Rewarding Learning

ADVANCED
General Certificate of Education
2025

Centre Number

Candidate Number

Further Mathematics

Assessment Unit A2 1

assessing

Pure Mathematics

[AFM11]

AFM11

WEDNESDAY 21 MAY, MORNING

TIME

2 hours 15 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all eleven** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Questions which require drawing or sketching should be completed using an HB pencil.
Show clearly the full development of your answers. **Answers without working may not gain full credit.**
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 150.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

1 Solve the differential equation

$$(x^2 + 4)\frac{dy}{dx} - 2xy = 3x^2 + 12$$

given that $y = 4$ when $x = 0$ [10]

[Turn over

2 **(a)** Evaluate the improper integral

$$\int_0^2 \ln x \, \mathrm{d}x$$

giving your answer in the form $a \ln b + c$

You may assume that $\lim_{t \to 0} (t \ln t) = 0$ [8]

(b) (i) Express

$$25x^2 + 10x - 8$$

in the form $(px + q)^2 + r$ where p, q and r are integers. [2]

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(ii) Hence show that

$$\int_{\frac{2}{5}}^{2} \frac{1}{\sqrt{25x^2 + 10x - 8}}\, dx = \frac{1}{5} \cosh^{-1}\left(\frac{11}{3}\right)$$ [5]

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3 **Fig. 1** below shows the design for an enamel badge.

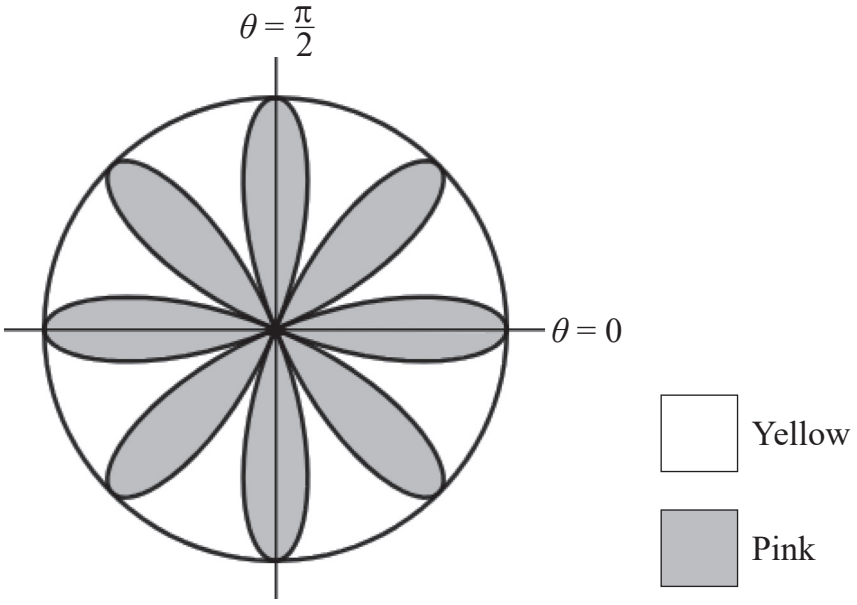


Fig. 1

The design is modelled by a curve enclosed within a circle, such that the curve touches the circle at multiple points, as shown.

The equation of the curve is $r = 6\cos 4\theta$

The shaded region is to be pink and the clear region is to be yellow.

By finding the area of the petal symmetric about the initial line ($\theta = 0$), show that an equal amount of each colour is required. [11]

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[Turn over

4 The distance x cm of a particle P from a fixed point O, at time t seconds, can be modelled by

$$\frac{d^2x}{dt^2} + 7\frac{dx}{dt} + 12x = 24$$

(i) Solve this differential equation given that $\frac{dx}{dt} = 13$ and $x = 0$ when $t = 0$ [13]

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(ii) Find the limiting value of x as t increases. [1]

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[Turn over

5 **(i)** Using the exponential definitions of $\sinh x$ and $\cosh x$ show that for $|x| < 1$

$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \qquad [6]$$



(ii) Hence, or otherwise, solve the equation

$$x = 4\tanh\left(\ln\sqrt{3x}\right)$$

where $0 < x < 4$ [6]

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[Turn over

6 **(i)** Show that

$$(r+1)^2 r^2 (2r+1) - r^2 (r-1)^2 (2r-1) \equiv 10r^4 + 2r^2 \qquad [4]$$

(ii) Hence show that

$$\sum_{r=1}^{n} r^4 = \frac{1}{30} n(n+1)(2n+1)(3n^2+3n-1) \qquad [8]$$

[Turn over

7 For each non-negative integer n let

$$I_n = \int_0^1 x^n \mathrm{e}^{-4x}\,\mathrm{d}x$$

(i) Show that for $n \geqslant 1$

$$I_n = -\frac{1}{4}\mathrm{e}^{-4} + \frac{1}{4}nI_{n-1} \qquad [7]$$

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(ii) Hence find the exact value of

$$\int_0^1 x^2 e^{-4x}\, dx$$ [5]

