Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2025

Further Mathematics

Assessment Unit AS 2
assessing
Applied Mathematics

[SFM21]

SFM21

FRIDAY 16 MAY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the **Answer Booklet** provided.
You must answer the questions in the dedicated spaces provided in the Answer Booklet. Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an HB pencil.
You must answer **all** questions from sections A and B **or** A and C **or** A and D **or** C and D.
You should spend equal time on each of the two sections.
Show clearly the full development of your answers. **Answers without working may not gain full credit**.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
The total mark for each section of this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

SECTION A Mechanics 1

Answer all five questions in this section.

1 **(a)** A particle P moves along a horizontal line through O.

The distance from O to P is x metres.

A variable force, $F = \sqrt{x}$ newtons, acts on P as it moves from $x = 1.44$ to $x = 5.76$

Find the work done by F. [3]

(b) A particle Q of mass 4 kg has initial speed $9\,\text{m}\,\text{s}^{-1}$

When acted on by the resultant force $\mathbf{G} = \begin{pmatrix} 5 \\ 12 \\ 2 \end{pmatrix}$ N, Q moves from point A to point B,

where $\overrightarrow{\text{OA}} = \begin{pmatrix} -11 \\ 8 \\ 0 \end{pmatrix}$ m and $\overrightarrow{\text{OB}} = \begin{pmatrix} -5 \\ 24 \\ 4 \end{pmatrix}$ m.

Find the speed of Q at B. [5]

2 A waterpark flume designer models a water slide ABCD as shown in **Fig. 1** below.

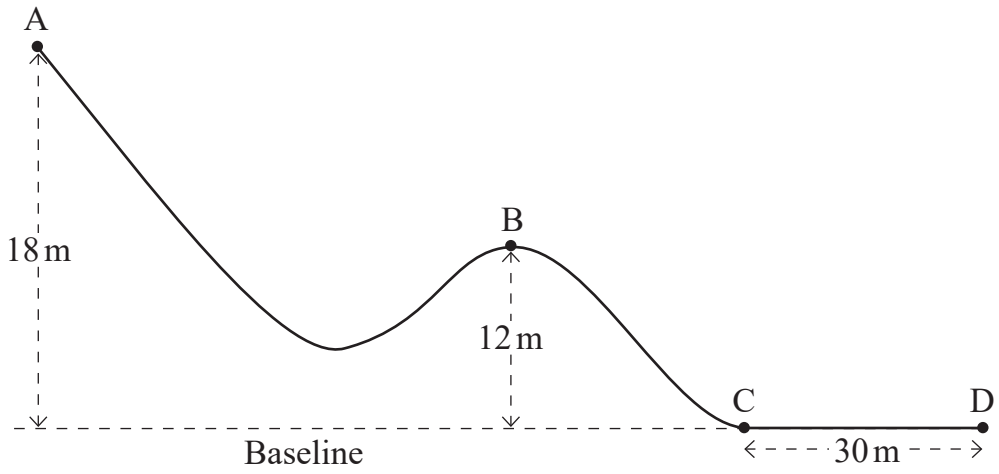


Fig. 1

The starting point A is 18 m vertically above the horizontal baseline.

The height of the local maximum B is 12 m vertically above the baseline.

The section ABC is frictionless.

The retardation section CD is 30 m long and is along the baseline.

A girl of mass 28 kg slides from rest at A.

Take the gravitational potential energy as zero at the baseline.

Find:

(i) the gravitational potential energy of the girl at A; [2]

(ii) the kinetic energy of the girl at B. [5]

The girl comes to rest at the point D.

(iii) Find, using the Work-Energy Principle, the value of the coefficient of friction between the girl and section CD. [4]

3 An aeroplane, of mass m kg, is travelling at a constant speed of $v\,\text{m}\,\text{s}^{-1}$ in a horizontal circular path of radius r metres.

The aeroplane banks at an angle of 30° to the horizontal.

The lift force, L newtons, acts perpendicular to the wings and through the centre of mass as shown in **Fig. 2** below.

