Rewarding Learning

ADVANCED
General Certificate of Education
2023

Further Mathematics

Assessment Unit A2 2

assessing

Applied Mathematics

[AFM21]

MONDAY 5 JUNE, AFTERNOON

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) FURTHER MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

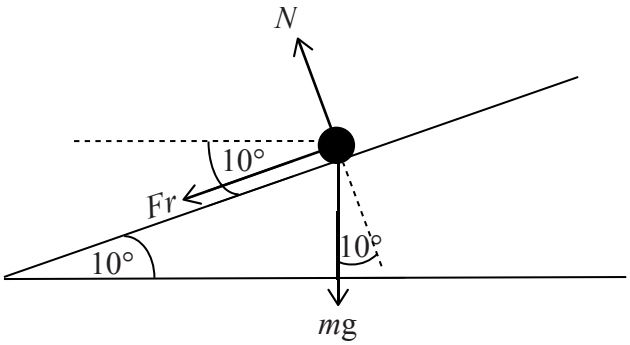
(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

Section A Mechanics 1

AVAILABLE MARKS

1



$(\leftarrow)$ $Fr\cos 10° + N\sin 10° = \frac{mv^2}{50}$ M2 W1

$(\uparrow)$ $N\cos 10° = Fr\sin 10° + mg$ M1W1

Also $Fr = \mu N$

$= 0.6N$ MW1

$\Rightarrow N\sin 10° + 0.6N\cos 10° = \frac{mv^2}{50}$

$N\cos 10° - 0.6N\sin 10° = mg$

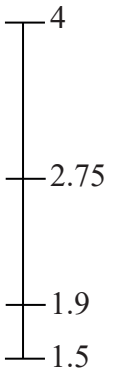
Dividing gives

$$\frac{\tan 10° + 0.6}{1 - 0.6\tan 10°} = \frac{v^2}{50g}$$ M1 W1

$$\Rightarrow v^2 = 50g\left(\frac{\tan 10° + 0.6}{1 - 0.6\tan 10°}\right)$$

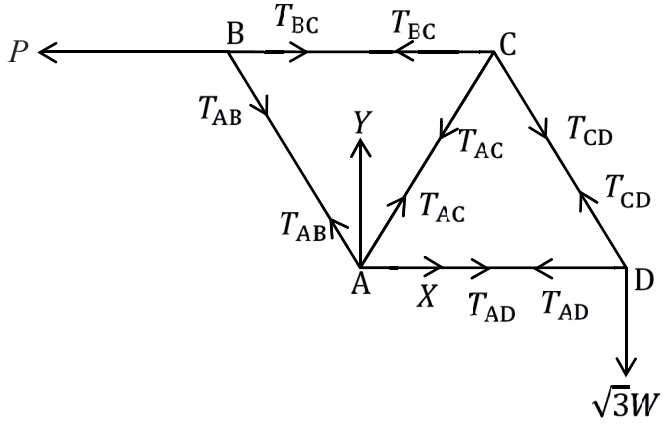
$v = 20.6\ \text{m s}^{-1}$ (3sf) W1 9

				AVAILABLE MARKS
2	**(i)**	$2 = 2\pi \sqrt{\frac{l_A}{9.81}}$	M1	
		$l_A = \frac{9.81}{\pi^2}$	W1	
		$= 0.994$ m (3sf)		
	(ii)	$T_B = 2\pi \sqrt{\frac{9.81}{\pi^2 \times 9.92}}$	M1	
		$= 1.988\ldots$	W1	
		At A: Number of oscillations per day $= \frac{24 \times 3600}{2}$	M1	
		$= 43\ 200$	W1	
		At B: Number of oscillations per day $= \frac{24 \times 3600}{1.988\ldots}$		
		$= 43\ 441.53$	MW1	
		Difference in number of oscillations $= 242$ (3sf)	W1	
	(iii)	$T_B = 2 = 2\pi \sqrt{\frac{l_B}{9.92}}$	M1	
		$l_B = \frac{9.92}{\pi^2}$	W1	
		$= \frac{9.92}{9.81}\ l_A$	M1	
		$= 1.01121\ l_A$		
		Hence the pendulum's length must be increased by 1.12% (3sf)	W1	12

				AVAILABLE MARKS
3	**(i)**	Amplitude = 0.5(4 – 1.5)	M1	
		= 1.25 m	W1	
		Period = 2(8.5 – 2)	M1	
		= 13 hours	W1	
	(ii)	$T = \frac{2\pi}{\omega}$	M1	
		$\omega = \frac{2\pi}{13}$	W1	
				
		$x = 2.75 - 1.9 = 0.85$	MW1	
		$x = r \cos \omega t$		
		$0.85 = 1.25 \cos\left(\frac{2\pi}{13}t\right)$	M1W1	
		$\frac{2\pi}{13}t = 0.823\ldots$		
		$t = 1.70286\ldots$		
		Earliest time is 3.43 am.	W1	10