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8 **(a)** Derive the Maclaurin expansion for

$$\cos^{-1} x$$

up to and including the term in x^3 [8]

(b) Using small angle approximations, show that

$$\frac{1 + 2\tan\theta}{13 + 5\sin\theta - 12\cos\theta} \approx 1 - 3\theta + 9\theta^2$$

when θ is small. [6]

9 The expression

$$\cos\theta - \sqrt{3}\sin\theta$$

can be written in the form

$$R\cos(\theta + \alpha) \quad \text{where} \quad 0 < \alpha < \frac{\pi}{2} \quad \text{and} \quad R > 0$$

(i) Find the values of R and α. [5]

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(ii) Hence prove by mathematical induction that

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left[\mathrm{e}^x\cos\left(\sqrt{3}x\right)\right] = 2^n\,\mathrm{e}^x\cos\left(\sqrt{3}x + \frac{n\pi}{3}\right)$$

for all integers $n \geqslant 1$ [11]

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10 (i) Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[x \sinh^{-1}(3x)\right] = \sinh^{-1}(3x) + \frac{3x}{\sqrt{1 + 9x^2}} \qquad [5]$$

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Fig. 2 below shows part of the graphs of $y = \sinh^{-1}(3x)$, $y = 0.5x$ and $x = 6$

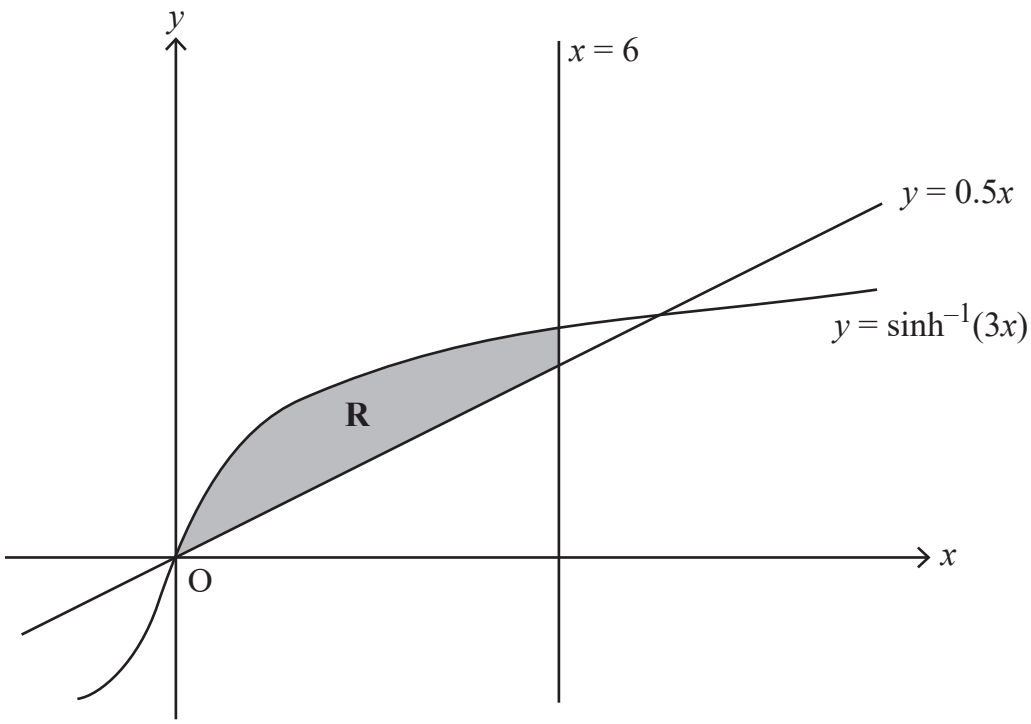


Fig. 2

(ii) Using part **(i)**, find the exact area of the shaded region R in **Fig. 2**, enclosed between the curve $y = \sinh^{-1}(3x)$ and the lines $y = 0.5x$ and $x = 6$ [9]

11 (a) Find, in the form $re^{i\theta}$, the values of the three roots of the equation

$$z^3 = \frac{-5 + 3i}{4 + i}$$ [6]

(b) (i) By expanding $(\cos\theta + \mathrm{i}\sin\theta)^6$ and applying de Moivre's theorem, prove that

$$\frac{\sin 6\theta}{\sin 2\theta} \equiv 2\cos 4\theta + 1 \quad \text{where} \quad 0 < \theta < \frac{\pi}{2} \qquad [10]$$

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(ii) Hence solve the equation

$$\sin 6\theta = 2\sin 2\theta \quad \text{for} \quad 0 < \theta < \frac{\pi}{2} \qquad [4]$$

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Question Number	**Marks**
1	
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Total Marks	

Examiner Number

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