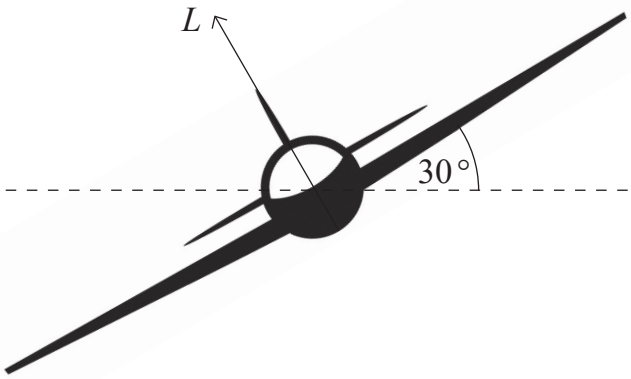


Fig. 2

Model the aeroplane as a particle.

(i) Show that $v^2 = \dfrac{rg}{\sqrt{3}}$ [7]

The aeroplane now flies at 45° to the horizontal in a circle of the same radius.

(ii) Find the new velocity in terms of v. [3]

4 A train of mass 200 tonnes moves on a level track.

When travelling at a constant speed of $72\,\text{km}\,\text{h}^{-1}$ it experiences a constant resistance of 750 newtons per tonne.

(i) Find the power output of the engine. [4]

The same train, operating at the same power, now ascends a hill inclined at θ° to the horizontal, where $\sin\theta = 0.05$

The resistance acting on the train is unchanged.

(ii) Find the train's deceleration when it is moving at $54\,\text{km}\,\text{h}^{-1}$ [5]

5 P and Q are particles of mass m_0 kg and m_1 kg, respectively, resting on a horizontal surface.

You may assume that P and Q occupy the same position.

They are connected by a light elastic string of natural length l metres and modulus of elasticity $2m_1 g$ newtons.

P is initially projected vertically upwards with speed $u\,\mathrm{m\,s^{-1}}$

For the first stage of the motion, Q is at rest on the surface and vertically below P as shown in **Fig. 3** below.

When the extension in the string is x metres, Q is about to move vertically.

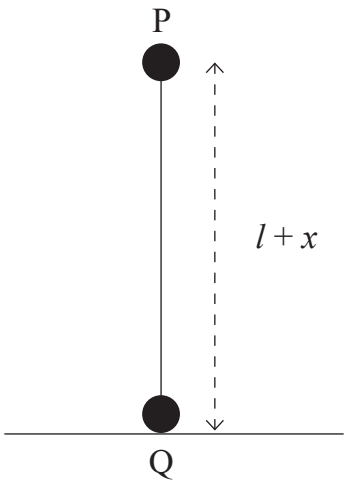


Fig. 3

(i) By considering the forces acting on Q, find the value of x in terms of l. [3]

P first comes to rest at the instant that Q is about to move vertically.

(ii) Using energy considerations, show that

$$u = \sqrt{gl\left(3 + \frac{m_1}{2m_0}\right)}$$ [9]

SECTION B Mechanics 2

Answer all five questions in this section.

1 **(i)** Write down the dimensions of force, F, and velocity, u. [1]

(ii) Find the dimensions of power, P, and work done, W. [3]

A researcher believes the work done, W, by the lifting unit of a grain machine may be related through the relationship $W = \mathrm{k}P^a u^b V^c$ where:

k is a dimensionless constant;

P is the power of its engine;

u is the velocity of the expelled grain;

V is the volume of its hopper.

(iii) Use the Method of Dimensions to find a formula for W. [7]

2 Two elastic strings, each of natural length l metres, are attached to two fixed points A and B on a sloping ceiling.

The free end of each string is attached to a particle P of weight W newtons, as shown in **Fig. 1** below.

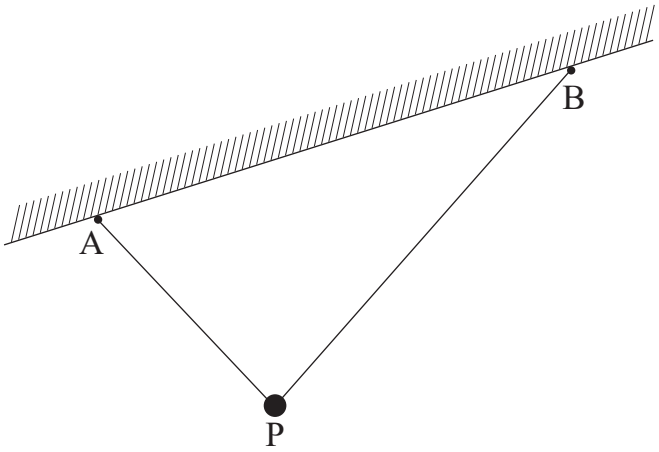


Fig. 1

P hangs in equilibrium.

