Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2025

Further Mathematics

Assessment Unit AS 2
assessing
Applied Mathematics

[SFM21]

FRIDAY 16 MAY, AFTERNOON

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) FURTHER MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

Section A Mechanics 1

		AVAILABLE MARKS

1 **(a)** Work done $= \int F \, dx$

$$= \int_{1.44}^{5.76} x^{\frac{1}{2}} \, dx \qquad \text{M1 W1}$$

$$= \left[\frac{2}{3} x^{\frac{3}{2}}\right]_{1.44}^{5.76}$$

$$= 8.06 \text{ J} \qquad \text{MW1}$$

(b) Displacement $= \overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB}$

$$= \begin{pmatrix} -5 \\ 24 \\ 4 \end{pmatrix} - \begin{pmatrix} -11 \\ 8 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 16 \\ 4 \end{pmatrix} \qquad \text{MW1}$$

Work done $= \mathbf{G} \bullet \overrightarrow{AB} = \begin{pmatrix} 5 \\ 12 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 6 \\ 16 \\ 4 \end{pmatrix} = 230 \text{ J}$ M1 W1

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = 230$ M1

$2(v^2 - 81) = 230$

$v = 14 \text{ m s}^{-1}$ W1 8

2 **(i)** PE at A $= mgh$

$= 28 \times 9.8 \times 18$ M1

$= 4939.2 \text{ J} = 4940 \text{ J (3sf)}$ W1

(ii) KE at A $= 0$ MW1

PE at B $= 3292.8$ J MW1

By conservation of mechanical energy

$KE_A + PE_A = KE_B + PE_B$

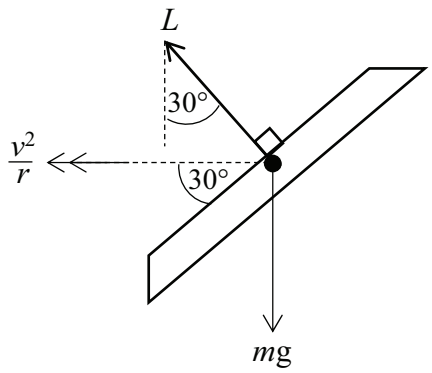
$0 + 4939.2 = KE_B + 3292.8$ M1W1

$KE_B = 1646.4 \text{ J} = 1650 \text{ J (3sf)}$ W1

(iii) WD by friction = Loss of KE (= PE at A) MW1

$\mu mg \times CD = 4939.2$ MW1 W1

$\mu = 0.6$ W1 11

				AVAILABLE MARKS
3	**(i)**			
		$\leftrightarrow$ Newton's 2nd Law $\quad \frac{mv^2}{r} = L \sin 30°$	M2 W1	
		$\updownarrow mg = L \cos 30°$	M1 W1	
		Dividing		
		$\tan 30° = \frac{v^2}{rg}$	M1	
		$v^2 = \frac{rg}{\sqrt{3}}$	W1	
	(ii)	$v_2^2 = rg \tan 45°$	M1 W1	
		$= rg$		
		$v^2 = \frac{rg}{\sqrt{3}}$		
		$\therefore v_2^2 = \sqrt{3}v^2$		
		$v_2 = 3^{\frac{1}{4}}v$	W1	10
4	**(i)**	72 km h^{-1} = 20 m s^{-1}	MW1	
		Tractive force = $\frac{P}{20}$	M1	
		Resistance = 750 × 200 = 150 000 N	MW1	
		Constant speed $\Rightarrow \frac{P}{v} = R$		
		P = 150 000 × 20 = 3 000 000 W = 3000 kW	W1	
	(ii)	Component of weight along plane = 200 000 g × 0.05		
		= 98 000 N	MW1	
		54 km h^{-1} = 15 m s^{-1}	MW1	
		Newton's 2nd Law		
		$200\,000a = \frac{3\,000\,000}{15} - 98\,000 - 150\,000$	M1W1	
		$a = -\frac{48\,000}{200\,000}$		
		$= -0.24\ \text{m s}^{-2}$		
		Deceleration = 0.24 ms^{-2}	W1	9

