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ADVANCED SUBSIDIARY (AS)
General Certificate of Education

Centre Number

Candidate Number

Mathematics

StemUp Model Solution

Assessment Unit AS 1
assessing
Pure Mathematics

SMT11

[SMT11]
Assessment

Assessment Level of Control:
Tick the relevant box (✓)

Controlled Conditions	
Other	

TIME
1 hour 45 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all ten** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an HB pencil.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions. **Answers without working may not gain full credit**.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

12365

1 **Fig. 1** below shows a sketch of the graph of the function $y = f(x)$.

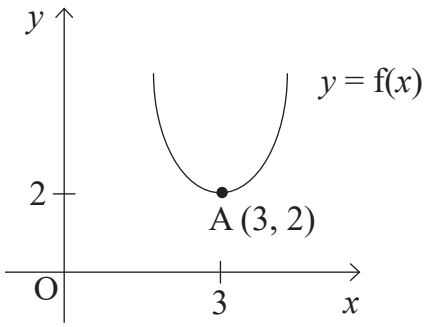


Fig. 1

Point A has coordinates (3, 2).

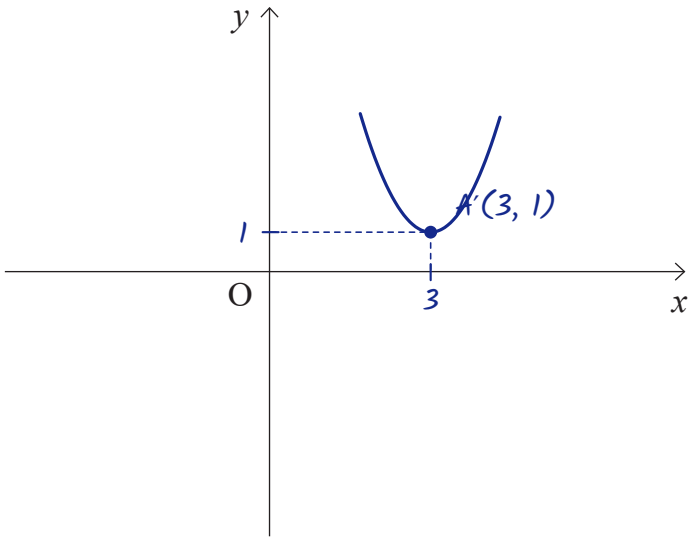
Sketch, on the axes opposite, the graphs of:

(i) $y = f(x) - 1$ [2]

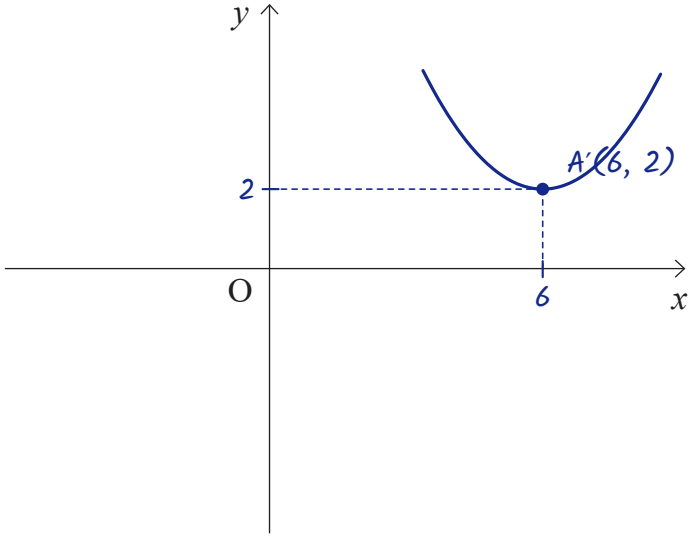
(ii) $y = f\left(\frac{x}{2}\right)$ [2]

clearly labelling the image of the point A in each case.

(i)



(ii)



[Turn over

2 **(a)** The vectors **a**, **b** and **c** are given by:

$$\mathbf{a} = 4\mathbf{i} + 3\mathbf{j}$$
$$\mathbf{b} = 6\mathbf{i} - \mathbf{j}$$
$$\mathbf{c} = x\mathbf{i} + 2\mathbf{j}$$

(i) Find $2\mathbf{a} - \mathbf{b}$ [2]

$2a - b = 2(4i + 3j) - (6i - j)$

$= 8i + 6j - 6i + j$

$= 2i + 7j$

(ii) If $|\mathbf{c}| = 6$, find the possible values of x. [4]

