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General Certificate of Education
2019

Centre Number

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Mathematics

StemUp Model Solution

Assessment Unit A2 2
assessing
Applied Mathematics

AMT21

[AMT21]
WEDNESDAY 5 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in black ink only. Questions which require drawing or sketching should be completed using an H.B. pencil.
Do not write with a gel pen.
Candidates must answer **all** questions from sections A and B.
Show clearly the full development of your answers.
Answers without working may not gain full credit.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100. The total available mark for each section of this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

11875

Answer all questions.

SECTION A

Mechanics

1 A uniform rod AB, of mass 15 kg, is held horizontally in equilibrium by two strings attached at C and D as shown in **Fig. 1** below.

The strings hang vertically and are light and inextensible.

AB = 6 m, BD = 1 m and CD = 2.5 m.

(i) Complete the diagram below showing all the external forces acting on the rod. [1]

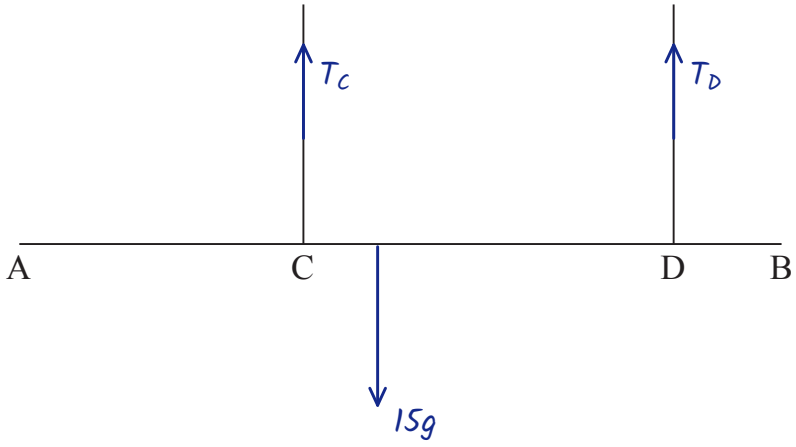


Fig. 1

(ii) Find the tension in each string. [5]

$AC = 6 - 1 - 2.5 = 2.5$ m; weight acts 3 m from A, 0.5 m from C

Moments about C: $T_D \times 2.5 = 15g \times 0.5$

$T_D = (15 \times 9.8 \times 0.5) \div 2.5 = 29.4$ N

$R(\uparrow)$: $T_C + T_D = 15g = 147$

$T_C = 147 - 29.4 = 117.6$ N

Tension at C = 118 N (3 s.f.), tension at D = 29.4 N

[Turn over

2 Two particles S and T, of masses $6m$ kg and $3m$ kg respectively, are travelling in a straight line when they collide.

Before the collision the particles are moving in opposite directions. S has a speed of $2u$ m s^{-1} and T has a speed of $6u$ m s^{-1}

After the collision the direction of each particle is reversed. S continues to travel with a speed of $2u$ m s^{-1} and T has a speed of ku m s^{-1} where k is a constant.

(i) Find the value of k. [4]

Take the initial direction of motion of S as positive.

Conservation of momentum: momentum before = momentum after

$6m(2u) + 3m(-6u) = 6m(-2u) + 3m(ku)$

$12mu - 18mu = -12mu + 3kmu$

$-6mu + 12mu = 3kmu$

$6mu = 3kmu$

$k = 2$

(ii) Find the impulse on T. [3]

Impulse on T = change in momentum of T

$I = m(v - u) = 3m(2u - (-6u))$

$I = 3m \times 8u$

$I = 24mu$ N s, in the direction opposite to the initial direction of motion of T (i.e. the direction S was moving).

[Turn over

3 The velocity v m s^{-1} of a particle travelling in a straight line at time t seconds is given by

$$v = 2t^2 - 9t + 4$$

The particle is instantaneously at rest on two different occasions.