			AVAILABLE MARKS
5	**(i)** Vertical force equilibrium for Q		
	$m_1 g = \frac{\lambda x}{l}$	M1 W1	
	$= \frac{2m_1 gx}{l}$		
	$x = \frac{1}{2}l$	W1	
	(ii) At start of motion of P		
	$KE_P = \frac{1}{2} m_0 u^2$		
	$KE_Q = PE_P = PE_Q = 0$	M1	
	At end, with extension x		
	$GPE_P = m_0 g(l + x)$	M1 W1	
	$EPE_P = \frac{\lambda x^2}{2l}$	M1	
	$= \frac{2m_1 gx^2}{2l}$	MW1	
	By conservation of mechanical energy		
	$\frac{1}{2} m_0 u^2 = 2m_1 g \left(\frac{l^2}{4}\right)\left(\frac{1}{2l}\right) + m_0 g\left(\frac{3l}{2}\right)$	M1 W2	
	$\frac{1}{2}u^2 = \left(\frac{1}{m_0} \times \frac{m_1}{4} + \frac{3}{2}\right) gl$		
	$u = \sqrt{gl\left(3 + \frac{m_1}{2m_0}\right)}$	W1	12
		Section A	**50**

Section B Mechanics 2

AVAILABLE MARKS

1 **(i)** $[F] = MLT^{-2}$
$[u] = LT^{-1}$ MW1

(ii) $[P] = [F]\,[u]$
$= M\,L^2\,T^{-3}$ MW1

$W = F \times d$
$[W] = [F][d]$
$= MLT^{-2} \times L = ML^2T^{-2}$ M1 W1

(iii) $W = k\,P^a\,u^b\,V^c$
$(ML^2T^{-2}) = (ML^2T^{-3})^a(LT^{-1})^b\,L^{3c}$

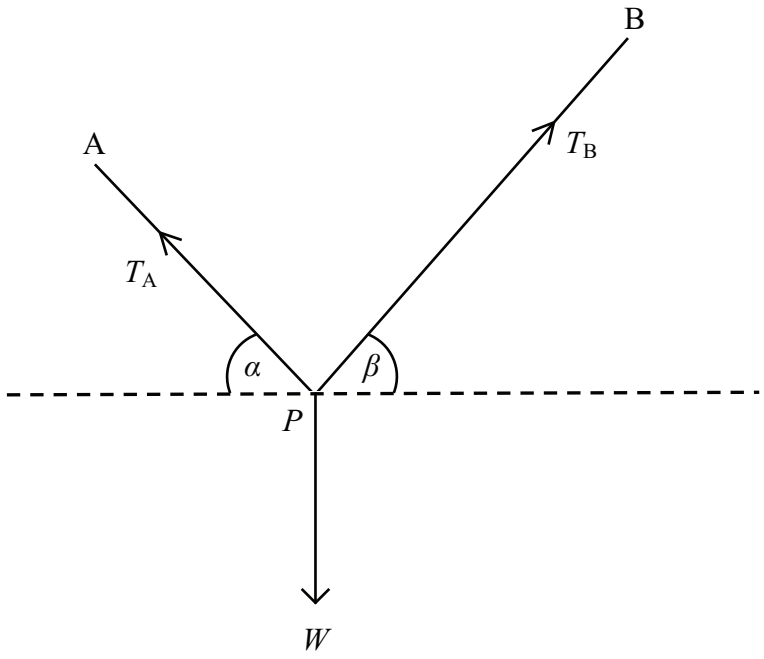
M: $1 = a$ M1 W1
T: $-2 = -3a - b$
L: $2 = 2a + b + 3c$ M1 W1

Solving $\Rightarrow a = 1, b = -1, c = \frac{1}{3}$ M1 W1

$W = \dfrac{kP\sqrt[3]{V}}{u}$ W1 11

2

AVAILABLE MARKS



(i) $\leftrightarrow$ At P $\quad T_A \cos\alpha = T_B \cos\beta$ MW1

$$\frac{5Wx}{l}\cos\alpha = \frac{7Wy}{l}\cos\beta \quad \text{MW1}$$

$$5x\cos\alpha = 7y\cos\beta \quad \text{W1}$$

(ii) $\updownarrow$ At P $\quad \frac{5Wx}{l}\sin\alpha + \frac{7Wy}{l}\sin 60 = W$ M1 W1

