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ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2019

Centre Number

Candidate Number

Mathematics

StemUp Model Solution

Assessment Unit AS 2
assessing
Applied Mathematics

SMT21

[SMT21]
WEDNESDAY 22 MAY, MORNING

TIME

1 hour 15 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in black ink only. Questions which require drawing or sketching should be completed using an HB pencil. **Do not write with a gel pen.**
Candidates must answer **all** questions from sections A and B.
Equal time should be spent on each section. Show clearly the full development of your answers.
Answers without working may not gain full credit.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 70. The total available mark for each section of this paper is 35.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

11876

Answer all questions.

SECTION A

Mechanics

1 **Fig. 1** below shows three forces acting on a particle.

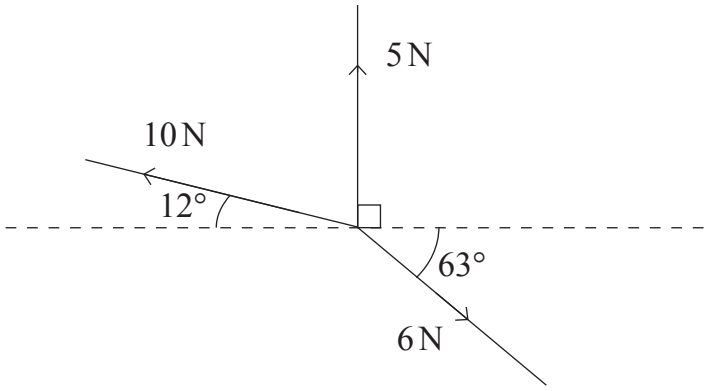


Fig. 1

Find the magnitude and direction of the resultant of these three forces. [6]

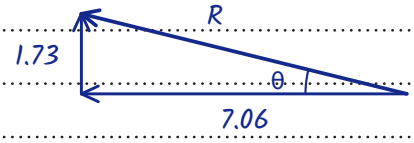
Resolving (↑): 5 + 10 sin 12° − 6 sin 63° = 1.7330... N

Resolving (→): 6 cos 63° − 10 cos 12° = −7.0575... N

$R = \sqrt{(7.0575...^2 + 1.7330...^2)}$

$= \sqrt{52.812...} = 7.2672...$

Magnitude R = 7.27 N (3 s.f.)



$\tan\theta = \frac{1.7330...}{7.0575...} = 0.24556...$

$\theta = \tan^{-1}(0.24556...) = 13.797...°$

Direction: 13.8° above the horizontal, to the left

(equivalently 76.2° to the upward vertical)



2 A drone is flown, with constant acceleration, in a horizontal plane.

When initially observed, at a point P, it has a velocity of $(2\mathbf{i} - 7\mathbf{j})\ \text{m s}^{-1}$

Four seconds later the drone is at a point Q and has a velocity of $(14\mathbf{i} + \mathbf{j})\ \text{m s}^{-1}$

(i) Find the acceleration of the drone in terms of **i** and **j**. [3]

$v = u + at$

$14i + j = (2i - 7j) + 4a$

$4a = 12i + 8j$

$a = (3i + 2j)\ m\ s^{-2}$

(ii) Find the distance PQ. [5]

$s = ut + \frac{1}{2}at^2$

$s = 4(2i - 7j) + \frac{1}{2}(3i + 2j)(4^2)$

$s = 8i - 28j + 24i + 16j$

$PQ = s = (32i - 12j)$ m

$PQ = \sqrt{(32^2 + (-12)^2)}$

$= \sqrt{1168} = 34.176...$ m

$PQ = 34.2$ m (3 s.f.)

3 **Fig. 2** below shows the velocity–time graph for an athlete running a 100 m race in 11.8 seconds.