4

AVAILABLE MARKS



(i) Let each rod have length l

M(A): $\sqrt{3}\,Wl = P \times l \cos 30°$ M1W1

$P = 2W$ W1

(ii) At B: (→) $T_{BC} + T_{AB} \cos 60° = 2W$ M1 W1

(↓) $T_{AB} \cos 30° = 0$ MW1

$T_{AB} = 0$

$T_{BC} = 2W$ newtons MW1

At C: (←) $2W + T_{AC} \cos 60° = T_{CD} \cos 60°$ MW1

(↓) $T_{AC} \cos 30° + T_{CD} \cos 30° = 0$ MW1

$T_{CD} = 2W$ newtons

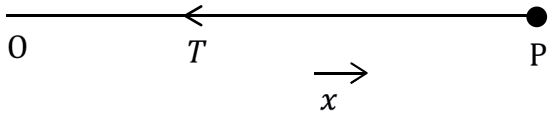
$T_{AC} = -2W$ newtons MW1

At D: (←) $T_{AD} + T_{CD} \cos 60° = 0$

$T_{AD} = -W$ newtons MW1

(iii) $T_{AB} = 0$ so therefore AB could be removed. MW1 12

5 (i)



AVAILABLE MARKS

$-T - kv = 0.16a$ M1 W2

$-\frac{0.36x}{0.25} - 0.96\frac{dx}{dt} = 0.16\frac{d^2x}{dt^2}$ MW2

$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$ MW1

(ii) AQE : $m^2 + 6m + 9 = 0$ MW1

$(m + 3)(m + 3) = 0$

$m = -3$ W1

$\Rightarrow x = (At+B)e^{-3t}$ MW1

$t = 0, x = 0.04 \Rightarrow 0.04 = B$ M1 W1

$\frac{dx}{dt} = -3(At + B)e^{-3t} + Ae^{-3t}$ MW1

$t = 0, \frac{dx}{dt} = 0$

$\Rightarrow 0 = -3(0.04) + A$

$A = 0.12$ MW1

$\Rightarrow x = (0.12t + 0.04)e^{-3t}$

$x = 0.04(3t + 1)e^{-3t}$ W1

(iii) AQE has equal roots. M1

Therefore the system is critically damped. W1 16

6 (i)

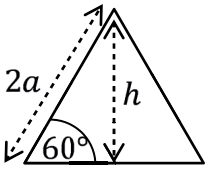
Point	B	F	G	H	Total
Mass	$4m$	m	$3m$	$2m$	$10m$
Dist of CoM from AH	$-a$	$-\frac{1}{2}a$	$\frac{1}{2}a$	0	$\bar{x}$

M1 W1

$\Rightarrow -4ma - \frac{1}{2}ma + \frac{3}{2}ma + 0 = 10m\,\bar{x}$ M1 W2

$\bar{x} = -0.3a$

i.e. $0.3a$ metres to left of AH W1



$h = 2a \sin 60°$
$= \sqrt{3}\,a$

Point	B	F	G	H	Total
Mass	$4m$	m	$3m$	$2m$	$10m$
Dist of CoM from BE	0	$\frac{\sqrt{3}}{2}a$	$\frac{\sqrt{3}}{2}a$	$-2a$	$\bar{y}$

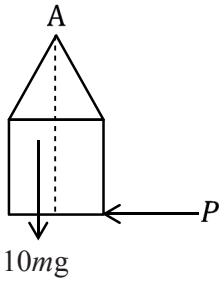
M1 W1

$\Rightarrow 0 + \frac{\sqrt{3}}{2}ma + \frac{3\sqrt{3}}{2}ma - 4ma = 10m\,\bar{y}$ MW1

$\bar{y} = -\frac{1}{5}(2 - \sqrt{3})a$

i.e. $\frac{1}{5}(2 - \sqrt{3})a$ metres below BE W1

(ii)