$$5x\sin\alpha + 7x\left(\frac{5\cos\alpha}{7\cos 60}\right)\sin 60 = l \quad \text{W1}$$

$$5x\left(\frac{1}{2}\sin\alpha + \frac{\sqrt{3}}{2}\cos\alpha\right) = \frac{l}{2} \quad \text{M1}$$

$$x = \frac{l}{5(\sin\alpha + \sqrt{3}\cos\alpha)} \quad \text{W1}$$

8

3 **(i)** On surface of Mars

AVAILABLE MARKS

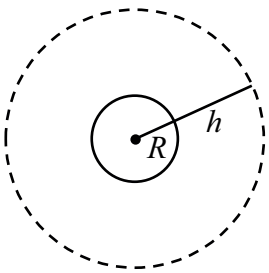
$$mg' = \frac{mMG}{R^2}$$ M1 M1

Hence

$$R = \sqrt{\frac{MG}{g'}} = \sqrt{\frac{6.42 \times 10^{23} \times 6.67 \times 10^{-11}}{3.7}}$$ W1

$= 3.40 \times 10^6$ m

(ii)



MW2

$m\omega^2 (R + h) = \frac{mMG}{(R+h)^2}$ W1

$(R + h)^3 = \frac{MG}{\omega^2}$

$\frac{2\pi}{\omega} = 88\,643$

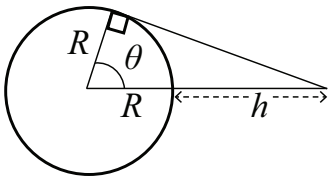
$\omega = \frac{2\pi}{88\,643}$ MW1

$h = \sqrt[3]{\frac{MG}{\omega^2}} - R$ M1

$= \left(6.42 \times 10^{23} \times 6.67 \times 10^{-11} \times \frac{88\,643^2}{4\pi^2}\right)^{\frac{1}{3}} - 3.4 \times 10^6$

$= 1.70 \times 10^7$ m W1

(iii)



$\theta = \cos^{-1}\left(\frac{R}{R+h}\right)$ M1

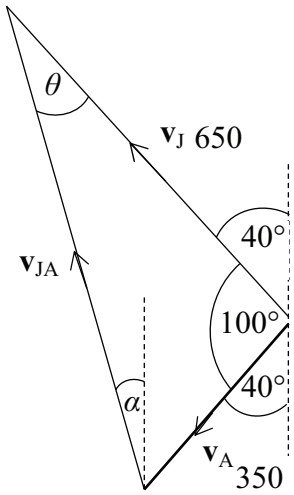
$= 80.4°$ W1

$360 \div 160.8 = 2.24$ W1

$\therefore$ 3 satellites needed W1 13

4

AVAILABLE MARKS



(i) Using velocity diagram

$\mathbf{v}_{JA} = -\mathbf{v}_A + \mathbf{v}_J$ M1 MW1

$v_{JA}^2 = 650^2 + 350^2 - 2 \times 350 \times 650 \cos 100°$ M1 W1

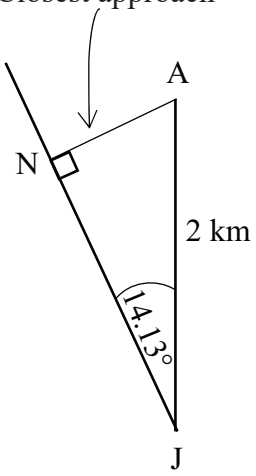
$v_{JA} = 789.943 \text{ km h}^{-1} = 790 \text{ km h}^{-1}$ W1

$\dfrac{\sin\theta}{350} = \dfrac{\sin 100°}{v_{JA}}$ M1

$\theta = \sin^{-1}\left(\dfrac{350 \sin 100°}{v_{JA}}\right) = 25.87°$

$\Rightarrow \alpha = 14.13° = 14.1°$ (N14.1°W) W1

(ii) Closest approach



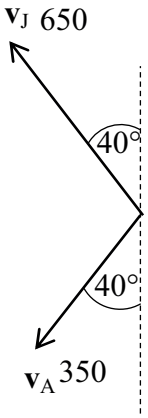
AN = 2 sin 14.13° M1

= 0.488 km (= 488 m) W1 9

4 Alternative Solution

AVAILABLE MARKS

(i)

