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ADVANCED SUBSIDIARY (AS)
General Certificate of Education

Centre Number

Candidate Number

Mathematics

StemUp Model Solution

Assessment Unit AS 2
assessing
Applied Mathematics

SMT21

[SMT21]
Assessment

Assessment Level of Control:
Tick the relevant box (✓)

Controlled Conditions	
Other	

TIME

1 hour 15 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in black ink only. Questions which require drawing or sketching should be completed using an HB pencil. **Do not write with a gel pen.**
Candidates must answer **all** questions from sections A and B.
Equal time should be spent on each section. Show clearly the full development of your answers.
Answers without working may not gain full credit.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 70. The total available mark for each section of this paper is 35.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all questions.

SECTION A

Mechanics

1 A ball is projected vertically upwards from ground level with a speed of 30.4 $\mathrm{m\,s^{-1}}$

(i) Find the speed of the ball after two seconds. [2]

$u = 30.4,\ a = -g = -9.8,\ t = 2,\ v = ?$

$v = u + at$

$v = 30.4 + (-9.8)(2)$

$v = 10.8$

Speed after 2 seconds = 10.8 $m\ s^{-1}$

(ii) Find the maximum height reached by the ball. [3]

At maximum height $v = 0$; $u = 30.4$, $a = -9.8$, $s = ?$

$v^2 = u^2 + 2as$

$0^2 = 30.4^2 + 2(-9.8)s$

$19.6s = 924.16$

$s = 47.151...$

Maximum height = 47.2 m

(iii) State one modelling assumption made in answering this question. [1]

There is no air resistance acting on the ball

(the ball is modelled as a particle).

[Turn over

2 A particle of mass 2 kg is held in equilibrium when acted upon by three forces, $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$

$$\mathbf{F_1} = (3\mathbf{i} + 6\mathbf{j}) \text{ N}$$
$$\mathbf{F_2} = (2\mathbf{i} - 7\mathbf{j}) \text{ N}$$

(i) Find $\mathbf{F_3}$ [3]

In equilibrium the resultant force is zero:

$F_1 + F_2 + F_3 = 0$

$(3i + 6j) + (2i - 7j) + F_3 = 0$

$5i - j + F_3 = 0$

$F_3 = (-5i + j)$ N

The force $\mathbf{F}_1$ is increased by a factor of 10

(ii) If $\mathbf{F}_2$ and $\mathbf{F}_3$ are unchanged, find the magnitude of the acceleration produced and the angle it makes with the vector **i**. [7]

New $F_1 = 10(3i + 6j) = (30i + 60j)$ N

Resultant force $= F_1 + F_2 + F_3$

$= (30i + 60j) + (2i - 7j) + (-5i + j)$

$= (27i + 54j)$ N

$F = ma$: $27i + 54j = 2a$

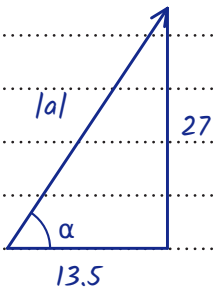
$a = (13.5i + 27j)$ m s^{-2}

$|a| = \sqrt{(13.5^2 + 27^2)} = \sqrt{911.25}$

$|a| = 30.186... = 30.2$ m s^{-2}

$\tan \alpha = 27 \div 13.5 = 2$

$\alpha = 63.434...° = 63.4°$



Acceleration 30.2 m s^{-2} at 63.4° to the vector i

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3 **Fig. 1** below shows crates P and Q resting on two planes each inclined at 45° to the horizontal.

P and Q are connected using a light, inextensible string passing over a smooth, light fixed pulley.