The string AP makes an acute angle α with the horizontal and has extension x metres.

The string BP makes an acute angle β with the horizontal and has extension y metres.

The modulus of elasticity of AP is $5W$ newtons and the modulus of elasticity of BP is $7W$ newtons.

(i) Show that

$$5x\cos\alpha = 7y\cos\beta$$ [3]

(ii) Given that $\beta = 60°$, show that the extension x is given by

$$x = \frac{l}{5\left(\sin\alpha + \sqrt{3}\cos\alpha\right)}$$ [5]

3 You may use the following approximations in this question:

The universal gravitational constant, $G = 6.67 \times 10^{-11}\,m^3\,kg^{-1}s^{-2}$

The mass of Mars, $M = 6.42 \times 10^{23}\,kg$

The value of acceleration due to gravity on the surface of Mars is $3.70\,m\,s^{-2}$

(i) Show that the radius of Mars, R, is $3.40 \times 10^6\,m$. [3]

A day on Mars is 88 643 seconds long.
A geostationary satellite orbits above the Martian equator with the same angular velocity as Mars.

(ii) Find the height of a geostationary orbit above the surface of Mars. [6]

The geostationary satellite shown in **Fig. 2** below can only be seen, along the Martian equator, from points on the arc PQ.

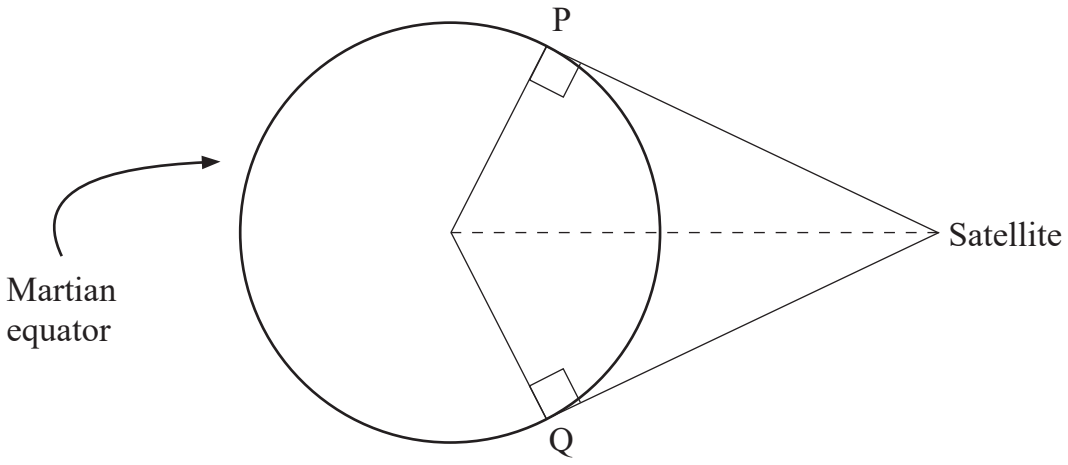


Fig. 2

(iii) Calculate the minimum number of geostationary satellites it takes for a satellite to be seen from every point on the equator. [4]

4 Aircraft A is travelling at $350\,\text{km}\,\text{h}^{-1}$ on a bearing of 220°

At noon, it passes 2 km north of fighter jet J travelling at $650\,\text{km}\,\text{h}^{-1}$ on a bearing of 320° as shown in **Fig. 3** below.

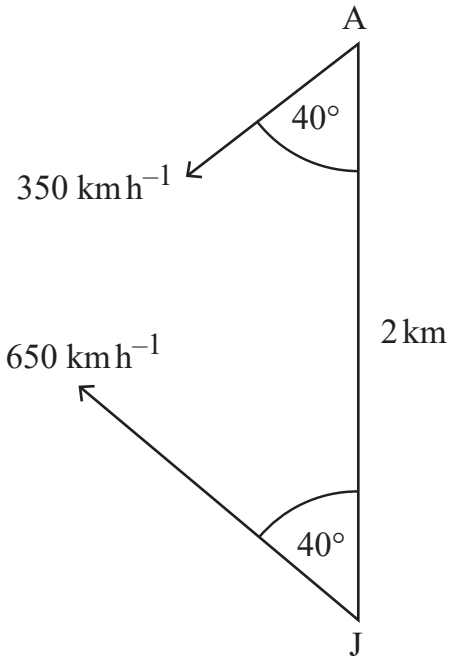


Fig. 3

(i) Find the magnitude and direction of the velocity of fighter jet J relative to aircraft A. [7]

(ii) Find the shortest distance between the jet and the aircraft in the subsequent motion. [2]

5 A weight is suspended at rest from a fixed point by a light inextensible string of length 3 m.

It is given an initial horizontal velocity $u\,\text{m}\,\text{s}^{-1}$ and subsequently moves in a vertical circle about the fixed point.