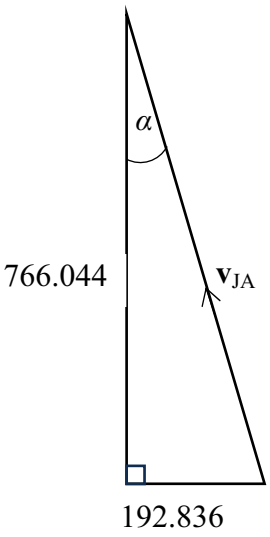


$\mathbf{v}_A = 350\begin{pmatrix} -\sin 40° \\ -\cos 40° \end{pmatrix}$ M1

$\mathbf{v}_J = 650\begin{pmatrix} -\sin 40° \\ \cos 40° \end{pmatrix}$ W1

$\mathbf{v}_{JA} = \mathbf{v}_J - \mathbf{v}_A = \begin{pmatrix} -650\sin 40° + 350\sin 40° \\ 650\cos 40° + 350\cos 40° \end{pmatrix}$ M1

$= \begin{pmatrix} -192.836 \\ 766.044 \end{pmatrix}$ W1



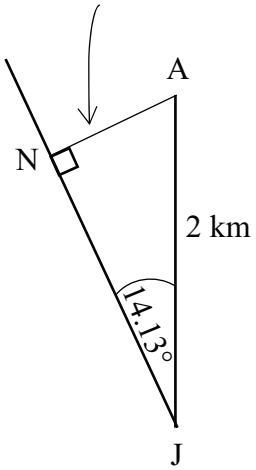
$\alpha = \tan^{-1}\left(\frac{192.836}{766.044}\right)$ M1

$= 14.13° = 14.1°$ (N14.1°W) W1

Magnitude $= \sqrt{(-192.836)^2 + 766.044^2}$

$= 789.942$ km h^{-1} $= 790$ km h^{-1} MW1

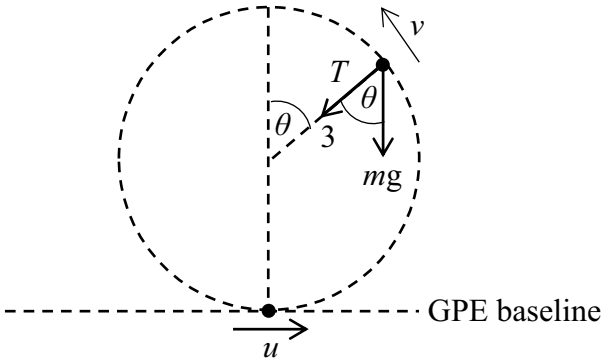
(ii) Closest approach



AN = 2 sin 14.13° M1

= 0.488 km (= 488 m) W1

5



		AVAILABLE MARKS
Resolving radially		
$\frac{mv^2}{3} = mg\cos\theta + T$	M1 MW1 W1	
Leaves path when $T = 0$	M1	
$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + 3mg(1 + \cos\theta)$	M1 W2	
$mu^2 = m(3g\cos\theta) + 6mg(1 + \cos\theta)$	MW1	
$u^2 = 6g + 9g\cos\theta$		
$u^2 = 6g + 9g\left(\frac{3}{5}\right)$		
$u^2 = \sqrt{111.72}$		
$u^2 = 10.6\ \text{m s}^{-1}$ (3sf)	W1	9

Note:

$PE_i = 0$ $\quad KE_i = \frac{1}{2}mu^2$

$PE_f = mg\left(3 + \frac{9}{5}\right)$ $\quad KE_f = \frac{1}{2}(3mg)\left(\frac{3}{5}\right)$