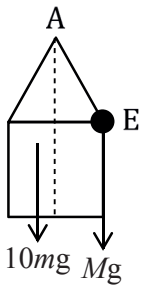


M(A): $P(2 + \sqrt{3})a = 10mg \times 0.3a$ M1 W1

$P = \frac{3}{2 + \sqrt{3}}\,mg$ W1

(iii) M(A): $Mga = 10mg \times 0.3a$ M1 W1

$M = 3m$ W1



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Section A **75**

Section B Mechanics 2

AVAILABLE MARKS

1 $m\dfrac{dv}{dt} = mg - 5mv$ M1 W1

$\dfrac{dv}{dt} = 9.8 - 5v$

$\displaystyle\int \frac{dv}{9.8 - 5v} = \int dt$ M1 W2

$-\dfrac{1}{5}\ln|9.8 - 5v| = t + c$ M1

$\ln|9.8 - 5v| = -5t - 5c$

$9.8 - 5v = Ae^{-5t}$ W1

$t = 0, v = 0 \Rightarrow 9.8 = A$ MW1

$v = 1.9, t = ? \Rightarrow 9.8 - 5 \times 1.9 = 9.8\,e^{-5t}$

$0.3 = 9.8\,e^{-5t}$

Time = 0.697 seconds (3sf) MW1

Alternative Solution

$\dfrac{dv}{dt} + 5v = 9.8$ MW1

IF $= e^{\int 5dt}$ M1

$= e^{5t}$ W1

$\Rightarrow e^{5t}\dfrac{dv}{dt} + 5ve^{5t} = 9.8e^{5t}$ M1

Integrate to give

$ve^{5t} = 1.96e^{5t} + c$

$v = 1.96 + ce^{-5t}$ W1

$t = 0, v = 0 \Rightarrow 0 = 1.96 + c$

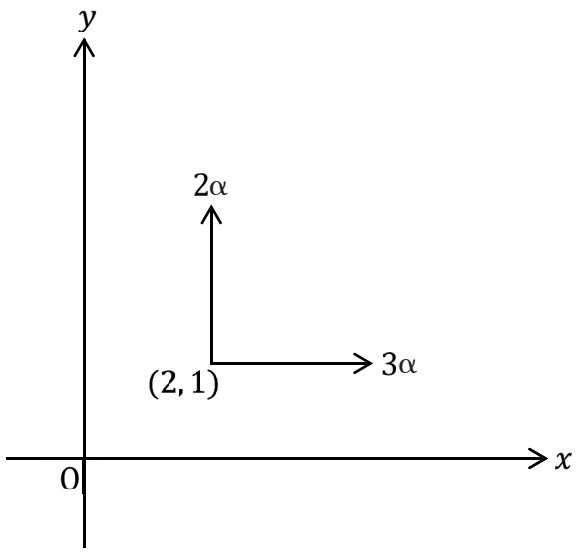
$c = -1.96$ MW1

$v = 1.9, t = ? \Rightarrow 1.9 = 1.96 - 1.96e^{-5t}$

$0.06 = 1.96e^{-5t}$

Time = 0.697 seconds (3sf) MW1 9

			AVAILABLE MARKS
2	**(i)** $\mathbf{a} = \frac{d\mathbf{v}}{dt}$		
	$= 18\cos 3t\,\mathbf{i} - 5\sin t\,\mathbf{j} + 18\sin 3t\,\mathbf{k}$	M1 W2	
	$t = \frac{\pi}{6} \Rightarrow \mathbf{a} = \left(-\frac{5}{2}\mathbf{j} + 18\mathbf{k}\right)\ \text{ms}^{-2}$	MW1	
	(ii) $\mathbf{s} = \int \mathbf{v}\,dt$		
	$= -2\cos 3t\,\mathbf{i} + 5\sin t\,\mathbf{j} - 2\sin 3t\,\mathbf{k} + \mathbf{c}$	M1 W2	
	$t = 0,\ \mathbf{s} = -2\mathbf{i}$		
	$\Rightarrow -2\mathbf{i} = -2\mathbf{i} + \mathbf{c}$		
	$\mathbf{c} = \mathbf{0}$	MW1	
	Distance $= \sqrt{4\cos^2 3t + 25\sin^2 t + 4\sin^2 3t}$	M1	
	$= \sqrt{4 + 25\sin^2 t}$ m	W1	
	Maximum distance $= \sqrt{29}$ m	MW1	
	This occurs when $\sin t = \pm 1$		
	First occurrence is at time $t = \frac{\pi}{2}$	MW1	12

			AVAILABLE MARKS
3	**(i)** $\mathbf{R} = (7 - 4 - P)\mathbf{i} + (1 - 2 + Q)\mathbf{j}$	M1	
	$= [(3 - P)\mathbf{i} + (Q - 1)\mathbf{j}]$ N	W1	
	(ii) M(O): $7 \times 3 - 1 \times 2 + 2 \times 1 + 4 \times 2 - P \times 2 + Q \times 3$	M1 W1	
	$= (29 - 2P + 3Q)$ Nm clockwise	W1	
	(iii) Equilibrium $\Rightarrow 3 - P = 0$		
	$Q - 1 = 0$	M1	
	$P = 3$	W1	
	$Q = 1$	W1	
	M(O) = 29 – 6 + 3 = 26 Nm	MW1	
	Since the total moment does not equal zero, then the system is not in equilibrium.	MW1	
	(iv) 		
	M(O): $3\alpha \times 1 - 2\alpha \times 2 = -\alpha$	MW1	
	$\Rightarrow 3 - P = 3\alpha$	M1	
	$Q - 1 = 2\alpha$	W1	
	$29 - 2P + 3Q = -\alpha$	W1	
	$\Rightarrow 29 + 6\alpha - 6 + 6\alpha + 3 = -\alpha$	M1W1	
	$26 + 13\alpha = 0$		
	$\alpha = -2$	W1	17