(i) Find the times when the particle is at rest. [3]

At rest when $v = 0$:

$2t^2 - 9t + 4 = 0$

$(2t - 1)(t - 4) = 0$

$t = \frac{1}{2}$ s or $t = 4$ s

(ii) Find the acceleration of the particle when $t = 3$ [4]

$a = \frac{dv}{dt} = 4t - 9$

When $t = 3$: $a = 4(3) - 9$

$a = 3 \text{ m s}^{-2}$

[Turn over

(iii) Find the total distance covered by the particle in the first 5 seconds of motion. [8]

The particle changes direction at $t = \frac{1}{2}$ and $t = 4$, so find the displacement at $t = \frac{1}{2}$, $t = 4$ and $t = 5$ separately.

$s = \int v\,dt = \int (2t^2 - 9t + 4)\,dt$

$s = \frac{2}{3}t^3 - \frac{9}{2}t^2 + 4t + c$

$t = 0, s = 0 \Rightarrow c = 0$

$t = \frac{1}{2}$: $s = \frac{2}{3}\left(\frac{1}{8}\right) - \frac{9}{2}\left(\frac{1}{4}\right) + 4\left(\frac{1}{2}\right) = \frac{23}{24}$ m

$t = 4$: $s = \frac{2}{3}(64) - \frac{9}{2}(16) + 4(4) = -\frac{40}{3} = -13\frac{1}{3}$ m

$t = 5$: $s = \frac{2}{3}(125) - \frac{9}{2}(25) + 4(5) = -\frac{55}{6} = -9\frac{1}{6}$ m

$0 \leqslant t \leqslant \frac{1}{2}$: distance $= \frac{23}{24}$ m

$\frac{1}{2} \leqslant t \leqslant 4$: distance $= \frac{23}{24} + \frac{40}{3} = \frac{343}{24}$ m

$4 \leqslant t \leqslant 5$: distance $= -\frac{55}{6} - \left(-\frac{40}{3}\right) = \frac{25}{6}$ m

Total distance $= \frac{23}{24} + \frac{343}{24} + \frac{100}{24} = \frac{466}{24} = \frac{233}{12}$

$= 19\frac{5}{12}$ m $= 19.4$ m (3 s.f.)

[Turn over

4 A footballer taking a free kick strikes the ball and it leaves the horizontal ground with speed u m s^{-1} at an angle α to the horizontal. The ball is projected from O, as shown in **Fig. 2** below, and is modelled as a particle. The ball remains in the same vertical plane and experiences no air resistance.

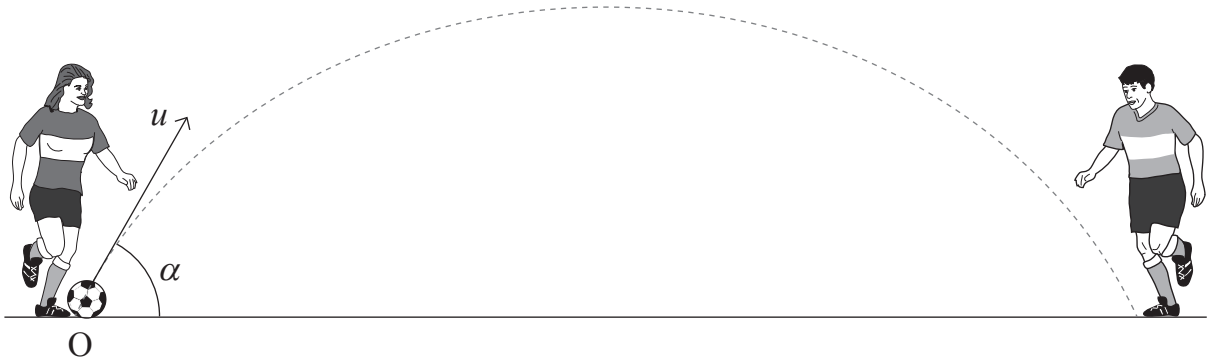


Fig. 2

(i) Show that the equation of trajectory for the ball is

$$y = x\tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha)$$

where x and y are the horizontal and vertical displacements of the ball from O. [7]

Horizontally ($a = 0$): initial velocity $u\cos\alpha$

$x = (u\cos\alpha)t$

$\Rightarrow t = \frac{x}{u\cos\alpha}$

Vertically ($a = -g$): initial velocity $u\sin\alpha$

$s = ut + \frac{1}{2}at^2$: $y = (u\sin\alpha)t - \frac{1}{2}gt^2$

Substitute for t:

