Show that

$$\frac{\tan x \cos x}{(1-\cos^2 x)^{\frac{1}{2}}} \equiv 1$$

[4]

$\sin^2 x + \cos^2 x \equiv 1 \quad \Rightarrow \quad 1 - \cos^2 x \equiv \sin^2 x$

so $(1 - \cos^2 x)^{1/2} \equiv \sqrt{(\sin^2 x)} \equiv \sin x$

$\tan x \cos x \equiv \frac{\sin x}{\cos x} \times \cos x \equiv \sin x$

$\therefore \quad LHS \equiv \frac{\sin x}{\sin x} \equiv 1 \equiv RHS$

**[Turn over**

**4** A curve is given by the equation $y = x^2 + x$

**(i)** Find the equation of the normal to the curve at the point where $x = 2$

Leave your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers. [7]

$y = x^2 + x \Rightarrow \frac{dy}{dx} = 2x + 1$

At $x = 2$: $y = 4 + 2 = 6$

gradient of tangent $= 2(2) + 1 = 5$

gradient of normal $= -\frac{1}{5}$

$y - 6 = -\frac{1}{5}(x - 2)$

$5y - 30 = -x + 2$

$x + 5y - 32 = 0$

**(ii)** Find the values of $k$ for which the line $y = kx - 4$ is a tangent to the curve $y = x^2 + x$ [7]

$x^2 + x = kx - 4$

$x^2 + (1 - k)x + 4 = 0$

Tangent $\Rightarrow$ equal roots $\Rightarrow b^2 - 4ac = 0$

$(1 - k)^2 - 4(1)(4) = 0$

$k^2 - 2k + 1 - 16 = 0$

$k^2 - 2k - 15 = 0$

$(k - 5)(k + 3) = 0$

$k = 5$ or $k = -3$

**[Turn over**

**5** **(a)** Integrate $5x^4 + \frac{1}{2x^3} - \frac{4}{\sqrt{x}}$ [4]

$\int (5x^4 + \frac{1}{2}x^{-3} - 4x^{-1/2})\,dx$

$= \frac{5x^5}{5} + \frac{x^{-2}}{2(-2)} - \frac{4x^{1/2}}{1/2} + c$

$= x^5 - \frac{1}{4x^2} - 8\sqrt{x} + c$

**(b)** Given that

$$\int_{-1}^{2} (x^2 - px + 3)\,dx = 8$$

find the value of $p$. [6]

$\int_{-1}^{2} (x^2 - px + 3)\,dx = \left[\frac{x^3}{3} - \frac{px^2}{2} + 3x\right]_{-1}^{2}$

$= \left(\frac{8}{3} - 2p + 6\right) - \left(-\frac{1}{3} - \frac{p}{2} - 3\right)$

$= 12 - \frac{3p}{2}$

$12 - \frac{3p}{2} = 8$

$\frac{3p}{2} = 4 \Rightarrow 3p = 8$

$p = \frac{8}{3}$

**[Turn over**

**6** $f(x) = qx^3 - 7x^2 + r$ where $q$ and $r$ are constants.

$f(x)$ has a factor $(x + 1)$

When $f(x)$ is divided by $(2x + 1)$ the remainder is 7

**(i)** Find the values of $q$ and $r$. [7]

$(x + 1)$ is a factor $\Rightarrow f(-1) = 0$

$-q - 7 + r = 0 \quad \Rightarrow \quad -q + r = 7 \qquad (1)$

Remainder 7 $\Rightarrow f\left(-\frac{1}{2}\right) = 7$

$-\frac{q}{8} - \frac{7}{4} + r = 7 \quad \Rightarrow \quad -q + 8r = 70 \qquad (2)$

