Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2022

Further Mathematics

Assessment Unit AS 2
assessing
Applied Mathematics

[SFM21]

THURSDAY 26 MAY, AFTERNOON

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) FURTHER MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

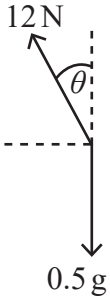
COVID-19 Context

Given the unprecedented circumstances presented by the COVID-19 public health crisis, senior examiners, under the instruction of CCEA awarding organisation, are required to train assistant examiners to apply the mark scheme in case of disrupted learning and lost teaching time. The interpretation and intended application of the mark scheme for this examination series will be communicated through the standardising meeting by the Chief or Principal Examiner and will be monitored through the supervision period. This paragraph will apply to examination series in 2021–2022 only.

SECTION A Mechanics 1

AVAILABLE MARKS

1 **(i)**



MW2

(ii) $12\cos\theta = 0.5\,g$ M1 W1

$$\cos\theta = \frac{0.5g}{12}$$

$\theta = 65.9°$ (3 s.f.) W1

$12\sin\theta = 0.5 \times 3^2 \times L\sin\theta$ M1 W1 W1

$$L = \frac{12}{0.5 \times 9}$$

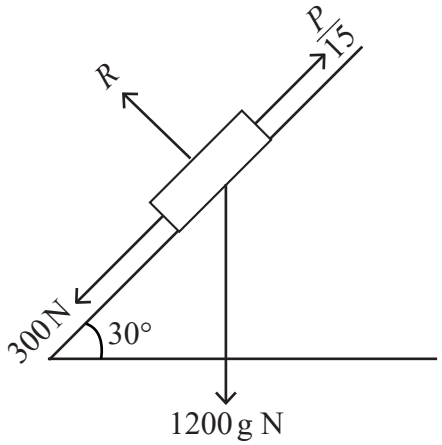
$L = \frac{8}{3}$ W1 9

2 **(i)** Driving force $= \frac{19\,500}{v}$ MW1

$\frac{19\,500}{v} - 300 = 1200\,(0.4)$ M1 W1

$v = 25\,\text{ms}^{-1}$ W1

(ii)

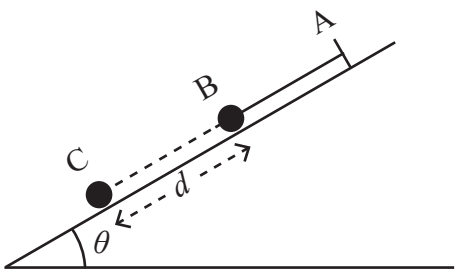


$\frac{P}{15} = 1200\,g\sin 30 + 300$ M1 MW1 W1

$P = 92.7\,\text{kW}$ W1 8

			AVAILABLE MARKS
3	$\int_0^3 (px+1)\mathrm{d}x = 3p^2$	M1 W1	
	$\left[\frac{px^2}{2}+x\right]_0^3 = 3p^2$	MW1	
	$\frac{9p}{2}+3 = 3p^2$		
	$6p^2-9p-6 = 0$		
	$2p^2-3p-2 = 0$		
	$(2p+1)(p-2)=0$		
	$p=-\frac{1}{2}\quad p=2$		
	$p=2$ (positive)	MW1	
	WD = change in KE	M1	
	$3p^2 = \frac{1}{2}mv^2$	W1	
	$12 = \frac{1}{2}\times 6\times v^2$		
	$v = 2\,\mathrm{m\,s^{-1}}$	W1	7
4	**(i)** At highest point:	M1	
	KE = 0		
	GPE = $80\,g\,d$		
	EPE = 0		
	$\Rightarrow$ Total ME = $80\,g\,d$ J	W1	
	After drop:	M1	
	KE = 0		
	GPE = $80\,g\,(-28)$		
	EPE = $\frac{18\,500\times 8^2}{2\times 20}$	M1 W1	
	$\Rightarrow$ Total ME = $29\,600 - 21\,952$		
	$= 7648$ J	W1	
	PCME $\Rightarrow 784\,d = 7648$	M1	
	$d = 9.76$ m (3 s.f.)	W1	
	(ii) At 25m below 0:		
	KE = $\frac{1}{2}(80)v^2$	MW1	
	GPE = $80\,g(-25)$	MW1	
	EPE = $\frac{18\,500\times 5^2}{2\times 20}$	MW1	
	$\Rightarrow$ Total ME = $40v^2 - 8037.5$	MW1	
	$40v^2 - 8037.5 = 7648$	M1	
	$v = 19.8\,\mathrm{m\,s^{-1}}$ (3 s.f.)	W1	
	(iii) Air resistance has been neglected	MW1	15