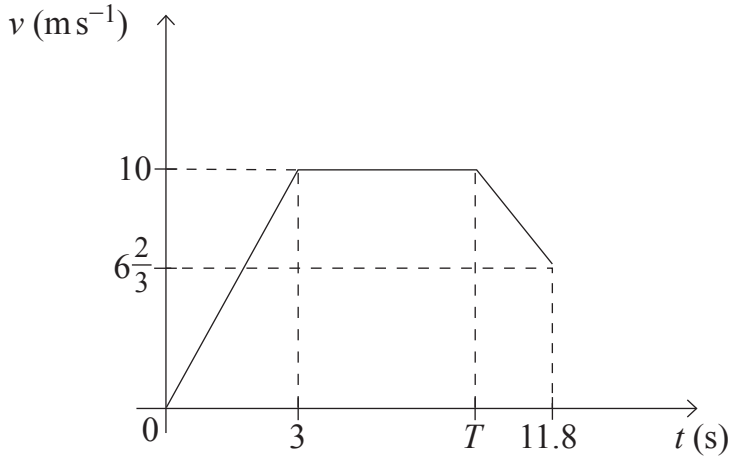


Fig. 2

(i) Describe the motion of the athlete between $t = 0$ and $t = 3$ [1]

The athlete accelerates uniformly (constant acceleration) from rest to 10 m s⁻¹ in the first 3 seconds.

The athlete starts to decelerate at time T.

(ii) Find the value of T. [5]

Distance = area under the graph = 100 m

$100 = \frac{1}{2}(3)(10) + 10(T - 3) + \frac{1}{2}(10 + 6\frac{2}{3})(11.8 - T)$

$100 = 15 + 10T - 30 + \frac{25}{3}(11.8 - T)$

$100 = 10T - 15 + \frac{295}{3} - \frac{25}{3}T$

$\times 3: \quad 300 = 30T - 45 + 295 - 25T$

$5T = 50$

$T = 10$



4 **Fig. 3** below shows two boxes, A and B, connected by a rope passing over a smooth, light, fixed pulley.

Box A is held at rest on a rough horizontal table 5 m from the pulley.

Box B hangs 2.5 m vertically above the floor.

A and B have masses 8 kg and 6 kg respectively.

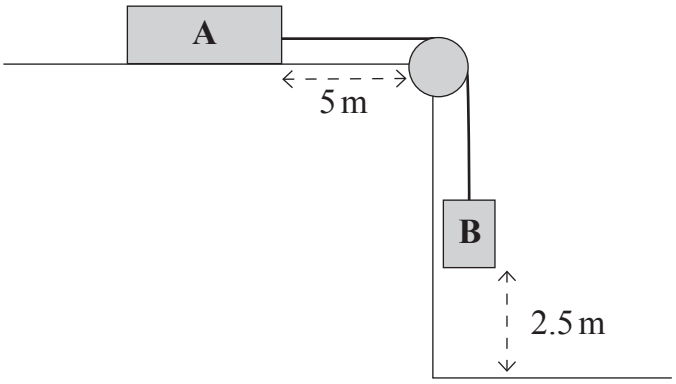
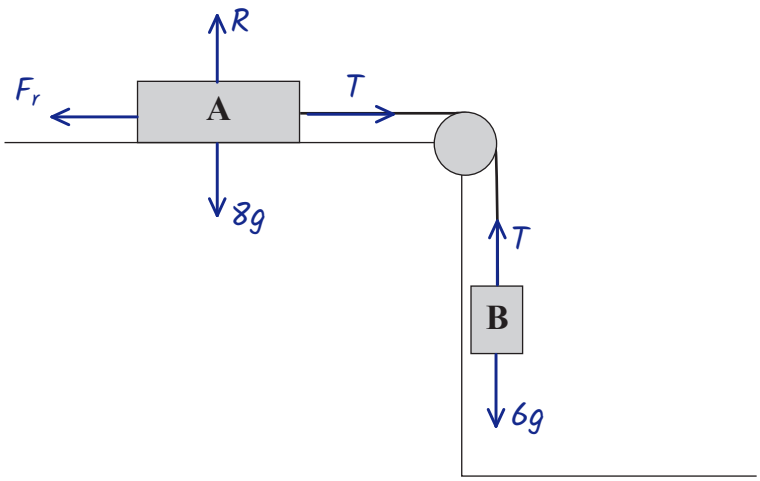


Fig. 3

Box A is released from rest.