4 **(i)** $m_A = m$ $u_A = u$ $v_A = ?$

$m_B = m$ $u_B = 0$ $v_B = ?$

Conservation of momentum

$mu + 0 = mv_A + mv_B$ M1 W1

$u = v_A + v_B$

Newton's law of restitution

$-eu = v_A - v_B$ M1 W1

$\Rightarrow u + eu = 2v_B$ M1

$v_B = \frac{1}{2}\, u(1+e)\ \text{m s}^{-1}$ W1

(ii) $m_B = m$ $u_B = \frac{1}{2}\, u(1+e)$ $v_B = \frac{6}{25}u$

$m_C = m$ $u_C = 0$ $v_C = ?$

$\frac{1}{2}\, mu(1+e) + 0 = \frac{6}{25}\, mu + mv_C$ MW1

$\Rightarrow \frac{1}{2}\, u(1+e) = \frac{6}{25}u + v_C$

$-\frac{1}{2}eu(1+e) - 0 = \frac{6}{25}u - v_C$ MW1

$\Rightarrow \frac{1}{2}u\,(1+e) - \frac{1}{2}\, eu\,(1+e) = \frac{12}{25}u$ M1

$25 + 25e - 25e - 25e^2 = 24$

$25e^2 = 1$

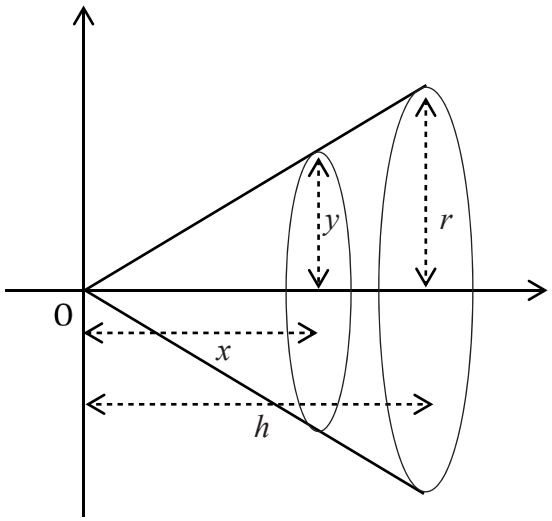
$e = \frac{1}{5}$ W1

AVAILABLE MARKS

		AVAILABLE MARKS
(iii) $v_A = \frac{1}{2}u(1-e)$		
$= \frac{2}{5}u$	M1 W1	
$v_C = \frac{1}{2}u(1+e) - \frac{6}{25}u$		
$= \frac{9}{25}u$	MW1	
Final KE $= \frac{1}{2}m\left[\left(\frac{2}{5}u\right)^2 + \left(\frac{6}{25}u\right)^2 + \left(\frac{9}{25}u\right)^2\right]$	M1 W1	
$= \frac{217}{1250}\,mu^2$	W1	
Initial KE $= \frac{1}{2}mu^2$	MW1	
Loss of KE $= \frac{204}{625}mu^2$ J	MW1	18

5 (i)

AVAILABLE MARKS



Let ρ = mass kg cm^{-3}
Mass of a single disc is $\pi y^2\, \delta x \rho$

$$\bar{x}\int_0^h \pi y^2 \rho \, dx = \int_0^h \pi y^2 x \rho \; dx$$ M1 W2

But $\frac{y}{x} = \frac{r}{h}$ M1

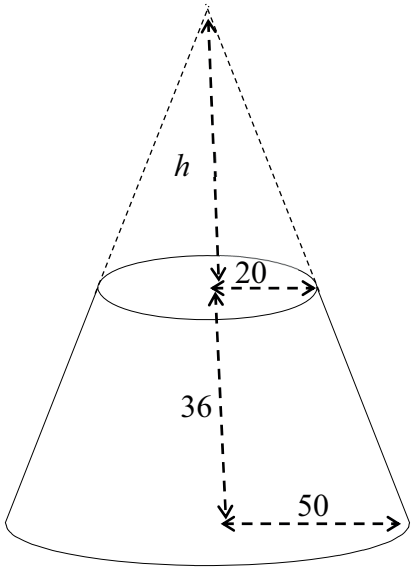
$$\Rightarrow \bar{x} \int_0^h \frac{r^2}{h^2} x^2 \; dx = \int_0^h \frac{r^2}{h^2} x^3 \, dx$$ W1

$$\frac{1}{3} h^3 \bar{x} = \frac{1}{4} h^4$$

$$\bar{x} = \frac{3}{4} h$$ W1

Therefore $d = \frac{1}{4} h$ W1

(ii)



$$\frac{20}{50} = \frac{h}{h+36} \qquad \text{M1}$$

$$20(h + 36) = 50h$$

$$h = 24 \qquad \text{W1}$$

Let ρ = mass kg cm^{-3}

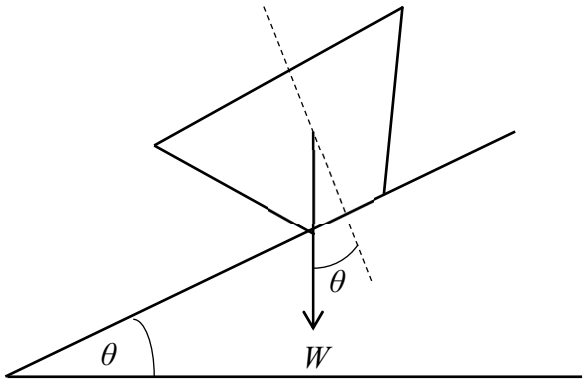
Solid	Large Cone	Small Cone	Frustrum	
Mass	$\frac{1}{3}\pi(50)^2(60)\rho$ $= 50\,000\pi\rho$	$\frac{1}{3}\pi(20)^2(24)\rho$ $= 3200\pi\rho$	$46\,800\pi\rho$	M1 W1
Distance of CoM from base	$\frac{1}{4}(60) = 15$	$\frac{1}{4}(24) + 36 = 42$	$\bar{y}$	M1W1