$|c| = \sqrt{(x^2 + 2^2)} = 6$

$x^2 + 2^2 = 6^2$

$x^2 + 4 = 36$

$x^2 = 32$

$x = \pm\sqrt{32}$

$x = 4\sqrt{2}$ or $x = -4\sqrt{2}$

(b) $f(x) = x^3 + x^2 - x + k$

$f(x)$ has a factor $(x - k)$

Find the possible values of k. [6]

$(x - k)$ is a factor $\Rightarrow$ $f(k) = 0$ (factor theorem)

$f(k) = k^3 + k^2 - k + k = 0$

$\Rightarrow k^3 + k^2 = 0$

$k^2(k + 1) = 0$

$k = 0$ or $k = -1$

[Turn over

3 **(a)** Solve the simultaneous equations

$$y - 3x - 2 = 0$$
$$x^2 + 2y = 20$$

[7]

$y - 3x - 2 = 0 \Rightarrow y = 3x + 2$

Substitute into $x^2 + 2y = 20$:

$x^2 + 2(3x + 2) = 20$

$x^2 + 6x + 4 = 20$

$x^2 + 6x - 16 = 0$

$(x + 8)(x - 2) = 0$

$x = -8$ or $x = 2$

$y = 3(-8) + 2 = -22$, $y = 3(2) + 2 = 8$

$x = -8, y = -22$ and $x = 2, y = 8$

(b) Solve $3^{2x-1} = 7^x$ [6]

$3^{2x-1} = 7^x$

$\log 3^{2x-1} = \log 7^x$

$(2x-1)\log 3 = x \log 7$

$2x \log 3 - \log 3 = x \log 7$

$2x \log 3 - x \log 7 = \log 3$

$x(2 \log 3 - \log 7) = \log 3$

$x = \dfrac{\log 3}{2 \log 3 - \log 7}$

$x = \dfrac{0.47712\ldots}{0.10914\ldots} = 4.37146\ldots$

$x = 4.37$ (3 s.f.)

[Turn over

4 **Fig. 2** below shows a sketch of a curve.

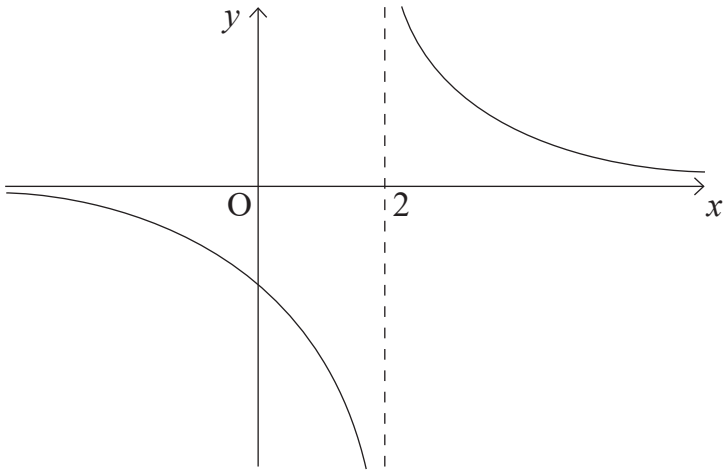


Fig. 2

The curve has equation $y = \frac{q}{x - p}$ where p and q are integers.

The curve has a horizontal asymptote with equation $y = 0$

The curve has a vertical asymptote with equation $x = 2$

(i) State the value of p. [1]

Vertical asymptote where $x - p = 0$, i.e. $x = p$

$p = 2$

The curve passes through the point (−1, −2).

(ii) Find the value of q. [3]

$y = \frac{q}{x-2}$ passes through $(-1, -2)$:

$-2 = \frac{q}{-1-2}$

$-2 = \frac{q}{-3}$

$q = 6$

(iii) Hence find the coordinates of the point where the curve cuts the y-axis. [2]

$y = \frac{6}{x-2}$, $x = 0$: $y = \frac{6}{0-2} = -3$

$(0, -3)$

[Turn over

5 **Fig. 3** below shows a sketch of a curve and a straight line.

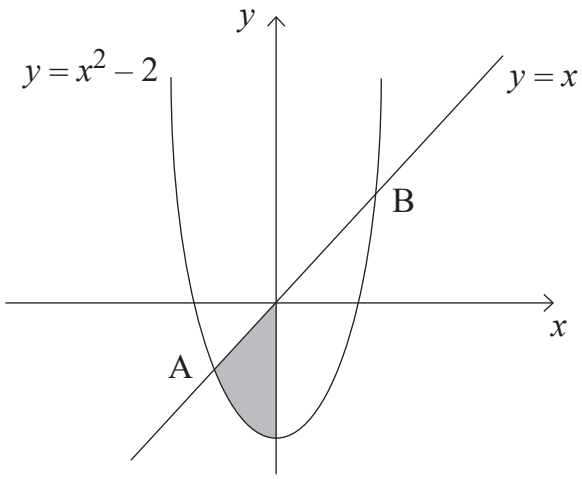


Fig. 3

The curve has equation $y = x^2 - 2$

The line has equation $y = x$

The curve and line intersect at the points A and B.