(i) Complete the diagram below showing all the external forces acting on A and B. [2]



B hits the floor with a speed of $3.7\,\text{m}\,\text{s}^{-1}$

(ii) Find the value of the coefficient of friction between A and the table. [8]

For B (falls 2.5 m from rest):

$v^2 = u^2 + 2as$

$3.7^2 = 0 + 2a(2.5)$

$13.69 = 5a \Rightarrow a = 2.738\ m\ s^{-2}$

Whole system, $F = ma$:

$6g - F_r = (8 + 6)a$

$58.8 - F_r = 14(2.738) = 38.332$

$F_r = 20.468\ N$

For A, resolving (↑): $R = 8g = 78.4\ N$

A is sliding, so $F_r = \mu R$

$20.468 = 78.4\mu$

$\mu = 0.26107...$

$\mu = 0.261$ (3 s.f.)

[Turn over



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(iii) Determine whether box A will collide with the pulley. [4]

When B hits the floor, A has moved 2.5 m, so A is
$5 - 2.5 = 2.5$ m from the pulley, moving at 3.7 m s^{-1}.
The rope goes slack, so only friction acts on A:
$-F_r = 8a \Rightarrow -20.468 = 8a \Rightarrow a = -2.5585$ m s^{-2}
$v^2 = u^2 + 2as$: $0 = 3.7^2 + 2(-2.5585)s$
$13.69 = 5.117s \Rightarrow s = 2.6754... = 2.68$ m
Since 2.68 m > 2.5 m, box A will collide with the pulley.

(iv) State one modelling assumption you have made about the rope. [1]

The rope is light and inextensible (so the tension is the same throughout and A and B have the same acceleration).

[Turn over

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SECTION B

Statistics

5 **(a)** Explain the difference between a census and a sample. [2]

A census collects data from every member of the population.

A sample collects data from only a subset (part) of the population, which is used to represent the whole population.

(b) A market researcher is interested to find out the opinions of nurses living in Northern Ireland.

To get a random sample, he questions women that he meets entering the Royal Victoria Hospital on a Monday morning after 9 am.

Give two reasons to explain why this sampling procedure is not satisfactory. [2]

1. Not everyone he questions will be a nurse (visitors, patients and other staff also enter the hospital), so he is not sampling the population of nurses.

2. He only questions women, so male nurses are excluded; he also uses only one hospital on one morning, ignoring nurses elsewhere and on other shifts, so the sample is biased.

[Turn over

6 **(a)** The weekly incomes, rounded to the nearest pound, of households on a certain street in Belfast are given below:

221 248 251 255 259 263 264 272 291 297 374

(i) Calculate the mean and standard deviation for these weekly incomes. [4]

$\Sigma x = 2995$, $\Sigma x^2 = 831067$, $n = 11$

$\bar{x} = \frac{\Sigma x}{n} = \frac{2995}{11} = 272.27...$

Mean = £272 (3 s.f.)

$\sigma = \sqrt{(\Sigma x^2/n - \bar{x}^2)}$

$= \sqrt{(831067/11 - 272.27...^2)}$

$= \sqrt{(75551.54... - 74132.43...)}$

$= \sqrt{1419.10...} = 37.671...$

Standard deviation = £37.7 (3 s.f.)

Outliers are defined to be more than $1.5 \times (Q_3 - Q_1)$ above Q_3 or
more than $1.5 \times (Q_3 - Q_1)$ below Q_1

(ii) Show that £374 is an outlier. [3]

Data are in order, n = 11:

Q_1 = 3rd value = 251, median = 263, Q_3 = 9th value = 291

IQR = $Q_3 - Q_1$ = 291 − 251 = 40

1.5 × 40 = 60

Q_3 + 60 = 291 + 60 = 351 (Q_1 − 60 = 191)

374 > 351, so £374 is an outlier.

Further investigations suggested that the weekly income of £374 was recorded incorrectly and should in fact have been £347

Without carrying out any further investigation, state what effect, if any, this change would have on:

(iii) the standard deviation, [1]

The standard deviation would decrease.

(iv) the interquartile range. [1]

The interquartile range would stay the same (no effect).