$(2) - (1)$: $7r = 63 \quad \Rightarrow \quad r = 9$

$q = r - 7 = 2$

$q = 2, \quad r = 9$

**(ii)** Factorise fully the expression f($x$) [3]

$2x^2 - 9x + 9$

$x + 1 \;\overline{)\; 2x^3 - 7x^2 + 0x + 9}$

$2x^3 + 2x^2$

$-9x^2 + 0x$

$-9x^2 - 9x$

$9x + 9$

$9x + 9$

$0$

$f(x) = (x + 1)(2x^2 - 9x + 9)$

$f(x) = (x + 1)(2x - 3)(x - 3)$

**[Turn over**

**(iii)** Hence solve $2(2^{3y}) - 7(2^{2y}) + 9 = 0$ [5]

Let $x = 2^y$: $2x^3 - 7x^2 + 9 = 0$

$(x + 1)(2x - 3)(x - 3) = 0 \Rightarrow x = -1, \frac{3}{2}, 3$

$2^y = -1$: no solution since $2^y > 0$

$2^y = \frac{3}{2} \Rightarrow y = \frac{\ln 1.5}{\ln 2} = 0.58496... = 0.585$

$2^y = 3 \Rightarrow y = \frac{\ln 3}{\ln 2} = 1.58496... = 1.58$

$y = 0.585$ or $y = 1.58$

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**(Questions continue overleaf)**

7 **(a)** Write down the first four terms in ascending powers of $x$ in the expansion of

$$(2 - x)^8$$

[4]

$(2 - x)^8 = 2^8 + {}^8C_1(2^7)(-x) + {}^8C_2(2^6)(-x)^2$
$+ {}^8C_3(2^5)(-x)^3 + \ldots$
$= 256 + 8(128)(-x) + 28(64)x^2 + 56(32)(-x^3) + \ldots$
$= 256 - 1024x + 1792x^2 - 1792x^3 + \ldots$

**(b)** The term independent of $x$ in the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405

Find the possible values of $k$. [5]

General term: ${}^{10}C_r\,(\sqrt{x})^{10-r}\left(-\frac{k}{x^2}\right)^r$

Power of x: $\frac{10-r}{2} - 2r = 0 \Rightarrow 10 - 5r = 0 \Rightarrow r = 2$

Term $= {}^{10}C_2\,(\sqrt{x})^8\left(-\frac{k}{x^2}\right)^2 = 45x^4 \times \frac{k^2}{x^4} = 45k^2$

$45k^2 = 405$

$k^2 = 9$

$k = 3$ or $k = -3$

**8** **(a)** Solve $0.7^x < 5$ [5]

$\log 0.7^x < \log 5$

$x \log 0.7 < \log 5$

$\log 0.7 < 0$, so dividing by it reverses the inequality:

$x > \frac{\log 5}{\log 0.7}$

$x > -4.5123...$

$x > -4.51$

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**(Questions continue overleaf)**

**(b) Fig. 1** below shows a sketch of a curve with equation $y = 8 - x^3$

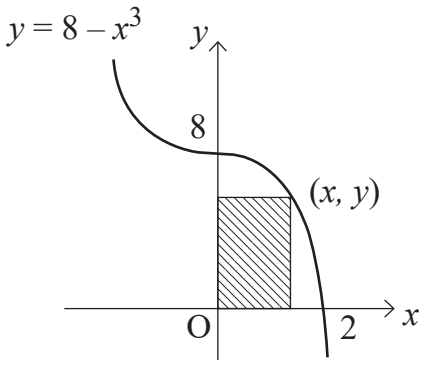



**Fig. 1**

Find the maximum area of a rectangle which can be inscribed in the closed region bounded by the curve and the positive $x$ and $y$ axes. [8]

Rectangle with vertex (x, y) on the curve, $0 < x < 2$:

$A = xy = x(8 - x^3)$

$A = 8x - x^4$

$\frac{dA}{dx} = 8 - 4x^3$

For a maximum $\frac{dA}{dx} = 0$: $8 - 4x^3 = 0$

$x^3 = 2 \Rightarrow x = \sqrt[3]{2}$ (= 1.2599...)