Section B | **50**

Section C Statistics

AVAILABLE MARKS

1 **(i)** Pressure MW1

(ii) $\bar{x} = \frac{\Sigma x}{7} = 0$ MW1

$\bar{y} = \frac{\Sigma y}{7} = \frac{388.7 + w}{7}$ MW1

$a = \bar{y} - b\bar{x}$

$66.4 = \frac{388.7 + w}{7}$ M1

$w = 76.1$ kPa W1

(iii) Summary statistics

$\sum x = 0 \qquad \sum y = 464.8 \qquad \sum xy = 14\,700$

$\sum x^2 = 70\,000 \qquad n = 7$ MW1

$S_{xx} = 70\,000 - \frac{0^2}{7} = 70\,000$

$S_{xy} = 14\,700 - \frac{0 \times 464.8}{7} = 14700$ MW1

$b = \frac{S_{xy}}{S_{xx}} = \frac{14\,700}{70\,000} = 0.21$ MW1

(iv) $y = 66.4 + 0.21x$

When $x = -200 \qquad y = 24.4$ kPa MW1

Unreliable as temperature of –200°C lies outside the range of the temperatures provided (extrapolation). MW1 10

			AVAILABLE MARKS
2	**(i)**	${}^{16}C_{10} = 8008$ MW1	
	(ii)	${}^{10}C_6 \times {}^{6}C_4 = 210 \times 15$ MW1 $= 3150$ MW1	
	(iii)	Circular seating arrangement forms a closed loop i.e. there is no 'starting point'. Therefore the number of different seating arrangements is given by: $(10 - 1)! = 9! = 362\,880$ MW1	
	(iv)	Taking the four female employees as a single entity, the number of arrangements is $(7 - 1)! = 720$ MW1 There are 4! arrangements of the four female employees. Therefore the total number of arrangements is given by: $(7 - 1)! \times 4! = 6! \times 4! = 720 \times 24 = 17280$ MW1 $P = \frac{17\,280}{362\,880} = \frac{1}{21}$ MW1	7
3	**(i)**	One of: Trials are independent Probability of a successful outcome is the same for each trial Only two outcomes for each trial: success and failure Continue until success achieved MW1	
	(ii)	$q^3p = \frac{8}{81}$ M1 W1 $q^6p = \frac{64}{2187}$ MW1 $\frac{q^6p}{q^3p} = \frac{\left(\frac{64}{2187}\right)}{\left(\frac{8}{81}\right)}$ M1 $q^3 = \frac{8}{27}$ $q = \frac{2}{3} \qquad p = \frac{1}{3}$ MW1	
	(iii)	$X \sim \text{Geo}\left(\frac{1}{3}\right) \quad p = \frac{1}{3}$ $E(X) = \frac{1}{p} = 3$ $\text{Var}(X) = \frac{1-p}{p^2} = \frac{\left(\frac{2}{3}\right)}{\left(\frac{1}{3}\right)^2} = 6$ MW1 $\text{Var}(Y) = a^2\,\text{Var}(X)$ $150 = 6a^2$ MW1 $a = 5$ W1 $E(Y) = aE(X) + b$ $9 = 5(3) + b$ MW1 $b = -6$ W1	11

4 **(i)** At $x = a$

$ba = \frac{4}{a^2} \Rightarrow b = \frac{4}{a^3}$ MW1

(ii) $\int f(x)\,dx = 1$ M1

$\int_0^a \frac{4}{a^3}x\,dx + \int_a^{\frac{16}{5}} \frac{4}{x^2}\,dx = 1$ M1 MW2

$\frac{2}{a^3}[x^2]_0^a + \left[-\frac{4}{x}\right]_a^{\frac{16}{5}} = 1$ MW1

$\frac{2}{a} + \frac{4}{a} = \frac{9}{4}$ M1 W1

$a = \frac{8}{3} \qquad b = \frac{27}{128}$ W1

(iii) $\int_0^a bx\,dx = \int_0^{\frac{8}{3}} \frac{27}{128}\,x\,dx = \frac{3}{4}$