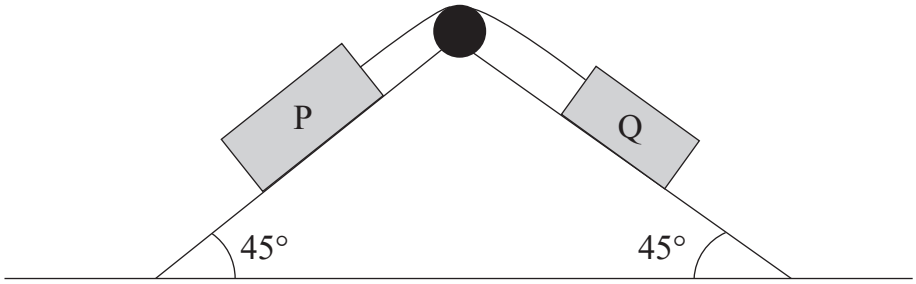
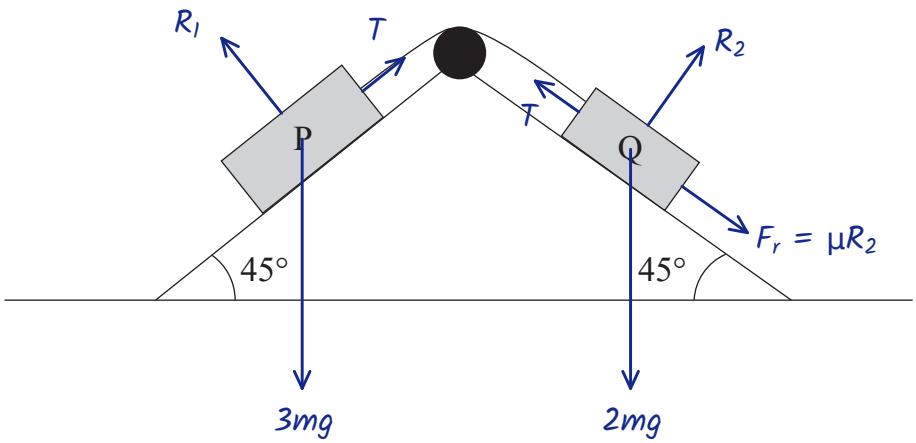


Fig. 1

P has mass $3m$ kg and rests on a smooth plane.
Q has mass $2m$ kg and rests on a rough plane with coefficient of friction μ.

The system is released from rest and motion occurs.

(i) Complete the diagram below to show all the external forces acting on P and Q. [2]



(ii) Find, in terms of μ and g, the acceleration produced. [7]

P, resolving down the plane: $3mg \sin 45° - T = 3ma$ (1)

Q, resolving up the plane: $T - F_r - 2mg \sin 45° = 2ma$ (2)

Q, perpendicular to the plane: $R_2 = 2mg \cos 45°$

Q is sliding, so $F_r = \mu R_2 = 2\mu mg \cos 45°$

(1) + (2): $3mg \sin 45° - 2\mu mg \cos 45° - 2mg \sin 45° = 5ma$

$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$: $\frac{3mg}{\sqrt{2}} - \frac{2\mu mg}{\sqrt{2}} - \frac{2mg}{\sqrt{2}} = 5ma$

$\div m$: $\frac{g}{\sqrt{2}} - \frac{2\mu g}{\sqrt{2}} = 5a$

$a = \frac{g - 2\mu g}{5\sqrt{2}} = \frac{g(1 - 2\mu)}{5\sqrt{2}}$ $m\,s^{-2}$

[Turn over

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The acceleration is found to be 1.3 $m\,s^{-2}$

(iii) Find the value of μ. [2]

$\frac{g(1-2\mu)}{5\sqrt{2}} = 1.3$

$9.8(1-2\mu) = 1.3 \times 5\sqrt{2} = 9.1923...$

$1-2\mu = 0.93800...$

$2\mu = 0.062001...$

$\mu = 0.0310$ (3 s.f.)