$y = 8 - 2 = 6$

$\frac{d^2A}{dx^2} = -12x^2 < 0$ at $x = \sqrt[3]{2} \Rightarrow$ maximum

Maximum area $A = 6\sqrt[3]{2} = 7.5595... = 7.56$ units$^2$

**9** **(a)** **Fig. 2** below shows a sketch of a circle with centre P(1,1).

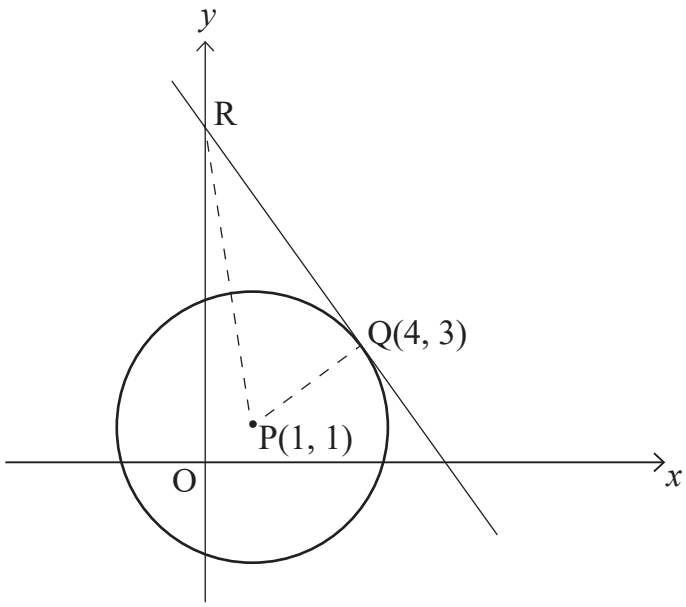



**Fig. 2**

The tangent to the circle at the point Q(4, 3) cuts the *y*-axis at point R.

Find the area of triangle PQR. [7]

Gradient PQ = $\frac{3-1}{4-1} = \frac{2}{3}$

Tangent ⊥ radius ⇒ gradient QR = $-\frac{3}{2}$

QR: $y - 3 = -\frac{3}{2}(x - 4) \Rightarrow y = -\frac{3}{2}x + 9$

$x = 0 \Rightarrow y = 9$, so R is (0, 9)

$PQ = \sqrt{(3^2 + 2^2)} = \sqrt{13}$

$QR = \sqrt{(4^2 + (-6)^2)} = \sqrt{52} = 2\sqrt{13}$

Angle PQR = 90° (tangent ⊥ radius)

Area PQR = $\frac{1}{2} \times \sqrt{13} \times 2\sqrt{13} = 13$ units$^2$

**(b)** **Fig. 3** below shows a sketch of a right-angled triangle ABC.

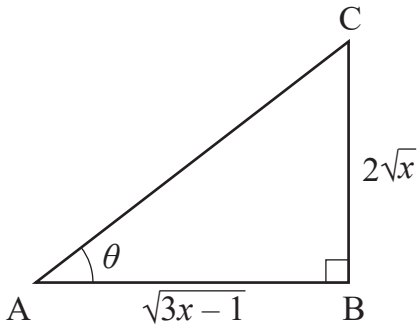



**Fig. 3**

AB has length $\sqrt{3x-1}$

BC has length $2\sqrt{x}$

$B\hat{A}C = \theta$

Given that $\cos^2\theta = \frac{1}{x}$, find the value of $x$. [8]

$AC^2 = AB^2 + BC^2 = (3x - 1) + 4x = 7x - 1$

$\cos\theta = \frac{AB}{AC} = \frac{\sqrt{(3x-1)}}{\sqrt{(7x-1)}}$

$\cos^2\theta = \frac{3x-1}{7x-1} = \frac{1}{x}$

$x(3x - 1) = 7x - 1$

$3x^2 - 8x + 1 = 0$

$x = \frac{8 \pm \sqrt{(64 - 4(3)(1))}}{6} = \frac{8 \pm \sqrt{52}}{6} = \frac{4 \pm \sqrt{13}}{3}$

$x = 2.5351...$ or $x = 0.13148...$

$AB = \sqrt{(3x - 1)}$ needs $3x - 1 > 0$, i.e. $x > 1/3$

so reject 0.131: $x = \frac{4 + \sqrt{13}}{3} = 2.54$

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| **Question Number** | **Marks** |
| 1 | |
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| **Total Marks** | |

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