5 (i)



Let d = distance the particle moves down the slope
Let GPE at C = 0

$KE_B + PE_B + EPE_B = KE_C + PE_C + EPE_C$ M1

$0 + 10g\,d\sin\theta + 0 = 0 + 0 + \frac{147d^2}{10}$ W2

$10g\,d\left(\frac{3}{5}\right) = \frac{147d^2}{10}$ M1

$588\,d = 147\,d^2$

$588 = 147\,d \; [d \neq 0]$

$d = 4$ metres W1

(ii) Frictional force $= \mu R$ M1

$= \mu \times 10\,g\cos\theta$ MW1

$= \mu \times 10\,g \times \left(\frac{4}{5}\right)$

$= 8\mu g$ N W1

$KE_B + PE_B + EPE_B + W_{IN} = KE_C + PE_C + EPE_C + W_{OUT}$ M1 W1

$0 + 10\,g\,d\sin\theta + 0 = 0 + 0 + \frac{147d^2}{10} + 8\mu g\,d$

$10\,g\,d\sin\theta = \frac{147d^2}{10} + 8\mu g\,d$

$10\,g\left(2 \times \frac{3}{5}\right) = \frac{147(2)^2}{10} + 16\mu g$

$117.6 = 58.8 + 156.8\mu$

$\mu = \frac{3}{8}$ W1 11

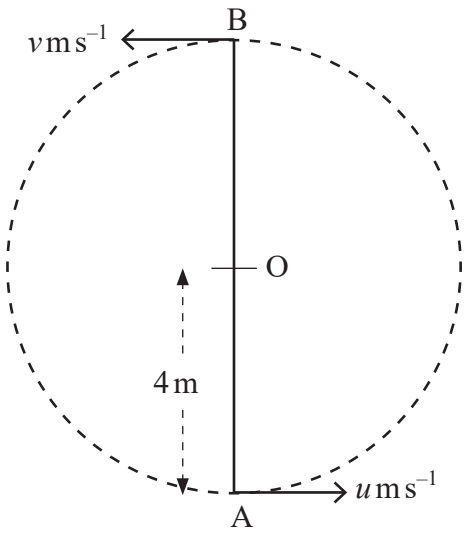
Section A **50**

AVAILABLE MARKS

Section B: Mechanics 2

				AVAILABLE MARKS
1	**(i)**	$[f] = T^{-1}$ $\quad [l] = L$		
		$[S] = MLT^{-2}$ $\quad [\rho] = ML^{-1}$	MW1	
			MW1	
		$T^{-1} = (MLT^{-2})^x (ML^{-1})^y L^z$	M1	
		Compare dimensions:		
		M: $0 = x + y$	M1	
		L: $0 = x - y + z$		
		T: $-1 = -2x$	W1	
		$\Rightarrow x = \frac{1}{2}, y = -\frac{1}{2}, z = -1$	W1	
	(ii)	$f_0 = \frac{k}{l}\sqrt{\frac{S}{\rho}}$		
		$f_1 = \frac{k}{0.85l}\sqrt{\frac{S}{\rho}}$	M1	
		$\Rightarrow f_1 = 1.176 f_0$		
		$\Rightarrow$ 17.6% increase in frequency	W1	8
2	**(i)**	$\frac{GMm}{r^2} = mr\omega^2$	M1 W1	
		$GMm = mr^3\omega^2$	M1 MW1 W1	
		$\omega^2 = \frac{GM}{r^3}$		
		$\omega = \sqrt{\frac{GM}{r^3}}$	MW1	
	(ii)	$\omega = \sqrt{\frac{6.674 \times 10^{-11} \times 5.683 \times 10^{26}}{(1.336 \times 10^8)^3}}$	M1	
		$= 1.261165965 \times 10^{-4}$	W1	
		$T = \frac{2\pi}{\omega}$		
		= 49 820.44776 seconds	M1	
		Period = 13.8 hours	W1	10