(i) Find the coordinates of A and B. [4]

$x^2 - 2 = x$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2$ or $x = -1$

$y = 2$ or $y = -1$

$A(-1, -1)$, $B(2, 2)$

(ii) Hence find the area of the shaded region in **Fig. 3**, which is bounded by the straight line, the curve and the y-axis. [6]

Area between curve and x-axis from $x = -1$ to $x = 0$:

$\int_{-1}^{0} (x^2 - 2)\,dx = \left[\frac{x^3}{3} - 2x\right]_{-1}^{0}$

$= (0 - 0) - \left(\frac{(-1)^3}{3} - 2(-1)\right)$

$= 0 - \frac{5}{3} = -\frac{5}{3}$

so this area is $\frac{5}{3}$ (below the x-axis)

Triangle between $y = x$, x-axis and $x = -1$: $\frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Shaded area $= \frac{5}{3} - \frac{1}{2}$

$= \frac{7}{6}$ units^2

[Turn over

6 **(a)** Solve $\sqrt{3}\sin 2x = \cos 2x$ $\quad 0° \leqslant x \leqslant 360°$ [6]

$\sqrt{3}\sin 2x = \cos 2x$

$\tan 2x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Let $y = 2x$, $0° \leqslant y \leqslant 720°$

$\tan y = \frac{1}{\sqrt{3}} \Rightarrow y = 30°$

$y = 30°, 210°, 390°, 570°$

$x = 15°, 105°, 195°, 285°$

(b) **Fig. 4** below shows a triangle ABC.

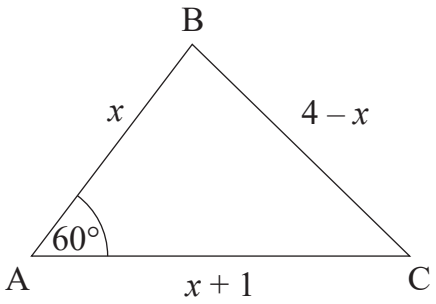


Fig. 4

$AB = x$ $BC = 4 - x$ $AC = x + 1$ $B\hat{A}C = 60°$

(i) Show that $x = \frac{5}{3}$ [3]

Cosine rule: $BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos 60°$

$(4 - x)^2 = x^2 + (x + 1)^2 - 2x(x + 1)\cos 60°$

$16 - 8x + x^2 = x^2 + x^2 + 2x + 1 - x^2 - x$

$16 - 8x + x^2 = x^2 + x + 1$

$16 - 8x = x + 1$

$9x = 15$

$x = \frac{15}{9} = \frac{5}{3}$ as required

(ii) Hence find the area of triangle ABC. [3]

Area $= \frac{1}{2}(AB)(AC)\sin 60° = \frac{1}{2} \times \frac{5}{3} \times \frac{8}{3} \times \sin 60°$

$= \frac{10\sqrt{3}}{9} = 1.92450...$

$= 1.92$ units^2 (3 s.f.)

[Turn over

7 **(a)** Differentiate $\frac{x}{5}+\frac{3}{x^2}-\frac{1}{4\sqrt{x}}$ [3]

$\frac{x}{5} + \frac{3}{x^2} - \frac{1}{4\sqrt{x}} = \frac{1}{5}x + 3x^{-2} - \frac{1}{4}x^{-1/2}$

$\frac{d}{dx}\left(\frac{x}{5} + 3x^{-2} - \frac{1}{4}x^{-1/2}\right) = \frac{1}{5} - 6x^{-3} + \frac{1}{8}x^{-3/2}$

$= \frac{1}{5} - \frac{6}{x^3} + \frac{1}{8x^{3/2}}$

(b) **Fig. 5** below shows a sketch of plans for a display area in a garden centre.

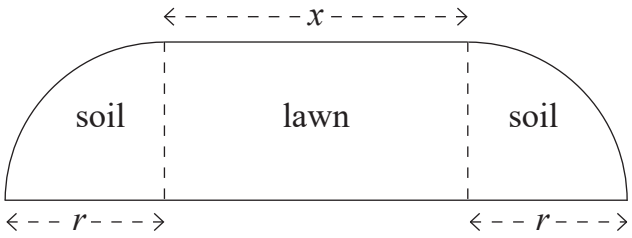


Fig. 5

The display area has a rectangular lawn with two quarter circles of soil, one on either side.