It leaves the path of this vertical circle when the string makes an angle θ with the upward vertical, where $\sin\theta = \frac{4}{5}$

Find the value of u. [9]

SECTION C Statistics

Answer all five questions in this section.

1 The relationship between pressure, y kPa, and temperature, x °C, is being investigated for a sample of a fixed volume of air.

A scientist measures the pressures of the sample at a number of set temperatures.

The data is given in **Table 1** below.

Table 1

Temperature (x, °C)	–150	–100	–50	0	50	100	150
Pressure (y, kPa)	33.5	46.2	56.3	68.3	w	86.1	98.3

From the data, the equation of the least squares regression line of pressure y on temperature x is found to be

$$y = 66.4 + bx$$

where b is a constant.

(i) State the response variable for this investigation. [1]

(ii) Show that $w = 76.1$ [4]

(iii) Hence find the value of b in the equation of the least squares regression line. [3]

(iv) Use the equation of the least squares regression line to estimate the pressure of the sample of air at a temperature of –200 °C.

Comment on the reliability of this estimate. [2]

2 A company consists of 16 employees.

Ten of the employees are male and six are female.

The company randomly chooses ten of the 16 employees to represent the company at a gala dinner.

(i) Find the number of different ways of selecting the group of ten employees for the dinner. [1]

(ii) Find the number of selections in which there are exactly six male employees among the ten chosen. [2]

The group selected to attend the gala dinner consists of six male and four female employees.

At the dinner the ten employees are randomly seated around a circular table with ten equally spaced chairs.

(iii) Find the number of different seating arrangements around the table. [1]

(iv) Find the probability that the four female employees are seated next to each other around the table. [3]

3 **(i)** State one necessary condition when modelling using the Geometric probability distribution. [1]

A biased die has each of its faces painted a different colour.

The probability that the die lands on its red face on any given throw is p.

The probability that the die lands on its red face for the first time on the fourth attempt is $\frac{8}{81}$

The probability that the die lands on its red face for the first time on the seventh attempt is $\frac{64}{2187}$

(ii) Show that $p = \frac{1}{3}$ [5]

Let the number of throws needed for the die to land on its red face for the first time be modelled by the random variable X.

The random variable Y is found directly from the random variable X using the formula

$$Y = aX + b$$

where a and b are constants.

Y has expected value 9 and variance 150

(iii) Given that a is positive, find the values of a and b.

$$\left[\text{If } X \sim \text{Geo}(p), \text{ then } \text{E}(X) = \frac{1}{p} \text{ and } \text{Var}(X) = \frac{1-p}{p^2}\right]$$

[5]

4 The random variable X has the continuous probability density function f(x) defined by

$$f(x) = \begin{cases} bx & 0 \leqslant x \leqslant a \\ \dfrac{4}{x^2} & a \leqslant x \leqslant \dfrac{16}{5} \\ 0 & \text{otherwise} \end{cases}$$

where a and b are positive constants.

(i) Show that $b = \dfrac{4}{a^3}$ [1]

(ii) Find the exact values of a and b. [8]

(iii) Find the lower quartile value of X. [4]

5 For a particular model of printer, faults occur one at a time at a random rate of 0.625 faults per month.

(i) Find the probability that a printer of this type will develop more than three faults over the course of one month. [4]

The company that supplies the printers offers a guarantee whereby it will, upon request, replace a printer if it develops more than n faults over a 12-month period.

The company wants the probability of having to replace a printer to be less than 1%.

(ii) Find the smallest value of n that satisfies this condition. [4]

The company has restricted the guarantee to the first three years after a new printer is purchased.

(iii) Suggest a reason for this restriction. [1]

SECTION D Discrete and Decision Mathematics

Answer all five questions in this section.

