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ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2019

Centre Number

Candidate Number

Mathematics

StemUp Model Solution

Assessment Unit AS 1
assessing
Pure Mathematics

SMT11

[SMT11]
WEDNESDAY 15 MAY, MORNING

TIME

1 hour 45 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all nine** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an H.B. pencil.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions. **Answers without working may not gain full credit**.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

1 **(a)** Solve the simultaneous equations

$$x + 2y + 3z = 9$$
$$2x - y + 4z = 17$$
$$3x + y - z = 2$$

[7]

$x + 2y + 3z = 9$ (1)

$2x - y + 4z = 17$ (2)

$3x + y - z = 2$ (3)

(1) + 2 × (2): $x + 2y + 3z + 4x - 2y + 8z = 9 + 34$

$5x + 11z = 43$ (4)

(2) + (3): $5x + 3z = 19$ (5)

(4) − (5): $8z = 24$

$z = 3$

Sub into (5): $5x + 9 = 19 \Rightarrow 5x = 10 \Rightarrow x = 2$

Sub into (3): $6 + y - 3 = 2 \Rightarrow y = -1$

Check in (1): $2 - 2 + 9 = 9$ correct

$\therefore x = 2, \quad y = -1, \quad z = 3$

[Turn over

(b) $\overrightarrow{OA} = 4\mathbf{i} - \mathbf{j}$

$\overrightarrow{OB} = 6\mathbf{i} + 2\mathbf{j}$

Find:

(i) the magnitude of the vector $\overrightarrow{AB}$, [3]

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -(4i - j) + (6i + 2j)$

$= 2i + 3j$

$|\overrightarrow{AB}| = \sqrt{2^2 + 3^2}$

$|\overrightarrow{AB}| = \sqrt{13} \quad (= 3.61)$

(ii) the angle $\overrightarrow{AB}$ makes with the positive x-axis. [2]

$\tan\theta = \frac{3}{2}$

$\theta = \tan^{-1}(1.5) = 56.309...°$

$\theta = 56.3°$

[Turn over

2 **Fig. 1** below shows a sketch of the graph of the curve given by the equation $y = f(x)$.

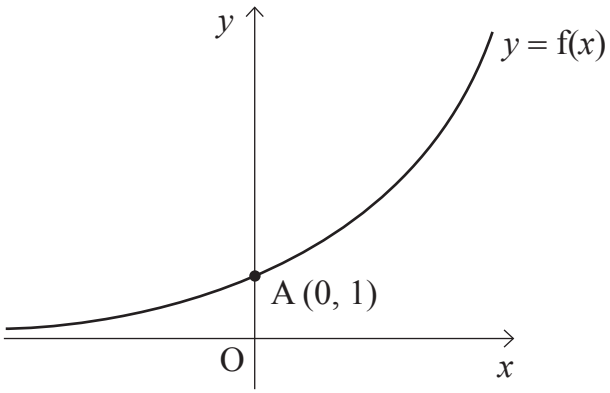


Fig. 1

Point A has coordinates (0, 1).

(a) Write down the coordinates of the point A under the following transformations:

(i) $y = f(x) + 1$ [2]

Translation of 1 unit up: (0, 1) → (0, 1 + 1)

A is (0, 2)

(ii) $y = f(x - 3)$ [2]

Translation of 3 units to the right: (0, 1) → (0 + 3, 1)

A is (3, 1)

(iii) $y = f(-x)$ [2]

Reflection in the y-axis: (0, 1) → (−0, 1)

A is (0, 1)

(b) $f(x)$ has an asymptote given by the equation $y = 0$

Write down the equation of the asymptote of the curve under the transformation

$$y + 2 = f(x).$$ [1]

y = f(x) − 2: translation of 2 units down

Asymptote is y = −2

3 **(a)** **(i)** Given that $\log_2 a = 3$ state the value of a. [1]

$a = 2^3$

$a = 8$

(ii) Hence solve the equation

$$\log_2 x - \log_2 (x - 1) = 3$$ [5]