3



AVAILABLE MARKS

Zero level for GPE is at A. M1

When the bob is at A:

KE = $\frac{1}{2} \times 4 \times u^2 = 2u^2$ MW1

GPE = 0 MW1

When the bob is at B:

KE = $\frac{1}{2} \times 4 \times v^2 = 2v^2$ MW1

GPE = $4\,g \times 8 = 32\,g$ MW1

Using conservation of energy:

$2u^2 = 2v^2 + 32\,g$ MW1

For complete circles, $v > 0$ M1

$\Rightarrow 2u^2 > 32\,g$

$\Rightarrow u > 12.5$ (3 s.f.) W1 8

4 Let **i** be a unit vector due E and **j** be a unit vector due N

AVAILABLE MARKS

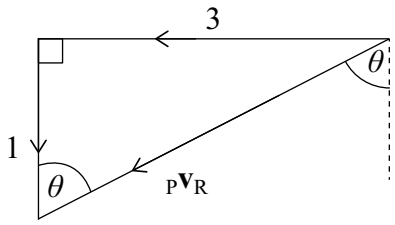
$\mathbf{v}_P = -4\mathbf{j}$ MW1

$\mathbf{v}_R = 3\sqrt{2}\cos 45°\mathbf{i} - 3\sqrt{2}\sin 45°\mathbf{j}$

$= 3\mathbf{i} - 3\mathbf{j}$ MW1

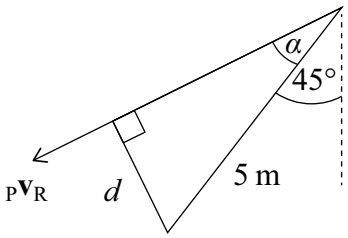
${}_P\mathbf{v}_R = -4\mathbf{j} - (3\mathbf{i} - 3\mathbf{j})$