$y = u\sin\alpha \times \frac{x}{u\cos\alpha} - \frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha}$

Q4 (i) continued

$y = x \tan \alpha - \frac{gx^2}{2u^2} \times \frac{1}{\cos^2 \alpha}$

Since $\frac{1}{\cos^2 \alpha} = \sec^2 \alpha = 1 + \tan^2 \alpha$:

$y = x \tan \alpha - \frac{gx^2}{2u^2}(1 + \tan^2 \alpha)$ as required

The ball is kicked towards a stationary teammate who is 15 m from O and on the same horizontal level. The initial speed of the ball is $14\,\text{m s}^{-1}$

(ii) Hence find the possible angles of projection for the ball to land at the foot of the other player. [5]

$u = 14$, $x = 15$ and $y = 0$ (ball lands at the teammate's feet):

$$0 = 15\tan\alpha - \frac{9.8 \times 15^2}{2 \times 14^2}(1 + \tan^2\alpha)$$

$$0 = 15\tan\alpha - 5.625(1 + \tan^2\alpha)$$

$$5.625\tan^2\alpha - 15\tan\alpha + 5.625 = 0$$

$\times 8/15$: $3\tan^2\alpha - 8\tan\alpha + 3 = 0$

$$\tan\alpha = \frac{8 \pm \sqrt{(64 - 36)}}{6} = \frac{8 \pm \sqrt{28}}{6}$$

$\tan\alpha = 2.2152\ldots$ or $\tan\alpha = 0.45142\ldots$

$\alpha = 65.7°$ or $\alpha = 24.3°$

[Turn over

5 A uniform ladder AB, of mass m_1 kg and length 6 m, rests with end A on rough horizontal ground.

The ladder is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{12}{13}$

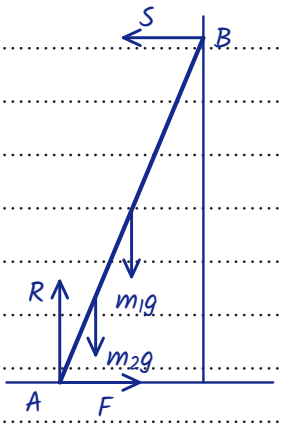
The other end B rests against a smooth vertical wall.

When John stands on the ladder, a distance of x m from A, the ladder is on the point of slipping. John has a mass m_2 kg.

If μ is the coefficient of friction between the ladder and the ground show that

$$x = \frac{72\mu(m_1 + m_2) - 15m_1}{5m_2}$$

[10]



$\sin\theta = \frac{12}{13} \Rightarrow \cos\theta = \frac{5}{13}$

R = normal reaction at A, F = friction at A,

S = normal reaction at B (smooth wall).

R(↑): $R = m_1g + m_2g = (m_1 + m_2)g$

R(→): $F = S$

On the point of slipping, friction is limiting:

$F = \mu R = \mu(m_1 + m_2)g$, so $S = \mu(m_1 + m_2)g$

Moments about A (clockwise = anticlockwise):

$S \times 6\sin\theta = m_1g \times 3\cos\theta + m_2g \times x\cos\theta$

$\mu(m_1 + m_2)g \times 6 \times \frac{12}{13} = 3m_1g \times \frac{5}{13} + m_2gx \times \frac{5}{13}$

Multiply by 13 and divide by g:

$72\mu(m_1 + m_2) = 15m_1 + 5m_2x$

$5m_2x = 72\mu(m_1 + m_2) - 15m_1$

$x = \frac{72\mu(m_1 + m_2) - 15m_1}{5m_2}$ as required



SECTION B

Statistics

6 **(a)** The owner of a local shop noticed that 40% of customers bought milk, 25% of customers bought bread and 55% of customers bought neither.