[Turn over

(b) The histogram in **Fig. 4** below shows the weekly incomes, in £, of a random sample of households in a neighbourhood.

Histogram showing weekly incomes

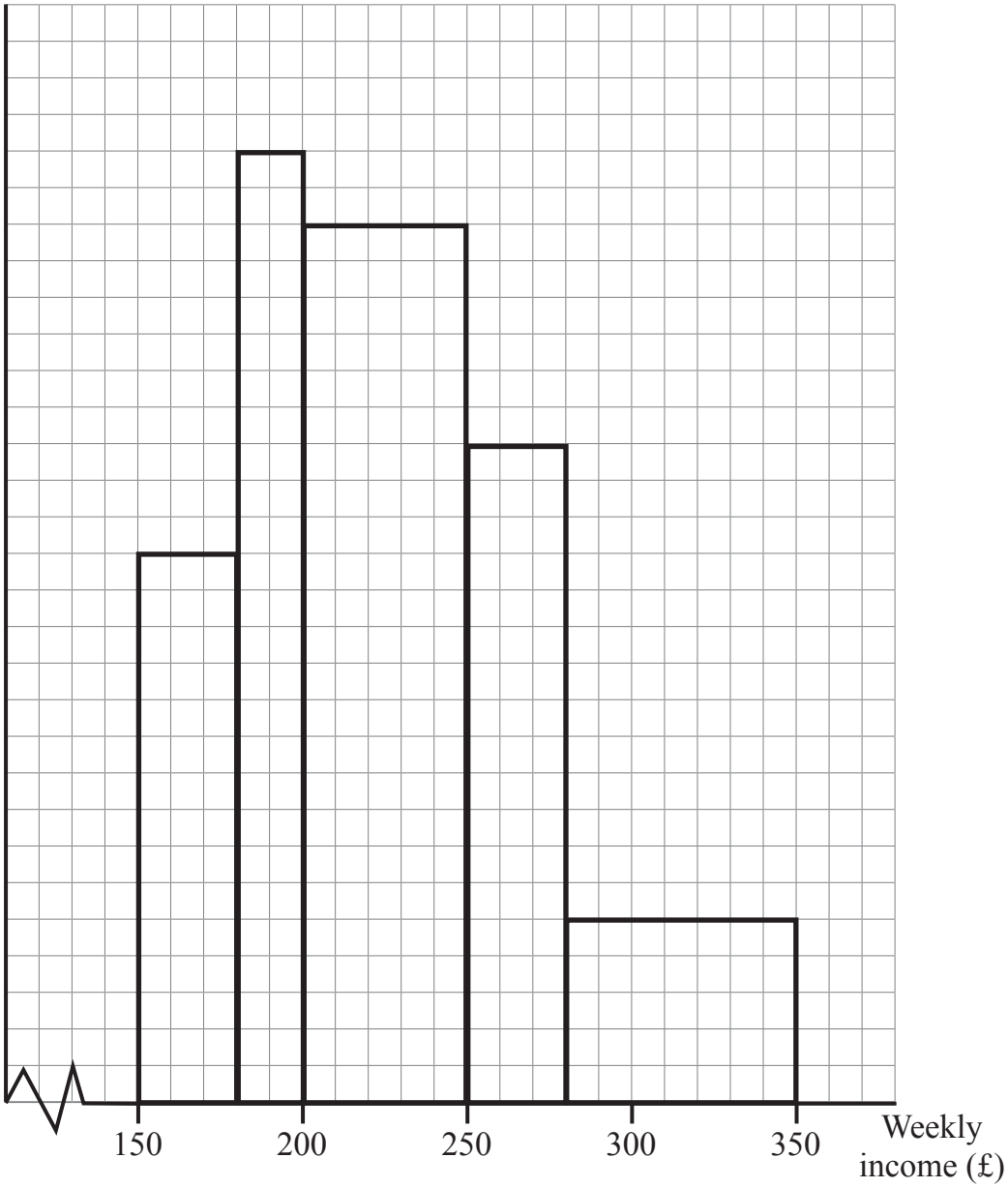


Fig. 4

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(i) Complete the table below. [1]