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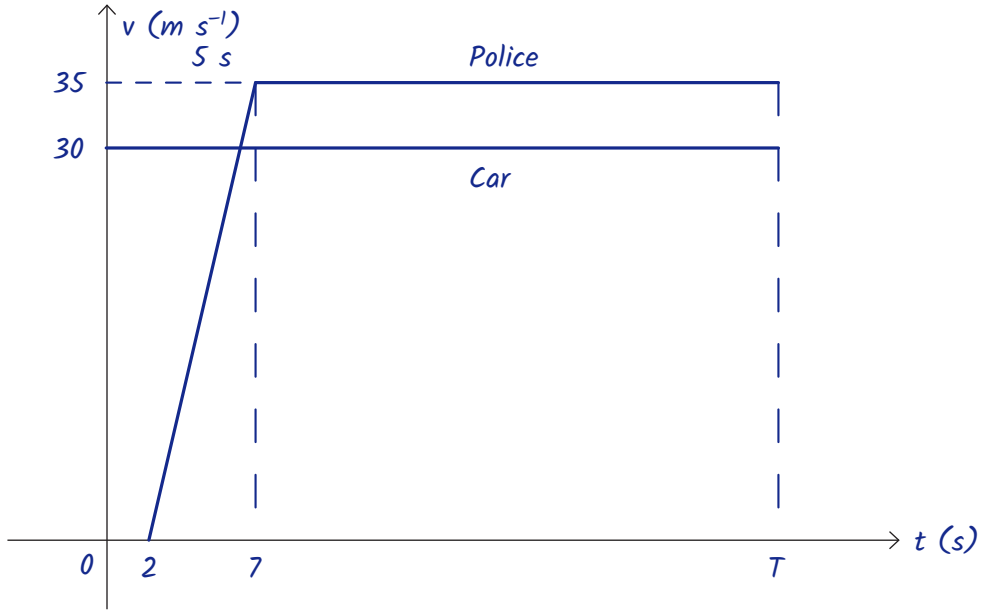
[Turn over

4 A car is travelling along a straight stretch of road at a constant speed of 30 $m\,s^{-1}$ when it passes a stationary police motorcycle at time $t = 0$ seconds.

Two seconds later the police motorcycle gives chase accelerating from 0 $m\,s^{-1}$ to 35 $m\,s^{-1}$ with constant acceleration of 7 $m\,s^{-2}$

The police motorcycle maintains this speed of 35 $m\,s^{-1}$ until it catches up with the car at time T.

By drawing a velocity–time graph find the value of T. [8]



Police: $v = u + at$, $35 = 0 + 7t \Rightarrow t = 5$ s, so it reaches 35 $m\,s^{-1}$ at $t = 7$

Distance travelled = area under the v–t graph

Car: $d = 30T$

Police: $d = \frac{1}{2}(T - 2 + T - 7)(35)$ (trapezium)

$d = 35T - 157.5$

Catches up when distances are equal: $30T = 35T - 157.5$

$5T = 157.5$

$T = 31.5$ s

[Turn over

SECTION B

Statistics

5 Over the past eight years, a farmer records the annual rainfall, x, in mm, and the annual yield of wheat, y, in bushels per hectare, on his farm.

Some of the summary values he calculated are:

$\Sigma xy = 181\,397.65$ $\quad \Sigma x^2 = 861\,132.58$ $\quad \Sigma y^2 = 39\,042.42$

$n = 8$ $\quad \Sigma x = 2512.8$ $\quad \Sigma y = 551.8$

(i) Calculate the product-moment correlation coefficient between the annual rainfall and the annual yield of wheat. [5]

$S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n} = 181397.65 - \frac{2512.8 \times 551.8}{8} = 8077.27$

$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 861132.58 - \frac{2512.8^2}{8} = 71862.1$

$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n} = 39042.42 - \frac{551.8^2}{8} = 982.015$

$r = \frac{S_{xy}}{\sqrt{(S_{xx} S_{yy})}} = \frac{8077.27}{\sqrt{(71862.1 \times 982.015)}}$

$r = 0.96151...$

$r = 0.962$ (3 s.f.)

(ii) Comment on what the value obtained in **(i)** suggests to the farmer. [1]

r is close to 1: strong positive correlation, so the greater the annual rainfall, the greater the annual yield of wheat.

Based on his data, the farmer calculates an equation of the y on x regression line to be

$$y = 33.7 + 0.112x$$

(iii) Give an interpretation, in context, of the value of the gradient of this regression line. [2]

For every additional 1 mm of annual rainfall, the annual yield of wheat increases by 0.112 bushels per hectare.

6 **(a)** The random variable X can take the values 3, 4, 5, 6 or 7

Table 1 below shows the probability that X takes each value.