1 **(a)** **(i)** Explain what is meant by an Eulerian circuit. [2]

(ii) Write down an Eulerian circuit for the graph K_5 shown in **Fig. 1** below. [1]

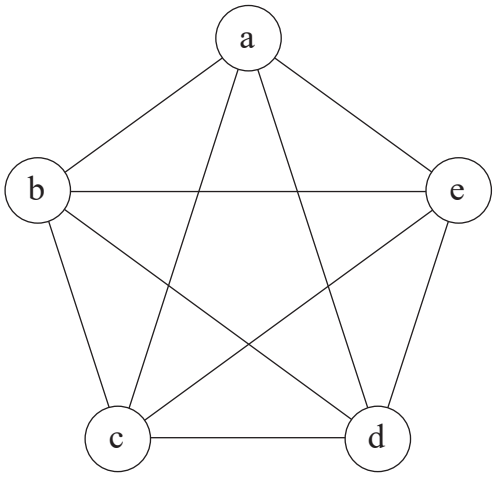


Fig. 1

(b) Consider the graph shown in **Fig. 2** below.

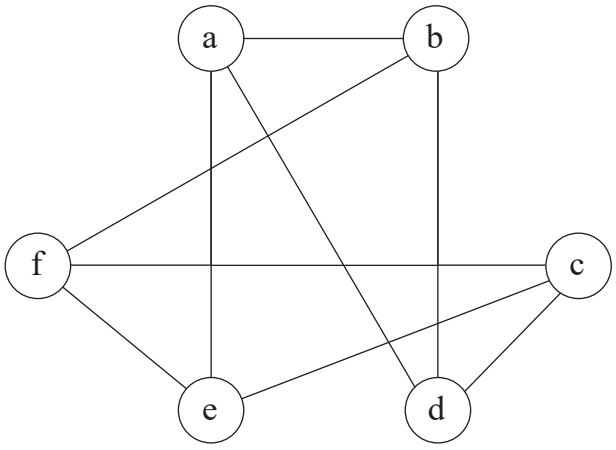


Fig. 2

Find how many edges need to be removed from the graph shown in **Fig. 2** to leave the minimum number of edges required to form:

(i) a spanning tree; [1]

(ii) a Hamiltonian cycle. [1]

(iii) Is the graph shown in **Fig. 2** planar?

If so, redraw it to demonstrate this. [2]

2 A sequence is defined by the recurrence relationship

$$s_{n+3} = s_{n+2} + 9s_{n+1} - 9s_n \qquad n \geqslant 0$$

where $s_0 = 7$, $s_1 = 25$ and $s_2 = 31$

(i) Show that the auxiliary equation is

$$x^3 - x^2 - 9x + 9 = 0$$ [2]

(ii) Hence show that

$$s_n = 5P^n - 2Q^n + R$$

where P, Q and R are integers to be determined. [7]

(iii) Find s_8 [1]

3 Cooking a dinner can be modelled by the activity network shown in **Fig. 3** below.

The activities are labelled A, B, C, …, M and are represented by the edges.

The number on each edge represents the estimated time, in minutes, required to complete that activity.