Weekly incomes (£)	Number of households
150 –	45
180 –	52
200 –	120
250 –	54
280 –	35
350 –	0

(ii) Estimate the probability that a household chosen at random received a weekly income of more than £230 [2]

Frequency density = frequency ÷ class width

150–180: 45 ÷ 30 = 1.5; 180–200: 52 ÷ 20 = 2.6

200–250: 120 ÷ 50 = 2.4; 280–350: 35 ÷ 70 = 0.5

250–280: frequency = 1.8 × 30 = 54

Total = 45 + 52 + 120 + 54 + 35 = 306

From £230 to £250: 2.4 × 20 = 48

Number above £230 ≈ 48 + 54 + 35 = 137

$P(\text{income} > £230) \approx \frac{137}{306} = 0.4477... = 0.448$ (3 s.f.)

[Turn over

7 The percentage unemployment rate (U) and percentage change in average wages (W) per year were recorded over a period of 10 years.

These are shown in **Table 1** below.

Table 1

% Unemployment Rate (U)	2.5	1.8	1.7	1.6	2.2	1.5	1.6	2.1	1.8	2.3
% Change in Wages (W)	3.2	3.6	4.6	4.4	2.7	3.5	4.3	2.7	4.1	2.9

(i) Calculate the product moment correlation coefficient between U and W. [5]

$\Sigma U = 19.1, \quad \Sigma W = 36, \quad n = 10$

$\Sigma U^2 = 37.53, \quad \Sigma W^2 = 134.26, \quad \Sigma UW = 67.13$

$S_{UW} = 67.13 - (19.1 \times 36)/10 = 67.13 - 68.76 = -1.63$

$S_{UU} = 37.53 - 19.1^2/10 = 37.53 - 36.481 = 1.049$

$S_{WW} = 134.26 - 36^2/10 = 134.26 - 129.6 = 4.66$

$r = \frac{S_{UW}}{\sqrt{(S_{UU} S_{WW})}} = \frac{-1.63}{\sqrt{(1.049 \times 4.66)}}$

$r = \frac{-1.63}{2.21096...} = -0.73723...$

$r = -0.737$ (3 s.f.)

(ii) Comment on the value obtained in part **(i)**. [1]

There is fairly strong negative correlation: the lower the unemployment rate, the higher the percentage change in wages (and vice versa).

[Turn over



8 A school sports club has 15 members. The probability of any member, chosen at random, being injured throughout the school year is p.

Let X be the random variable "the number of members injured throughout the school year".

(i) Find an expression, in terms of p, for $P(X = 1)$ [3]

$X \sim B(15, p)$

$P(X = 1) = {}^{15}C_1\, p^1(1 - p)^{14}$

$P(X = 1) = 15p(1 - p)^{14}$

It is known that $P(X = 2) = 3P(X = 1)$

(ii) Prove that $p = 0.3$ [4]

$P(X = 2) = {}^{15}C_2\, p^2(1 - p)^{13} = 105p^2(1 - p)^{13}$

$P(X = 2) = 3P(X = 1)$:

$105p^2(1 - p)^{13} = 3 \times 15p(1 - p)^{14} = 45p(1 - p)^{14}$

Divide by $15p(1 - p)^{13}$ $(p \neq 0, 1)$:

$7p = 3(1 - p)$

$7p = 3 - 3p$

$10p = 3$

$\therefore p = 0.3$ as required

(iii) Hence find the probability that at least three members are injured throughout the school year. [6]

$X \sim B(15, 0.3)$; at least three means $X \geq 3$

$P(X \geqslant 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2)$

$P(X = 0) = {}^{15}C_0(0.3)^0(0.7)^{15} = 0.0047475...$

$P(X = 1) = {}^{15}C_1(0.3)^1(0.7)^{14} = 0.0305200...$

$P(X = 2) = {}^{15}C_2(0.3)^2(0.7)^{13} = 0.0915601...$

$P(X \leqslant 2) = 0.1268277...$

$P(X \geqslant 3) = 1 - 0.1268277... = 0.8731722...$

$P(X \geqslant 3) = 0.873$ (3 s.f.)

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