$\log_2 x - \log_2(x-1) = \log_2\left(\frac{x}{x-1}\right)$

$\log_2\left(\frac{x}{x-1}\right) = 3$

$\frac{x}{x-1} = 2^3 = 8$

$x = 8(x-1)$

$x = 8x - 8$

$7x = 8$

$x = \frac{8}{7}$

(b) Find the exact solution of the equation

$$3e^{2x} - 4 = 0$$ [4]

$3e^{2x} = 4$

$e^{2x} = \frac{4}{3}$

$2x = \ln(\frac{4}{3})$

$x = \frac{1}{2}\ln(\frac{4}{3})$

[Turn over

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4 **(a)** Find the range of values of k for which the equation

$$x^2 + kx + 16 = 0$$

has no real roots. [6]

No real roots $\Rightarrow$ $b^2 - 4ac < 0$

$a = 1,\ b = k,\ c = 16$

$k^2 - 4(1)(16) < 0$

$k^2 - 64 < 0$

Critical values: $k^2 = 64 \Rightarrow k = 8$ or $k = -8$

$(k - 8)(k + 8) < 0$, the quadratic in k is negative between the roots

$\therefore\ -8 < k < 8$

(b) Find the centre and area of the circle given by the equation

$$x^2 - 4x + y^2 + 6y - 3 = 0$$ [6]

$x^2 - 4x + y^2 + 6y - 3 = 0$

$(x - 2)^2 - 4 + (y + 3)^2 - 9 - 3 = 0$

$(x - 2)^2 + (y + 3)^2 = 16$

Centre $(2, -3)$

Radius $= \sqrt{16} = 4$

Area $= \pi r^2 = \pi \times 4^2$

Area $= 16\pi$ units^2

[Turn over

5 **(a)** Given that

$$\int_1^k 2\sqrt{x}\,dx = \frac{28}{3}$$

find the value of k. [6]

$\int_1^k 2x^{1/2}\,dx = \frac{28}{3}$

$\left[\frac{2x^{3/2}}{3/2}\right]_1^k = \left[\frac{4x^{3/2}}{3}\right]_1^k = \frac{28}{3}$

$\frac{4k^{3/2}}{3} - \frac{4(1)^{3/2}}{3} = \frac{28}{3}$

$4k^{3/2} - 4 = 28$

$4k^{3/2} = 32$

$k^{3/2} = 8$

$k = 8^{2/3} = 2^2$

$k = 4$

(b) Solve the equation

$$2\cos^2\theta = 1 - \sin\theta$$

for $0 \leqslant \theta \leqslant 360°$ [7]

$2\cos^2\theta = 1 - \sin\theta$

Using $\cos^2\theta = 1 - \sin^2\theta$:

$2(1 - \sin^2\theta) = 1 - \sin\theta$

$2 - 2\sin^2\theta = 1 - \sin\theta$

$0 = 2\sin^2\theta - \sin\theta - 1$

$0 = (2\sin\theta + 1)(\sin\theta - 1)$

$\sin\theta = -\frac{1}{2}$ or $\sin\theta = 1$

$\sin\theta = 1$: $\theta = 90°$

$\sin\theta = -\frac{1}{2}$: sin is negative in the 3rd and 4th quadrants

$\theta = 180° + 30° = 210°$ or $\theta = 360° - 30° = 330°$

$\therefore \theta = 90°, 210°, 330°$

[Turn over

6 An open tin in the shape of a cylinder is shown in **Fig. 2** below.

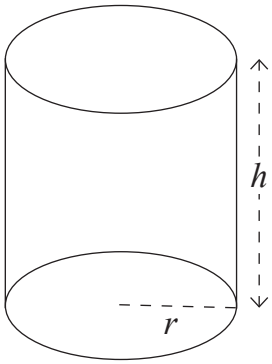


Fig. 2

The open tin has base radius r cm and height h cm.
The total surface area of the tin is $300\pi\,\text{cm}^2$