$= -3\mathbf{i} - \mathbf{j}$ MW1



$\theta = \tan^{-1}\left(\frac{3}{1}\right)$ M1

$= 71.565°\ldots$ W1



$\alpha = 71.565°\ldots - 45°$ M1

$= 26.565°\ldots$ MW1

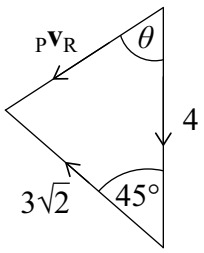
Shortest distance

$d = 5\sin\alpha$ M1

$= 2.24$ m (3 s.f.) W1

Alternative Solution

${}_P\mathbf{v}_R = \mathbf{v}_P - \mathbf{v}_R$



M1 W1

${}_P\mathbf{v}_R^2 = 4^2 + (3\sqrt{2})^2 - 2(4)(3\sqrt{2})\cos 45°$ M1

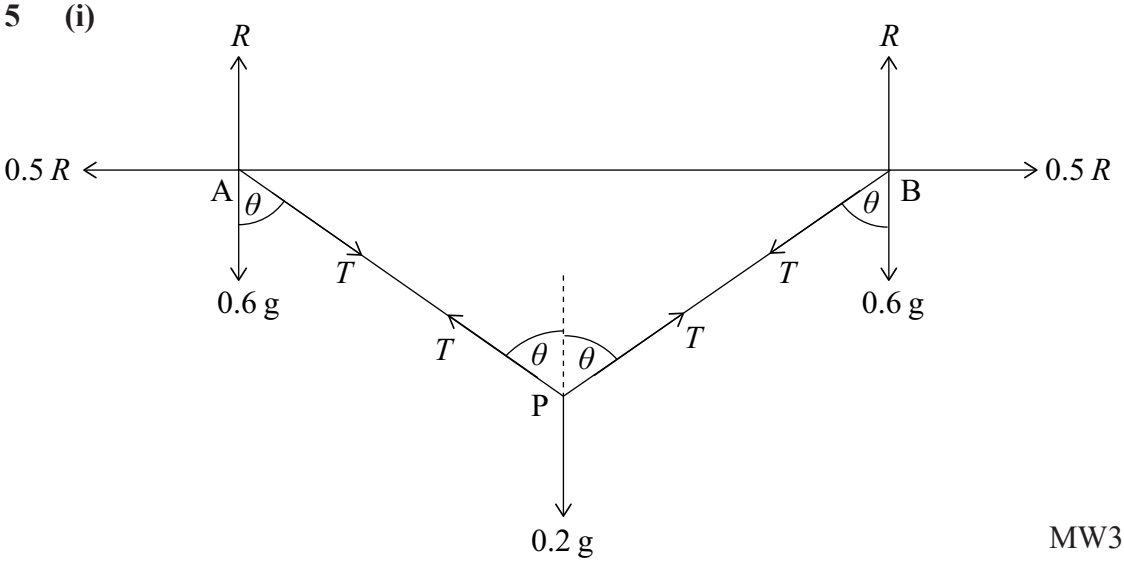
${}_P\mathbf{v}_R = \sqrt{10}$ W1

$\frac{\sqrt{10}}{\sin 45°} = \frac{3\sqrt{2}}{\sin\theta}$

$\theta = 71.565°\ldots$ MW1 9

Method continues as in original mark scheme for final 4 marks.

5 (i)



MW3

(ii) At A: (↑) $R = 0.6g + T\cos\theta$ M1 W1

At P: (↑) $2T\cos\theta = 0.2g$ MW1

$T\cos\theta = 0.1g$

$\Rightarrow R = 0.6\,g + 0.1\,g$

$R = 0.7\,g$ MW1

(iii) Rings just about to slip

$Fr = 0.5R$ M1

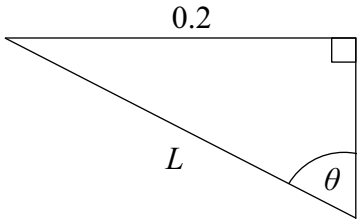
At A: (→) $T\sin\theta = 0.5R$ M1

$T\sin\theta = 0.35\,g$ W1

But also $T\cos\theta = 0.1g$ from (ii) M1

Divide to give $\tan\theta = 3.5$ M1

$\theta = 74.0546°\ldots$ W1



$\sin\theta = \dfrac{0.2}{L}$ M1

$L = \dfrac{0.2}{\sin\theta}$

$L = 0.208$ (3 s.f.) W1 15

Section B **50**

AVAILABLE MARKS

Section C Statistics

AVAILABLE MARKS

1 **(i)** Quota sampling MW1

(ii) Advantage: Interviewer chooses who to interview in order to fill the quota. This represents a cheaper and faster way of obtaining a representative sample than other techniques (e.g. stratified).
Avoids the cost and time associated with constructing a sampling frame. MW2

Disadvantage: Introduces interviewer bias into the selection process.
Non-responses are not recorded and are simply replaced. MW2 5

2 **(i)** $S_{xx} = \sum x^2 - \frac{(\Sigma x)^2}{n} = 480 - \frac{60^2}{10} = 120$ M1 W1

$S_{yy} = \sum y^2 - \frac{(\Sigma y)^2}{n} = 62.4 - \frac{24^2}{10} = 4.8$ MW1

$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

$0.9 = \frac{S_{xy}}{\sqrt{120 \times 4.8}}$ M1

$S_{xy} = 21.6$ W1

$\bar{x} = \frac{\Sigma x}{n} = \frac{60}{10} = 6$

$\bar{y} = \frac{\Sigma y}{n} = \frac{24}{100} = 2.4$ MW1

$b = \frac{S_{xy}}{S_{xx}} = \frac{21.6}{120} = 0.18$ MW1

$a = \bar{y} - b\bar{x} = 2.4 - 0.18 \times 6 = 1.32$ MW1

$y = 1.32 + 0.18x$ W1

(ii) **(a)** Estimate lies outside the range of values for x (extrapolation), which can be unreliable. MW1