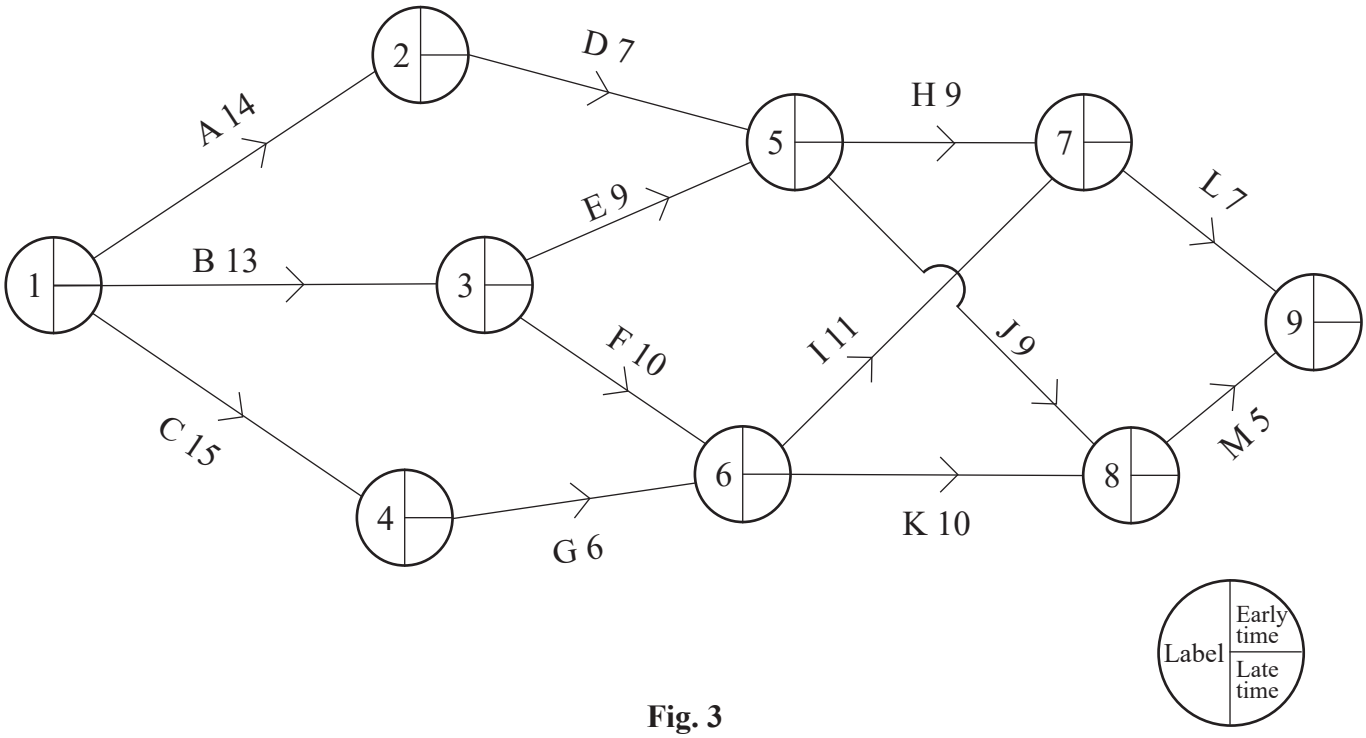


Fig. 3

(i) Calculate the early and late times for each event.

Write your answers in the spaces provided in the copy of **Fig. 3** in the **Answer Booklet**. [5]

(ii) Determine:

(a) the critical path; [1]

(b) the time to complete the activities in the critical path. [1]

(iii) (a) What is meant by the float time for activity D? [1]

(b) Calculate the float time for activity D. [2]

(c) How many activities have float times? [1]

4 Let p, q and r be propositional statements.

(i) Use truth tables to prove

$$p \text{ and } (q \text{ or } r) \equiv (p \text{ and } q) \text{ or } (p \text{ and } r)$$ [6]

(ii) Hence, write in a simpler form the statement:

"Sally likes pavlova and ice cream or peaches and ice cream." [2]

(iii) Without using truth tables, use the statement in part **(i)** to simplify

$$\sim p \text{ and } [\ (p \text{ and } q) \text{ or } (p \text{ and } \sim q)\ ]$$ [3]

5 The symmetry transformations of a square are shown in **Fig. 4** below.

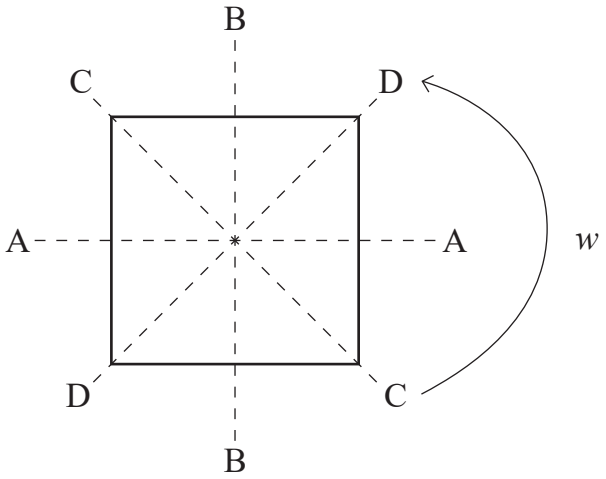


Fig. 4

A, B, C and D represent reflections of the square in the lines labelled correspondingly.

w represents an anticlockwise rotation of the square, in its plane, through 90° about its centre.

The composition of transformations is denoted by the operator *o*.