$$50\,000\pi\rho \times 15 = 3200\pi\rho \times 42 + 46\,800\pi\rho\bar{y} \qquad \text{M1}$$

$$\bar{y} = \frac{171}{13}\text{ cm} \qquad \text{W1}$$

$$= 13.2\text{ cm (3 sf)}$$

(iii)



When about to topple $\tan\theta = \dfrac{20}{36 - \left(\frac{171}{13}\right)}$ M2 W1

$$\theta = 41.2° \text{ (3sf)} \qquad \text{W1}$$

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Section B **75**

Section C Statistics

AVAILABLE MARKS

1 **(i)** Let $W = S_1 + 3S_2 - 4T_1 + T_2$

$E(W) = 36 + 3(36) - 4(42) + 42 = 18$ M1 W1

$Var(W) = 7 + (3^2 \times 7) + (4^2 \times 3) + 3 = 121$ M1 MW2

(ii) $W \sim N(18, 11^2)$

$P(S_1 + 3S_2 < 4T_1 - T_2) = P(W < 0)$ M1

$= 0.0509$ (3sf) M1 W1 — 8

2 **(i)** $\overline{X} \sim N\left(100, \frac{\sigma^2}{49}\right)$ M1 W1

$P(\overline{X} < 98) = 0.25$ M1

$P(Z < z) = 0.25$ where $z = \frac{98 - 100}{\left(\frac{\sigma}{7}\right)}$ M1 W1

$\Phi(z) = 0.25$

$\Phi(-z) = 0.75$

$-z = 0.674$

$z = -0.674$ M1 W1

$-0.674 = \frac{98 - 100}{\left(\frac{\sigma}{7}\right)}$

$\sigma = 20.8$ MW1

(ii) $\frac{\sigma}{\sqrt{n}} = \frac{20.8}{7} = \frac{104}{35} = 2.97$ (3sf) MW1

(iii) Central Limit Theorem MW1

Large sample size $n \geqslant 30$ MW1 — 11

3 **(i)** Let the difference between the results of the non-smoking and smoking twins, r_n and r_s, be modelled by the random variable D, where $D = R_n - R_s$.

H_0: $\mu_d = 0$ MW1

H_1: $\mu_d > 0$ MW1

Small sample :. t – test (upper one-tailed) M1

$v = 12 - 1 = 11$ $\therefore$ $T \sim t(11)$ MW1

where $T = \dfrac{\overline{D} - \mu_d}{\dfrac{S_d}{\sqrt{n}}}$

Critical value at 0.5% with $v = 11$ is $t_c = 3.106$ MW1

Twin pair	1	2	3	4	5	6	7	8	9	10	11	12
d	5	9	12	–6	–6	16	14	18	–1	13	21	7
d^2	25	81	144	36	36	256	196	324	1	169	441	49

M1 W1

$\sum d = 102$ $\sum d^2 = 1758$ MW1

$\overline{d} = \dfrac{\sum d}{n} = \dfrac{102}{12} = 8.5$ MW1

$S_d^2 = \dfrac{n}{n-1}\left(\dfrac{\sum d^2}{n} - \bar{d}^2\right) = \dfrac{12}{11}\left(\dfrac{1758}{12} - 8.5^2\right)$ M1

$S_d^2 = 81$ W1

If H_0 is true then $\mu_d = 0$, and the test value is given by

$t_{test} = \dfrac{\overline{d}}{\dfrac{S_d}{\sqrt{n}}} = \dfrac{8.5}{\left(\dfrac{\sqrt{81}}{\sqrt{12}}\right)}$ M1 MW1

$t_{test} = 3.272$ W1

$t_{test} > t_c$:. reject H_0 MW1

i.e. there is sufficient evidence, at the 0.5% level, to suggest that smoking regularly for at least five years reduces lung function. MW1

(ii) Weight, pre-existing medical conditions. MW1

(iii) The fact that the twins are identical provides a common link between the two sets of results, so that a paired t-test can be used. MW1

Genetics and environmental factors in early childhood can influence lung function. Using identical twins minimises the effects of these factors so that, if a difference does exist between the two sets of values, it is more likely to be due to the factor being investigated i.e. smoking. MW1