(i) Express h in terms of r. [4]

Open tin: surface area = base + curved surface

$\pi r^2 + 2\pi rh = 300\pi$

$r^2 + 2rh = 300$

$2rh = 300 - r^2$

$h = \frac{300 - r^2}{2r}$

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(ii) Hence show that the volume of the tin can be expressed as

$$V = 150\pi r - \frac{\pi r^3}{2}$$ [3]

$V = \pi r^2 h$

$V = \pi r^2 \times \frac{300 - r^2}{2r}$

$V = \frac{\pi r(300 - r^2)}{2}$

$V = \frac{300\pi r}{2} - \frac{\pi r^3}{2}$

$\therefore\ V = 150\pi r - \frac{\pi r^3}{2}$ as required

[Turn over



The volume of the tin is

$$V = 150\pi r - \frac{\pi r^3}{2}$$

(iii) Using calculus, find the values of r and h for which the volume of the tin is a maximum. [7]

$\frac{dV}{dr} = 150\pi - \frac{3\pi r^2}{2}$

For a stationary point:

$\frac{dV}{dr} = 0 \Rightarrow \frac{3\pi r^2}{2} = 150\pi$

$3r^2 = 300$

$r^2 = 100$

$r = 10 \quad (r > 0)$

$h = \frac{300 - 10^2}{2 \times 10} = \frac{200}{20} = 10$

$\frac{d^2V}{dr^2} = -3\pi r$

When $r = 10$: $\frac{d^2V}{dr^2} = -30\pi < 0 \Rightarrow$ maximum

Volume is a maximum when $r = 10$ and $h = 10$

[Turn over

7 Solve the equation

$$2x^3 - 8x^2 + 3x + 10 = 0$$ [7]

$f(x) = 2x^3 - 8x^2 + 3x + 10$

$f(2) = 2(8) - 8(4) + 3(2) + 10$

$= 16 - 32 + 6 + 10 = 0$

$\Rightarrow (x - 2)$ is a factor

$$\begin{array}{r} 2x^2 - 4x - 5 \\ x - 2 \overline{)\,2x^3 - 8x^2 + 3x + 10} \\ 2x^3 - 4x^2 \\ \hline -4x^2 + 3x \\ -4x^2 + 8x \\ \hline -5x + 10 \\ -5x + 10 \\ \hline 0 \end{array}$$

$\Rightarrow (x - 2)(2x^2 - 4x - 5) = 0$

$x - 2 = 0 \quad$ or $\quad 2x^2 - 4x - 5 = 0$

$x = 2$

$x = \frac{4 \pm \sqrt{((-4)^2 - 4(2)(-5))}}{2(2)}$

$x = \frac{4 \pm \sqrt{56}}{4} = \frac{4 \pm 2\sqrt{14}}{4}$

$x = \frac{2 \pm \sqrt{14}}{2} \quad (= 2.87 \text{ or } -0.871)$

$\therefore x = 2, \quad x = \frac{2 + \sqrt{14}}{2}, \quad x = \frac{2 - \sqrt{14}}{2}$



8 The points A and B have coordinates $(2a, a^2)$ and $(a+1, a)$ respectively, where $a \neq 0$

(i) A straight line, perpendicular to AB and passing through the point A, cuts the x-axis at the point P.

Find, in terms of a, the coordinates of the point P. [8]

$A(2a, a^2)$, $B(a+1, a)$

Gradient of AB $= \frac{a - a^2}{(a+1) - 2a} = \frac{a(1-a)}{1-a} = a$

Perpendicular gradient $= -\frac{1}{a}$

Line through A: $y = mx + c$

$a^2 = -\frac{1}{a}(2a) + c$

$a^2 = -2 + c \quad \Rightarrow \quad c = a^2 + 2$

$y = -\frac{x}{a} + a^2 + 2$

Cuts the x-axis where $y = 0$:

$\frac{x}{a} = a^2 + 2$

$x = a^3 + 2a$

$\therefore$ P is $(a^3 + 2a, 0)$

[Turn over

(ii) The midpoint of the line AB has equal x and y ordinates.