The lawn has a length of x metres.
Each patch of soil has a radius of r metres.

The perimeter of the display is 32 m.

(i) Express x in terms of r. [4]

Perimeter $= x + 2r + x + 2 \times \frac{1}{4}(2\pi r)$

$= 2x + 2r + \pi r$

$2x + 2r + \pi r = 32$

$2x = 32 - 2r - \pi r$

$x = \frac{32 - 2r - \pi r}{2}$

(ii) Hence show that the area of the lawn, $A\,\text{m}^2$, is such that

$$A = 16r - r^2 - \frac{1}{2}\pi r^2$$ [3]

Area of lawn $A = xr$

$A = \left(\frac{32 - 2r - \pi r}{2}\right) r$

$A = \frac{32r - 2r^2 - \pi r^2}{2}$

$A = 16r - r^2 - \frac{1}{2}\pi r^2$ as required

(iii) Using calculus, find the exact value of r for which the area of the lawn is a maximum. [6]

$A = 16r - r^2 - \frac{1}{2}\pi r^2$

$\frac{dA}{dr} = 16 - 2r - \pi r$

For a stationary point $\frac{dA}{dr} = 0$:

$16 - 2r - \pi r = 0$

$r(2 + \pi) = 16$

$r = \frac{16}{2 + \pi}$

$\frac{d^2A}{dr^2} = -2 - \pi < 0 \Rightarrow$ maximum

Area is a maximum when $r = \frac{16}{2 + \pi}$ m

[Turn over

8 Find the equation of the tangent to the circle given by the equation

$$x^2 + y^2 - 2x - 6y - 3 = 0$$

at the point (4, 5).

Leave your answer in the form $ax + by + c = 0$, where a, b and c are integers. [9]

$x^2 - 2x + y^2 - 6y - 3 = 0$

$(x - 1)^2 - 1 + (y - 3)^2 - 9 - 3 = 0$

$(x - 1)^2 + (y - 3)^2 = 13$

Centre (1, 3)

Gradient of radius to (4, 5) $= \frac{5 - 3}{4 - 1} = \frac{2}{3}$

Tangent is perpendicular to radius: gradient $= -\frac{3}{2}$

$y - 5 = -\frac{3}{2}(x - 4)$

$2y - 10 = -3x + 12$

$3x + 2y - 22 = 0$

[Turn over

9 Given that

$$(1 - x)(1 + ax)^n \equiv 1 + 23x + 228x^2 + \ldots$$

find the values of a and n. [10]

$(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$

$= 1 + nax + \frac{n(n-1)a^2}{2}x^2 + \ldots$

$(1 - x)(1 + ax)^n = 1 + nax + \frac{n(n-1)a^2}{2}x^2 - x - nax^2 + \ldots$

$= 1 + (na - 1)x + \left(\frac{n(n-1)a^2}{2} - na\right)x^2 + \ldots$

Compare coefficients with $1 + 23x + 228x^2$:

x: $na - 1 = 23 \Rightarrow na = 24$

x^2: $\frac{n(n-1)a^2}{2} - na = 228$

$a = \frac{24}{n}$: $\frac{n(n-1)}{2} \times \frac{576}{n^2} - 24 = 228$

$\frac{288(n-1)}{n} = 252$

$288n - 288 = 252n$

$36n = 288$

$n = 8$, $a = \frac{24}{8} = 3$

[Turn over

10 Solve $(2 - e^x)(4 - e^x) < 1$ [8]

$(2 - e^x)(4 - e^x) < 1$

$8 - 2e^x - 4e^x + e^{2x} < 1$

$e^{2x} - 6e^x + 7 < 0$

Let $y = e^x$: $y^2 - 6y + 7 < 0$

Critical values: $y^2 - 6y + 7 = 0$

$(y - 3)^2 - 9 + 7 = 0$

$(y - 3)^2 = 2$

$y = 3 \pm \sqrt{2}$

$e^x = 3 \pm \sqrt{2} \quad \Rightarrow \quad x = \ln(3 \pm \sqrt{2})$

Quadratic in y is negative between the roots:

$3 - \sqrt{2} < e^x < 3 + \sqrt{2}$

$\ln(3 - \sqrt{2}) < x < \ln(3 + \sqrt{2})$

(i.e. $0.461 < x < 1.48$)

THIS IS THE END OF THE QUESTION PAPER

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For Examiner's use only	
Question Number	**Marks**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total Marks	

Examiner Number

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