$\therefore \int_0^a bx\,dx > \frac{1}{4}$

$\therefore$ LQ lies in interval $0 \leqslant x \leqslant \frac{8}{3}$

Let lower quartile be Q

$\int_0^Q \frac{27}{128}x\,dx = \frac{1}{4}$ M1 MW1

$\frac{27}{256}Q^2 = \frac{1}{4}$ MW1

$Q = 1.53960... = 1.54$ (to 3sf) W1 13

$= \frac{8}{3\sqrt{3}}$

AVAILABLE MARKS

		AVAILABLE MARKS
5 **(i)**	Let X be the random variable 'number of faults per month for one printer'	
	$X \sim \text{Po}(0.625)$ $\quad \lambda = 0.625$ faults/month — MW1	
	Solution 1 (with calculator statistical distribution facility)	
	$\text{P}(X > 3) = 1 - \text{P}(X \leqslant 3)$ — M1	
	$= 1 - 0.99612$ — M1	
	$= 0.00388$ — W1	
	Solution 2 (without calculator statistical distribution facility)	
	$\text{P}(X > 3) = 1 - \text{P}(X \leqslant 3)$ — M1	
	$= 1 - [\text{P}(X = 0) + \text{P}(X = 1) + \text{P}(X = 2) + \text{P}(X = 3)]$	
	$= 1 - e^{-0.625}\left(\frac{0.625^0}{0!} + \frac{0.625^1}{1!} + \frac{0.625^2}{2!} + \frac{0.625^3}{3!}\right)$ — M1	
	$= 1 - 0.99612$	
	$= 0.00388$ — W1	
(ii)	Let Y be the random variable 'number of faults per year for one printer'	
	$\lambda = 0.625 \times 12 = 7.5$ faults/year $\quad Y \sim \text{Po}(7.5)$ — MW1	
	$\text{P}(Y > n) < 0.01$	
	$1 - \text{P}(Y \leqslant n) < 0.01$	
	$\text{P}(Y \leqslant n) > 0.99$ — M1	
	From Poisson Tables (with $\lambda = 7.5$)	
	$\text{P}(Y \leqslant 14) = 0.9897$ — M1	
	$\text{P}(Y \leqslant 15) = 0.9954$	
	The smallest value of n is 15 — W1	
(iii)	Faults are more likely to occur with older printers. — MW1	9
	Section C	**50**

Section D Discrete and Decision Mathematics

AVAILABLE MARKS

1 **(a)** **(i)** An Eulerian circuit is

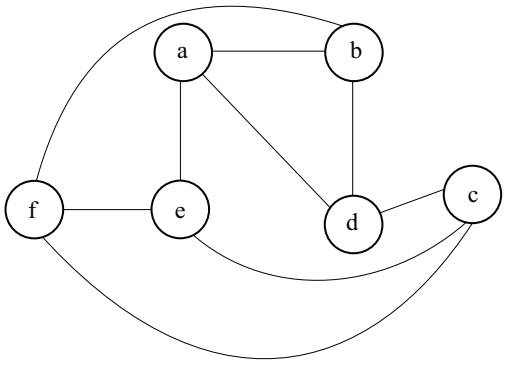
- a closed tour of edges — M1
- visiting every edge once — M1

(ii) $a - b - c - d - e - a - c - e - b - d - a$ — W1

(b) **(i)** 4 — W1

(ii) 3 — W1

(iii) Yes – planar — M1



W1 — 7

2 **(i)** Substitute $s_n = kx^n$

to give $kx^{n+3} = kx^{n+2} + 9kx^{n+1} - 9kx^n$ — M1

$x^n(x^3 - x^2 - 9x + 9) = 0$

$x^3 - x^2 - 9x + 9 = 0$ — W1

($x = 0$ does not satisfy starting values)