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			AVAILABLE MARKS
4	**(i)** $\bar{x} = \frac{\sum x}{n} = \frac{3696}{7} = 528\,\text{cm}$	MW1	
	$\overline{X} \sim \text{N}\left(\mu, \frac{\sigma^2}{n}\right)$	M1	
	Critical value for a 99% CI is 2.576	MW1	
	99% CI for population mean μ is given by		
	$99\%\ \text{CI} = 528 \pm 2.576 \times \frac{12}{\sqrt{7}}$	M1 W1	
	$= (516, 540)$ (3sf)	W1	
	(ii) Maximum width of 10		
	$\Rightarrow 2 \times 2.576 \times \frac{12}{\sqrt{n}} < 10$	M1 W1	
	$\sqrt{n} > 6.1824$		
	$n > 38.2$	W1	
	Minimum sample size is 39	MW1	
	(iii) $\bar{y} = \frac{\sum y}{n} = \frac{21\,200}{40} = 530\,\text{cm}$	MW1	
	$S^2 = \frac{40}{39}\left(\frac{11\,242\,222}{40} - 530^2\right)$	M1	
	$S^2 = \frac{2074}{13} = 160\,\text{cm}^2$ (3sf)	W1	
	(iv) Large sample $\therefore \overline{X} \sim \text{N}\left(\mu, \frac{\sigma^2}{n}\right)$	M1	
	95% CI for population mean μ is given by		
	$95\%\ \text{CI} = 530 \pm 1.96 \times \frac{\sqrt{\frac{2074}{13}}}{\sqrt{40}}$	MW2	
	$= (526, 534)$ (3sf)	W1	
	(v) Mean and variance are estimates from a sample **or** The CI is an interval for the population mean as opposed to individual results	MW1	18

5 **(i)** From observed data

AVAILABLE MARKS

$\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{130}{52} = 2.5$ M1 W1

Let X be the number of speeding fines issued in a single week
$X \sim \text{Po}(2.5)$ M1

$\text{P}(X = 3) = \text{e}^{-2.5}\dfrac{2.5^3}{3!} = 0.2138$ M1 W1

$a = 0.2138 \times 52 = 11.1$ weeks MW1
$b = 52 - (4.3 + 10.7 + 13.3 + 11.1 + 6.9)$
$b = 52 - 46.3 = 5.7$ MW1

(ii) Let X be the number of speeding fines issued in a single week
$\text{H}_0 : X \sim \text{Po}(2.5)$ MW1
$\text{H}_1 : X$ is not modelled by a Po(2.5) distribution MW1

x	O	E
0	5	4.3
1	4	10.7
2	16	13.3
3	16	11.1
4	9	6.9
5 or more	2	5.7

Group $x = 0$ and $x = 1$ classes so that combined expected frequency is at least 5 M1

Test statistic: $\chi^2 = \sum \dfrac{(O-E)^2}{E}$

x	O	E	$\dfrac{(O-E)^2}{E}$
0 or 1	9	15.0	2.4
2	16	13.3	0.5481
3	16	11.1	2.1631
4	9	6.9	0.6391
5 or more	2	5.7	2.4018

W1 M1 W1

$\chi^2 = 8.152$ W1

$v = 5 - 2 = 3$ MW1

Critical value: $\chi^2_{5\%}(3) = 7.815$ MW1

$\chi^2 > \chi^2_{5\%}(3)$ $\therefore$ reject H_0 MW1
i.e. there is sufficient evidence, at the 5% level, to reject the hypothesis that the results can be modelled by a Po(2.5) distribution. MW1

(iii) Very small value $\therefore$ query reliability of the observed data or the calculation of the test statistic. MW1 19

Section C **75**

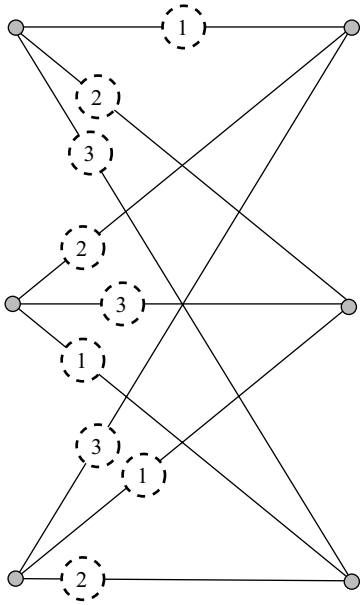
Section D Discrete and Decision Mathematics

AVAILABLE MARKS

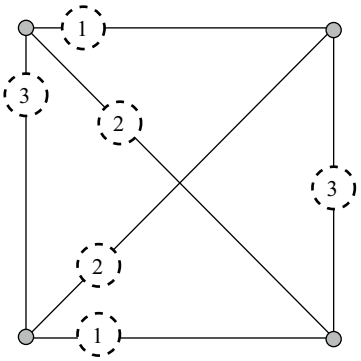
1 (a) (i) $K_{3,3}$ number of colours in optimum vertex colouring = 2 MW1

K_4 number of colours in optimum vertex colouring = 4 MW1

(ii)



$K_{3,3}$



K_4

M1 W2

(b) (i)

Pupil	Aoife	Clodagh	Eunice	Gwen
Possible Partners	Janice Katherine	Katherine Laura	Janice Katherine	Janice Laura