The partially completed group table for the symmetry group (S, o), where S is the set of transformations of the square, is shown in **Fig. 5** below.

For example, $w \; o \; A = D$

o	i	w	w^2	w^3	A	B	C	D
i	i	w	w^2	w^3	A	B	C	
w	w	w^2	w^3	i	D	C	A	
w^2	w^2	w^3	i	w	B	A	D	
w^3	w^3	i	w	w^2	C	D		
A	A	C	B	D		w^2	w	
B	B	D	A	C	w^2		w^3	
C	C		D	A	w^3	w		
D								

Fig. 5

(i) Complete the copy of this group table in the **Answer Booklet**. [3]

(ii) Determine whether or not (S, o) is commutative, justifying your answer. [2]

(iii) Complete the table in the **Answer Booklet**, entering the period of each element. [3]

(iv) List all the subgroups of (S, o) [3]

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Centre Number

Candidate Number

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ANSWER BOOKLET

Subject

Further Mathematics

Unit

2

Component Code

SFM21

SFM21

Applied Mathematics

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Question Number	SECTION A Mechanics 1
A1 (a)	

Question Number	SECTION A Mechanics 1
A1 (b)	

[Turn over

Question Number	SECTION A Mechanics 1
A2 (i)	
A2 (ii)	

Question Number	SECTION A Mechanics 1
A2 (iii)	

Question Number	SECTION A Mechanics 1
A3 (i)	

Question Number	SECTION A Mechanics 1
A3 (ii)	

Question Number	SECTION A Mechanics 1
A4 (i)	

Question Number	SECTION A Mechanics 1
A4 (ii)	

Question Number	SECTION A Mechanics 1
A5 (i)	
A5 (ii)	

Question Number	SECTION A Mechanics 1

[Turn over

Question Number	SECTION B Mechanics 2
B1 (i)	
B1 (ii)	

Question Number	SECTION B Mechanics 2
B1 (iii)	

Question Number	SECTION B Mechanics 2
B2 (i)	

Question Number	SECTION B Mechanics 2
B2 (ii)	

Question Number	SECTION B Mechanics 2
B3 (i)	
B3 (ii)	

Question Number	SECTION B Mechanics 2
B3 (iii)	

Question Number	SECTION B Mechanics 2
B4 (i)	

Question Number	SECTION B Mechanics 2
B4 (ii)	

Question Number	SECTION B Mechanics 2
B5	

Question Number	SECTION B Mechanics 2

[Turn over

Question Number	SECTION C Statistics
C1 (i)	
C1 (ii)	

Question Number	SECTION C Statistics
C1 (iii)	
C1 (iv)	

Question Number	SECTION C Statistics
C2 (i)	
C2 (ii)	

Question Number	SECTION C Statistics
C2 (iii)	
C2 (iv)	

Question Number	SECTION C Statistics
C3 (i)	
C3 (ii)	

Question Number	SECTION C Statistics
C3 (iii)	

Question Number	SECTION C Statistics
C4 (i)	
C4 (ii)	

Question Number	SECTION C Statistics
C4 (iii)	

Question Number	SECTION C Statistics
C5 (i)	

Question Number	SECTION C Statistics
C5 (ii)	
C5 (iii)	

Question Number	SECTION D Discrete and Decision Mathematics
D1 (a) (i)	
D1 (a) (ii)	

Question Number	SECTION D Discrete and Decision Mathematics
D1 (b) (i)	
D1 (b) (ii)	
D1 (b) (iii)	

Question Number	SECTION D Discrete and Decision Mathematics
D2 (i)	
D2 (ii)	

Question Number	SECTION D Discrete and Decision Mathematics
D2 (iii)	

Question Number	SECTION D Discrete and Decision Mathematics
D3 (i)	