(ii) $x^3 - x^2 - 9x + 9 = 0$

$(x - 1)(x^2 - 9) = 0$ — M1

$x = 1, 3, -3$ — W1

$s_n = A(3^n) + B(-3)^n + C$ — MW1

$n = 0 \quad A + B + C = 7$

$n = 1 \quad 3A - 3B + C = 25$ — M1

$n = 2 \quad 9A + 9B + C = 31$ — W1

$2A - 4B = 18$ — M1

$6A + 12B = 6$

$12A = 60$

$A = 5, B = -2, C = 4$ — W1

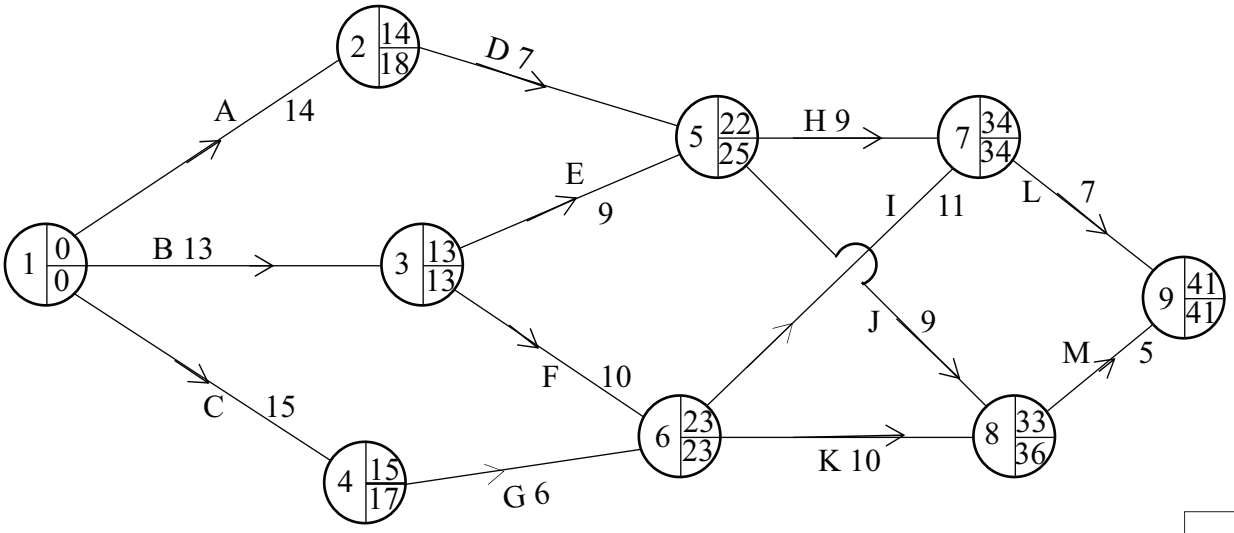
$P = 3, Q = -3, R = 4$

(iii) $s_8 = 5(3)^8 - 2(-3)^8 + 4$

$= 19687$ — W1 — 10

3 (i)

AVAILABLE MARKS



E_i = early time at node i
L_i = late time at node i

E_1, E_2, E_3, E_4	MW1	
$E_5, \ldots E_9$	MW1	
$L_7, \ldots L_9$	MW1	
L_5, L_6	MW1	
$L_1, \ldots L_4$	MW1	

(ii) (a) Critical path is B F I L MW1

(b) 41 minutes W1

(iii) (a) Float time for activity D is the maximum number of minutes after the end of the preceding activities by which activity D must be started in order to finish on time. M1

(b) Float time for activity D = 25 – 7 – 14 M1
= 4 minutes W1

(c) 13 – 4 = 9 activities MW1 11

4 **(i)**

p	q	r	q or r	p and (q or r)	p and q	p and r	(p and q) or (p and r)
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F
	M1		MW1	MW1	MW1	MW1	MW1

AVAILABLE MARKS

(ii) Sally likes ice cream and either pavlova or peaches MW2

(iii) ~p and [(p and q) or (p and ~q)]
~p and [p and (q or ~q) MW1
~p and p MW1
False MW1 11

5 **(i)**

o	i	w	w^2	w^3	A	B	C	D
i	i	w	w^2	w^3	A	B	C	D
w	w	w^2	w^3	i	D	C	A	B
w^2	w^2	w^3	i	w	B	A	D	C
w^3	w^3	i	w	w^2	C	D	B	A
A	A	C	B	D	i	w^2	w	w^3
B	B	D	A	C	w^2	i	w^3	w
C	C	B	D	A	w^3	w	i	w^2
D	D	A	C	B	w	w^3	w^2	i

MW3

(ii) Not commutative — M1
e.g. $C \; o \; w = B$ but $w \; o \; C = A$ — W1

(iii)

Element	i	w	w^2	w^3	A	B	C	D
Period	1	4	2	4	2	2	2	2

MW3

(iv) $\{i\}$ and $\{i, w, w^2, w^3, A, B, C, D\}$ — MW1
$\{i, w, w^2, w^3\}$ and $\{i, w^2\}$ — MW1
$\{i, A\}$, $\{i, B\}$, $\{i, C\}$ and $\{i, D\}$ — MW1

11

Section D **50**

Total **100**

AVAILABLE MARKS