(i) Calculate the probability that a customer selected at random bought bread and no milk. [2]

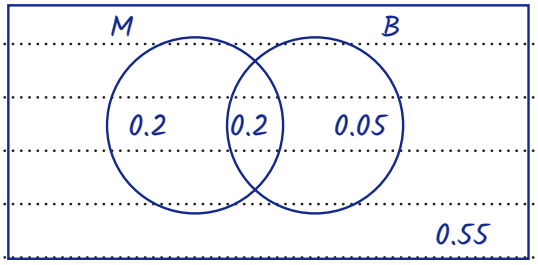
$P(M \cup B) = 1 - 0.55 = 0.45$

$P(M \cap B) = P(M) + P(B) - P(M \cup B)$

$= 0.4 + 0.25 - 0.45 = 0.2$

$P(B \cap M') = P(B) - P(M \cap B) = 0.25 - 0.2$

$= 0.05$



(ii) Calculate the probability that a customer selected at random bought bread given that they bought milk. [2]

$P(B \mid M) = \frac{P(B \cap M)}{P(M)} = \frac{0.2}{0.4} = \frac{1}{2} = 0.5$

(iii) Determine whether the event "buying milk" is statistically independent of the event "buying bread". [2]

$P(M \cap B) = 0.2$

$P(M) \times P(B) = 0.4 \times 0.25 = 0.1$

$0.2 \neq 0.1$, so $P(M \cap B) \neq P(M) \times P(B)$

$\therefore$ buying milk and buying bread are not statistically independent.

[Turn over

(b) Jenny has two bags. Bag A contains 5 white marbles and 2 black marbles.

Bag B contains 2 white marbles and 6 black marbles.

One marble is selected at random from bag A and placed in bag B.

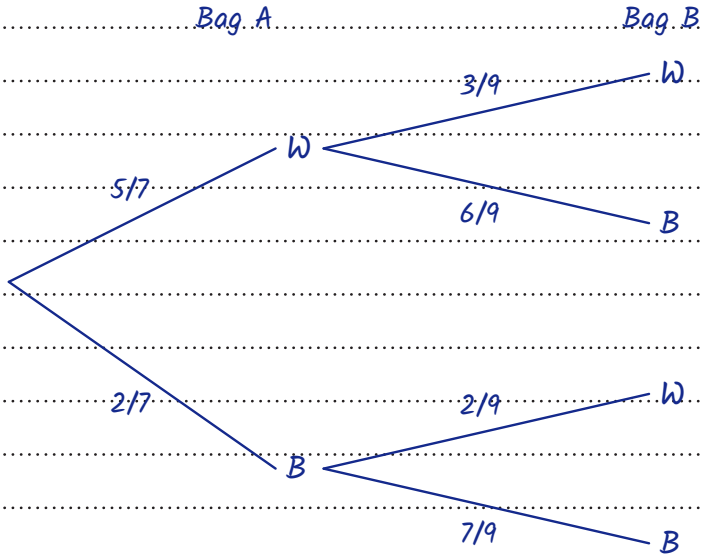
One marble is then selected at random from bag B.

(i) Find the probability that the selected marbles are both white. [2]

White from A: bag B then has 3 white, 6 black (9 marbles)

Black from A: bag B then has 2 white, 7 black (9 marbles)

$P(W, W) = \frac{5}{7} \times \frac{3}{9} = \frac{15}{63} = \frac{5}{21}$



(ii) Find the probability that the second marble is white. [2]

$P(2nd\ W) = P(W, W) + P(B, W)$

$= \frac{5}{21} + \frac{2}{7} \times \frac{2}{9} = \frac{15}{63} + \frac{4}{63} = \frac{19}{63}$

(iii) Find the probability that the first marble is black given that the second is white. [3]

$P(1st\ B \mid 2nd\ W) = \frac{P(B, W)}{P(2nd\ W)} = \frac{4/63}{19/63} = \frac{4}{19}$

[Turn over

7 **(a)** The length, in cm, of a copper pipe has a normal distribution with mean μ and variance σ^2

It is found that 12% of the pipes exceed 17.5 cm and that 25% of the pipes are less than 16.8 cm.

(i) Find the values of μ and σ^2 [8]

$X \sim N(\mu, \sigma^2)$

$P(X > 17.5) = 0.12 \Rightarrow P(Z < z) = 0.88 \Rightarrow z = 1.175$

$\frac{17.5 - \mu}{\sigma} = 1.175 \Rightarrow 1.175\sigma = 17.5 - \mu \quad (1)$

$P(X < 16.8) = 0.25 \Rightarrow z = -0.674$

$\frac{16.8 - \mu}{\sigma} = -0.674 \Rightarrow -0.674\sigma = 16.8 - \mu \quad (2)$

$(1) - (2)$: $1.849\sigma = 0.7$

$\sigma = 0.37858...$

$\mu = 17.5 - 1.175 \times 0.37858... = 17.0552...$

$\mu = 17.1$ (3 s.f.)