These 4 pupils have only 3 possible preferred partners between them. M1 W1

Thus, by Hall's Marriage Theorem, no complete matching is possible. M1

(ii) Possible answers are:

Pupil	Janice	Katherine	Laura	Meaghan	Niamh	Olive	Patricia	Quetzal
Matched Partner	Aoife	Eunice	Felicity	Beatrice	Clodagh	Gwen	Debbie	Harriet
Alternative	Clodagh	Eunice	Debbie	Aoife	Harriet	Gwen	Beatrice	Felicity

M1 W1

(c) (i) a, b, e, d, c M1 W1

(ii) Hamiltonian path with minimum weight = b, a, c, e, d W1

(iii) The Nearest Neighbour Algorithm is a Greedy Algorithm. M1

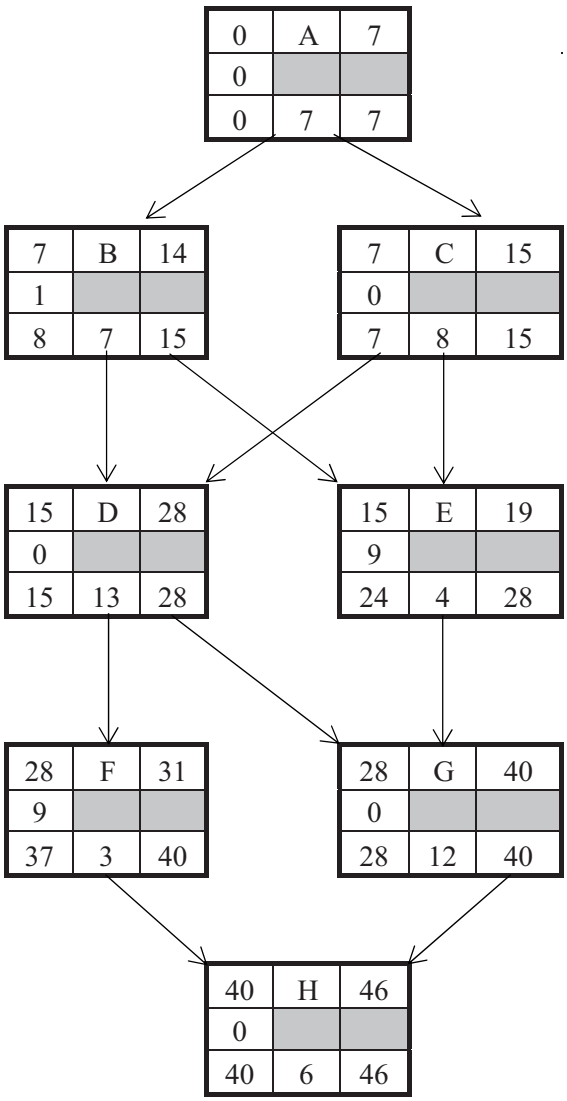
(Thus, it is restricted at any step to only choose an optimal adjacent vertex which may not give a minimal path.)

14

2 **(i)** 4, 3, 12, 6 MW2

AVAILABLE MARKS

(ii)



ES and EF times MW1
LS times MW1
LF times MW1
Slack times MW1

(iii) Critical path = A C D G H MW1

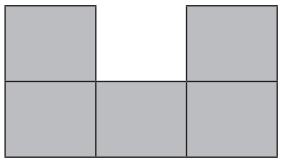
(iv) Expected time = 7 + 8 + 13 + 12 + 6 = 46 MW1
Project variance = (4 + 9 + 25 + 36 + 4)/36 = $2.16\dot{}$ M2 W1
$T \sim N(46, 2.16\dot{})$ M1
$P(T > 48) = 1 - P(T < 48)$
$= 1 - 0.9129$ W1
$= 0.0871$ W1

Alternative solution

$z = (48 - 46)/\sqrt{2.16\dot{}} = 1.3587$ W1
$P(Z > 1.3587) = 0.0871$ W1 14

3 **(i)** 0 rooks 1 way
1 rook 5 ways
2 rooks 4 ways M1
So the rook polynomial is $1 + 5x + 4x^2$ W1

(ii) Rook polynomial for



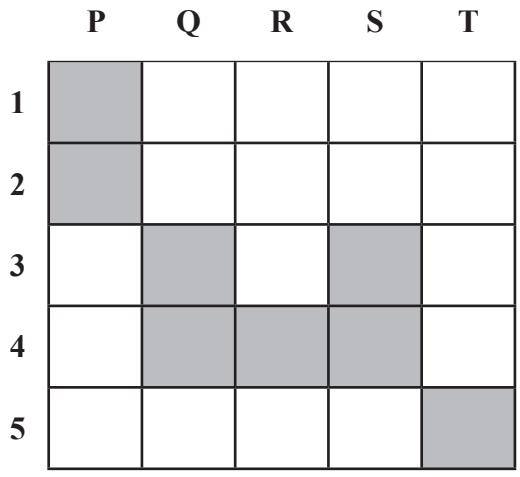
is $1 + 5x + 4x^2$ M1
Rook polynomial for  is $1 + 2x$ MW1