Find the possible values of a in their simplest surd form. [5]

Midpoint of AB = $\left(\frac{2a + a + 1}{2}, \frac{a^2 + a}{2}\right) = \left(\frac{3a + 1}{2}, \frac{a^2 + a}{2}\right)$

x and y ordinates equal: $\frac{3a + 1}{2} = \frac{a^2 + a}{2}$

$3a + 1 = a^2 + a$

$0 = a^2 - 2a - 1$

$0 = (a - 1)^2 - 2$

$(a - 1)^2 = 2$

$a - 1 = \pm\sqrt{2}$

$a = 1 + \sqrt{2}$ or $a = 1 - \sqrt{2}$

[Turn over

9 **Fig. 3** below shows a diagram of a circle with centre O.

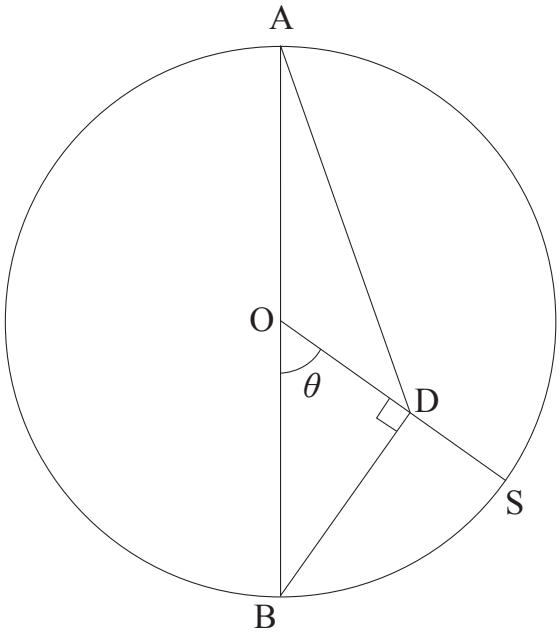


Fig. 3

AB is a diameter of the circle.
S lies on the circumference of the circle.
D is the foot of the perpendicular from B to OS.

The acute angle BOS is θ

$OA = OB = r$

$OD = x$

(i) By applying the cosine rule to triangle AOD, show that

$$AD^2 = r^2 (1 + 3\cos^2\theta)$$ [7]

In triangle AOD: OA = r, OD = x, ∠AOD = 180° − θ

$AD^2 = r^2 + x^2 - 2rx\cos(180° - \theta)$

Since $\cos(180° - \theta) = -\cos\theta$:

$AD^2 = r^2 + x^2 + 2rx\cos\theta$

In right-angled triangle OBD (∠ODB = 90°, OB = r):

$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$

Substituting $x = r\cos\theta$:

$AD^2 = r^2 + r^2\cos^2\theta + 2r(r\cos\theta)\cos\theta$

$= r^2 + r^2\cos^2\theta + 2r^2\cos^2\theta$

$= r^2 + 3r^2\cos^2\theta$

$\therefore\ AD^2 = r^2(1 + 3\cos^2\theta)$ as required

[Turn over

(ii) When BD bisects OS,

$$\text{AD} = \frac{r\sqrt{k}}{2}$$

Find the value of k, where k is a positive integer. [5]

BD bisects OS $\Rightarrow$ D is the midpoint of OS

$OD = x = \frac{r}{2}$

From triangle OBD: $\cos\theta = \frac{x}{r} = \frac{r}{2} \div r = \frac{1}{2}$

$AD^2 = r^2[1 + 3(\frac{1}{2})^2] = r^2(1 + \frac{3}{4})$

$AD^2 = \frac{7}{4}r^2$

$AD = \frac{\sqrt{7}}{2}r = \frac{r\sqrt{7}}{2}$

$\therefore\ k = 7$

THIS IS THE END OF THE QUESTION PAPER

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Question Number	**Marks**
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total Marks	

Examiner Number

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