$\sigma^2 = (0.37858...)^2 = 0.14332...$

$\sigma^2 = 0.143$ (3 s.f.)

[Turn over

The copper pipe required for a certain bathroom unit must be less than 17.2 cm.

(ii) Calculate the percentage of copper pipes that will be too long for the bathroom unit. [5]

$P(X > 17.2) = P\left(Z > \frac{17.2 - 17.0552}{0.37858}\right)$

$= P(Z > 0.383)$

$= 1 - \Phi(0.383)$

$= 1 - 0.6491 = 0.3509$

Percentage of pipes too long = 35.1%

(b) Give a reason why the normal distribution is important in statistics. [1]

Many natural and practical quantities (e.g. lengths, heights, measurement errors) are approximately normally distributed, so it can be used to model them.

[Turn over



8 It has been established that 40% of children at a particular school reach a higher level of reading by the start of Year 5.

Ten children are chosen at random from those starting Year 5 at the school.

(i) Calculate the probability that at least one of these children has reached the higher level of reading. [4]

Let X = number of the 10 children at the higher level

$X \sim B(10, 0.4)$

$P(X \geqslant 1) = 1 - P(X = 0)$

$P(X = 0) = {}^{10}C_0\,(0.4)^0\,(0.6)^{10} = 0.0060466...$

$P(X \geqslant 1) = 1 - 0.0060466... = 0.99395...$

$= 0.994$ (3 s.f.)

Following a new reading incentive introduced by the school, a teacher believes that there has been an improvement in the percentage of children achieving the higher reading level.

The teacher randomly selects 10 children who are starting Year 5.
It was found that 7 of them had reached the higher level of reading.

(ii) Perform a suitable hypothesis test to determine whether the teacher's belief is true at the 5% level. [9]

Let p = proportion of children reaching the higher level

H_0: $p = 0.4$

H_1: $p > 0.4$ (one-tailed test, 5% level)

Under H_0, $X \sim B(10, 0.4)$

Reject H_0 if $P(X \geq 7) < 0.05$

$P(X \geq 7) = 1 - P(X \leq 6)$

$= 1 - 0.9452$

$= 0.0548$

$0.0548 > 0.05$, so 7 is not in the critical region

($P(X \geq 8) = 0.0123 < 0.05$, so critical region is $X \geq 8$)

Do not reject H_0: there is insufficient evidence at the 5% level to reject H_0 in favour of H_1.

The evidence does not support the teacher's belief that the percentage reaching the higher level has improved.

[Turn over

(iii) How could the test be improved? [1]

The teacher could take a larger sample of children (more than 10) so the test is more reliable.

[Turn over



9 A retiring parkrun organiser informs the new organiser that the time taken by the participants of the regular 5 kilometre run is normally distributed with mean 28 minutes and standard deviation 7.5 minutes.

The new organiser believes that the times for the month of December will have a different mean. Subsequently, he randomly selects 30 of the times recorded for participants during December. The mean time of this sample is 24.5 minutes.

(i) Carry out a hypothesis test and show that the new organiser's belief can be accepted at the 10% significance level. [8]

Let μ = mean December time (minutes)

H_0: $\mu = 28$ H_1: $\mu \neq 28$ (two-tailed test)

Under H_0, $\bar{X} \sim N(28, 7.5^2/30)$

10% level, two-tailed: critical values $z = \pm 1.645$

Reject H_0 if $z < -1.645$ or $z > 1.645$

$$z = \frac{24.5 - 28}{7.5 \div \sqrt{30}} = \frac{-3.5}{1.36931\ldots}$$

$z = -2.556$

$-2.556 < -1.645$, so z lies in the critical region.

Reject H_0 and accept H_1.

There is sufficient evidence at the 10% level that the mean December time is different from 28 minutes, so the new organiser's belief can be accepted.

(ii) In this context, give a reason why the new organiser's belief could be true. [1]

In December the colder weather may mean only the more dedicated (faster) runners take part, so the mean time could be lower than usual.

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