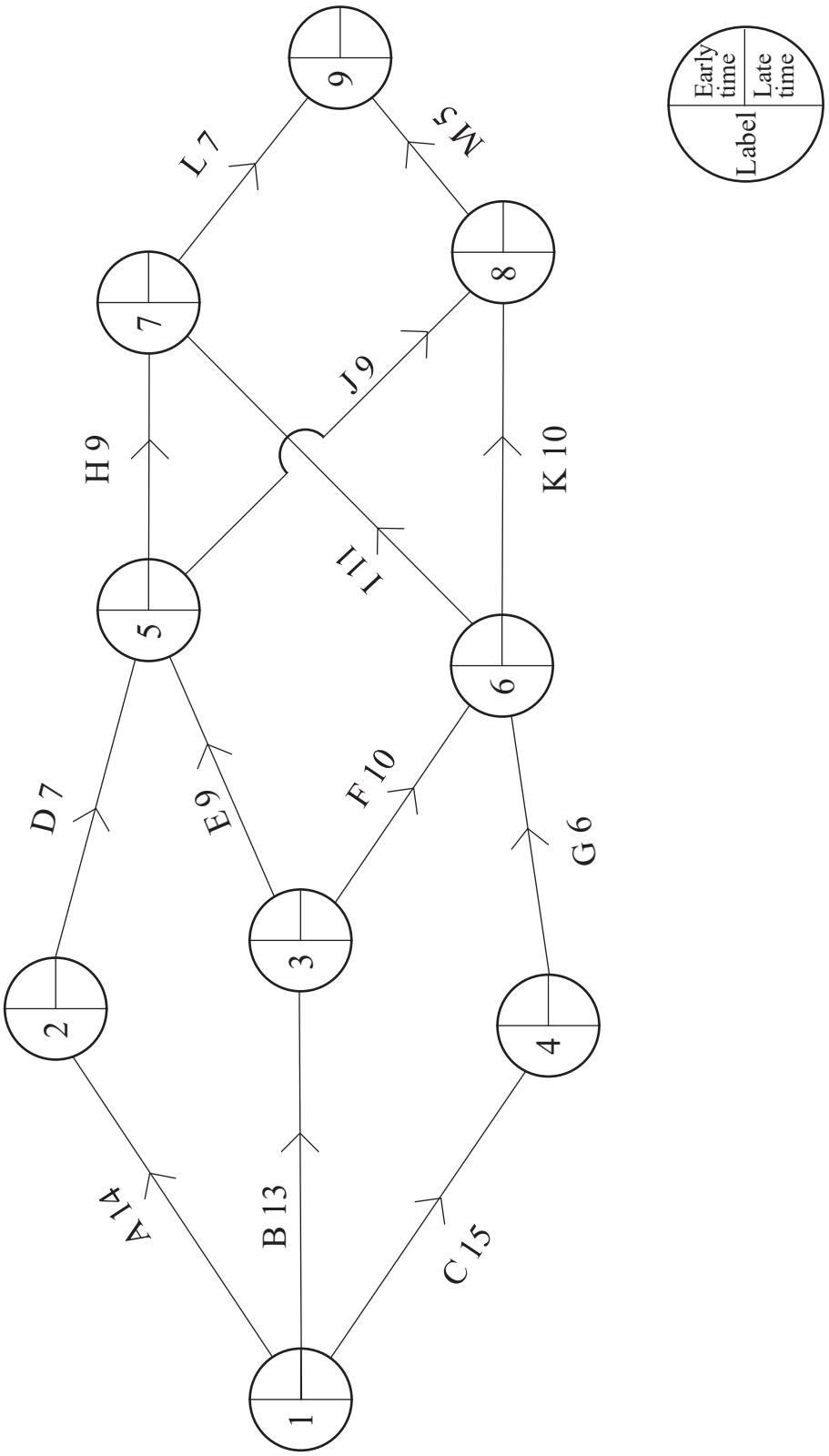


Fig. 3

Question Number	SECTION D Discrete and Decision Mathematics
D3 (ii) (a)	
D3 (ii) (b)	
D3 (iii) (a)	
D3 (iii) (b)	
D3 (iii) (c)	

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Question Number	SECTION D Discrete and Decision Mathematics
D4 (i)	
D4 (ii)	
D4 (iii)	

Question Number	SECTION D Discrete and Decision Mathematics
D5 (i)	

o	i	w	w^2	w^3	A	B	C	D
i	i	w	w^2	w^3	A	B	C	
w	w	w^2	w^3	i	D	C	A	
w^2	w^2	w^3	i	w	B	A	D	
w^3	w^3	i	w	w^2	C	D		
A	A	C	B	D		w^2	w	
B	B	D	A	C	w^2		w^3	
C	C		D	A	w^3	w		
D								

Fig. 5

Question Number	SECTION D Discrete and Decision Mathematics
D5 (ii)	

D5 (iii)

Element	i	w	w^2	w^3	A	B	C	D
Period								

D5 (iv)

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