Table 1

x	3	4	5	6	7
$P(X = x)$	0.2	c	0.47	0.09	$3c$

(i) Find the value of c. [2]

$\Sigma P(X = x) = 1$

$0.2 + c + 0.47 + 0.09 + 3c = 1$

$4c = 1 - (0.2 + 0.47 + 0.09) = 0.24$

$c = 0.06$

(ii) Find P($X < 5$). [2]

$P(X < 5) = P(X = 3) + P(X = 4)$

$= 0.2 + 0.06$

$= 0.26$

[Turn over

(b) For breakfast each day Ian eats either cereal or toast, and also drinks either fruit juice, tea or coffee.

The probability that Ian has toast is 0.4

The probability that he has fruit juice is 0.64 and he is equally likely to have either tea or coffee.

On a randomly chosen morning, find the probability that Ian

(i) has coffee; [1]

P(tea) = P(coffee) = (1 − 0.64) ÷ 2

P(coffee) = 0.36 ÷ 2 = 0.18

(ii) does not have toast or tea. [2]

P(not toast) = 1 − 0.4 = 0.6

P(not tea) = 1 − 0.18 = 0.82

P(no toast and no tea) = 0.6 × 0.82

= 0.492

[Turn over

7 The manager of a large call centre is concerned that the mean time taken to answer calls is longer than the target of 15 seconds.

One Tuesday morning, she records the time taken, t seconds, to answer each of the first 800 calls to the centre.

Some summary values are:

$$n = 800 \qquad \Sigma t = 13\,600 \qquad \Sigma t^2 = 366\,600$$

(i) Calculate the mean of the times taken to answer these calls. [2]

mean $= \frac{\Sigma t}{n} = \frac{13600}{800}$

mean = 17 seconds

(ii) What might this value suggest to the manager? [1]

The mean time of 17 s is more than the 15 second target, so the manager's concern appears to be justified.

(iii) Calculate an estimate of the variance of the times taken to answer a call. [2]

$s^2 = \frac{1}{n-1}\left(\Sigma t^2 - \frac{(\Sigma t)^2}{n}\right)$

$s^2 = \frac{1}{799}\left(366600 - \frac{13600^2}{800}\right)$

$s^2 = 169.4618...$

$s^2 = 169 \text{ seconds}^2$

The manager remembers that, on the morning she collected the data, some staff were involved in training and were not able to answer calls, so she decides to take another sample.

(iv) Suggest three improvements to the manager's strategy for sampling. [3]

1. Take times from a range of different days, not just Tuesday.
2. Take times spread throughout the day, not just the morning.
3. Take times from days over a number of weeks, on days when all staff are available to answer calls.

8 In a manufacturing process it is known that 3% of the components produced are faulty. Twenty components are randomly chosen and inspected.

(i) Find the probability that exactly two of the components in the batch are faulty. [3]

X = number of faulty components in the batch

$X \sim B(20, 0.03)$

$P(X = 2) = {}^{20}C_2 (0.03)^2 (0.97)^{18}$

$= 190 \times 0.0009 \times 0.97^{18}$

$= 0.098829...$

$= 0.0988$ (3 s.f.)

[Turn over

(ii) Find the probability that fewer than two components in the batch are faulty. [3]

$X \sim B(20, 0.03)$

$P(X < 2) = P(X = 0) + P(X = 1)$

$= (0.97)^{20} + {}^{20}C_1 (0.03)^1 (0.97)^{19}$

$= 0.543794... + 0.336367...$

$= 0.88016...$

$= 0.880$ (3 s.f.)

A batch contains n components which have been randomly selected.

The probability that no components are faulty in batches of this size is 0.2957 to 4 decimal places.

(iii) Find the value of n. [6]

Y = number of faulty components in the batch, $Y \sim B(n, 0.03)$

$P(Y = 0) = {}^nC_0 (0.03)^0 (0.97)^n = 0.97^n$

$0.97^n = 0.2957$

$\log 0.97^n = \log 0.2957$

$n \log 0.97 = \log 0.2957$

$n = \dfrac{\log 0.2957}{\log 0.97}$

$n = 40.001...$

$n = 40$

[Turn over

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For Examiner's use only	
Question Number	**Marks**
1	
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