Rook polynomial for  is $1 + x$ MW1

Three shapes are non-interfering. M1

Combined rook polynomial is $(1 + x)(1 + 2x)(1 + 5x + 4x^2)$ M1
$= (1 + 3x + 2x^2)(1 + 5x + 4x^2)$
$=1 + 5x + 4x^2 + 3x + 15x^2 + 12x^3 + 2x^2 + 10x^3 + 8x^4$ W1
$= 1 + 8x + 21x^2 + 22x^3 + 8x^4$ W1

(iii)



M1 W1

Rook polynomial for excluded positions = $1 + 8x + 21x^2 + 22x^3 + 8x^4$ MW1
IE principle M1
Number of different ways
$= 5! - 8 \times 4! + 21 \times 3! - 22 \times 2! + 8 \times 1! - 0 \times 0!$ M1 W1
= 18 ways W1 16

AVAILABLE MARKS

			AVAILABLE MARKS
4	**(a)**	$P_G = \frac{1}{24}(x_1^{12} + 3x_2^6 + 6x_1^2x_2^5 + 6x_4^3 + 8x_3^4)$	
		Replace x_i by 4 M1	
		Number of possible pendants is	
		$\frac{1}{24}(4^{12} + 3 \times 4^6 + 6 \times 4^2 \times 4^5 + 6 \times 4^3 + 8 \times 4^4)$ MW1	
		$= \frac{1}{24}(16\,777\,216 + 12\,288 + 98\,304 + 384 + 2048)$	
		$= 703\,760$ pendants W1	
	(b) **(i)**	$P_G = \frac{1}{12}(x_1^6 + 8x_3^2 + 3x_1^2x_2^2)$	
		Replace x_i by $B^i + G^i$ M1	
		$P_G = \frac{1}{12}[(B+G)^6 + 8(B^3+G^3)^2 + 3(B+G)^2(B^2+G^2)^2]$ M1 W1	
	(ii)	Terms in B^3G^3: M1	
		$(B+G)^6$ gives $\binom{6}{3}B^3G^3 = 20B^3G^3$	
		$8(B^3+G^3)^2$ gives $8\binom{2}{1}B^3G^3 = 16B^3G^3$ MW1	
		$3(B+G)^2(B^2+G^2)^2 = 3(B^2+2BG+G^2)(B^4+2B^2G^2+G^4)$ gives $12B^3G^3$ M1 W1	
		Number of colourings is $\frac{1}{12}(20 + 16 + 12) = 4$ colourings MW1	11
5	**(i)**	$a_1, 2a_2, 3a_3, 4a_4, \ldots$ MW1	
	(ii)	By differentiating $\frac{1}{1-t} = 1 + t + t^2 + t^3 + t^4 + \ldots$ M1	
		Obtain $\frac{1}{(1-t)^2} = 1 + 2t + 3t^2 + 4t^3 +$ W1	
		Hence the generating function is $\frac{1}{(1-t)^2}$ W1	
	(iii)	Even positive integers 2, 4, 6, 8 have the generating function	
		$2 + 4t + 6t^2 + 8t^3 + \ldots$ M1	
		Comparing with part (ii), this is given by $2 + 4t + 6t^2 + 8t^3 + \ldots = \frac{2}{(1-t)^2}$ W1	
	(iv)	$(1 + 3t + 5t^2 + 7t^3 + \ldots) = (2 + 4t + 6t^2 + 8t^3 + \ldots) - (1 + t + t^2 + t^3 + \ldots)$ MW1	
		Replacing with closed form of generating functions: M1	
		$(1 + 3t + 5t^2 + 7t^3 + \ldots) = \frac{2}{(1-t)^2} - \frac{1}{1-t}$	
		$= \frac{1+t}{(1-t)^2}$ W1	9

6 **(i)** Introduce slack variables: M1

$4x + y + u = 16$
$2x + 5y + v = 26$
$-3x - 2y + P = 0$ M1 W1

(ii)

x	y	u	v	P	
4	1	1	0	0	16
2	5	0	1	0	26
–3	–2	0	0	1	0

M1 W2

(iii) First iteration:

Identify pivot column – column (1) (as –3)

Identify pivot row – row (1) (as ratio = 4) MW1

Divide pivot row by 4 MW1

Reduce other entries in pivot column to zero by subtracting k × pivot row

Obtaining:

x	y	u	v	P	
1	0.25	0.25	0	0	4
0	4.5	–0.5	1	0	18
0	–1.25	0.75	0	1	12

W1

(iv)

x	y	u	v	P	
1	0	$0.2\dot{7}$	$-0.0\dot{5}$	0	3
0	1	$-0.\dot{1}$	$0.\dot{2}$	0	4
0	0	$0.6\dot{1}$	$0.2\dot{7}$	1	17

Optimal solution: $x = 3$ $y = 4$ $P = 17$ M1 W1 11

Section D **75**

Total **150**

AVAILABLE MARKS