(b) The explanatory variable in this model is x.
This regression line should not be used to estimate x for a given y.
The regression line of x on y would be required for this purpose. MW1 11

3 **(i)**

	1	2	3	4
1	0	1	2	3
2	1	0	1	2
3	2	1	0	1
4	3	2	1	0

M1W1

P(difference = 0) = $\frac{4}{16}$ P(difference = 1) = $\frac{6}{16}$

P(difference = 2) = $\frac{4}{16}$ P(difference = 3) = $\frac{2}{16}$ MW1

P($X = 0$) = $\frac{8}{16} = \frac{1}{2}$ P($X = 1$) = $\frac{6}{16} = \frac{3}{8}$ P($X = 5$) = $\frac{2}{16} = \frac{1}{8}$

x	0	1	5
P($X = x$)	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{8}$

M1 W1

(ii) $E(X) = \sum xp(x)$

$= \left(0 \times \frac{1}{2}\right) + \left(1 \times \frac{3}{8}\right) + \left(5 \times \frac{1}{8}\right)$ M1

$= 1$ W1

Charge for single attempt = 1 + 0.50 = £1.50 MW1

(iii) $E(X^2) = \sum x^2 p(x)$

$= \left(0^2 \times \frac{1}{2}\right) + \left(1^2 \times \frac{3}{8}\right) + \left(5^2 \times \frac{1}{8}\right)$ M1

$= 3.5$ W1

$\text{Var}(X) = E(X^2) - E^2(X)$
$= 3.5 - 1^2$
$= 2.5$ MW1

(iv) $E(Y) = E(3X - 4) = 3E(X) - 4$
$= 3 \times 1 - 4$
$= -1$ MW1

$\text{Var}(Y) = \text{Var}(3X - 4) = 3^2 \times \text{Var}(X)$ M1
$= 9 \times 2.5$
$= 22.5$ W1 14

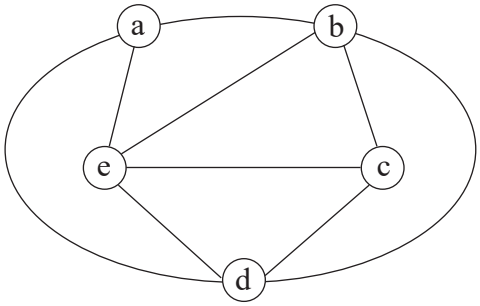
AVAILABLE MARKS

				AVAILABLE MARKS
4	**(i)**	X is the random variable 'number of throws of coin before it first lands on tails'		
		$X \sim \text{Geo}(p)$		
		$\text{P}(X = 2) = 0.24$		
		$qp = 0.24$	M1	
		$(1 - p)p = 0.24$	MW1	
		$p^2 - p + 0.24 = 0$	MW1	
		$p = 0.6$ $\quad p = 0.4$	W1	
	(ii)	$X \sim \text{Geo}(0.4)$ $\quad p = 0.4$ $\quad q = 0.6$	MW1	
		$\text{P}(X = 8) = q^7 p$		
		$= 0.6^7 \times 0.4$	MW1	
		$= 0.0112$ (3 s.f.)	W1	
	(iii)	$\text{P}(X \geqslant 5) = \text{P}$(failure on first four attempts)		
		$= q^4$	MW1	
		$= 0.6^4$		
		$= 0.130$ (3 s.f.)	W1	9
5	**(i)**	${}^7\text{C}_4 = 35$	MW1	
	(ii)	3 consonants × 1 vowel	M1	
		${}^5\text{C}_3 \times {}^2\text{C}_1 = 10 \times 2 = 20$	MW1	
		$\frac{20}{35} = \frac{4}{7} = 0.571$ (3 s.f.)	MW1	
	(iii)	$7! = 5040$	MW1	
	(iv)	$2 \times 6! = 1440$	M1 W1	
		$\frac{1440}{5040} = \frac{2}{7} = 0.286$ (3 s.f.)	MW1	
	(v)	$\frac{2 \times (n-1)!}{n!}$	M1 W1	
		$= \frac{2 \times (n-1)(n-2)(n-3)...3.2.1}{n(n-1)(n-2)(n-3)...3.2.1}$		
		$= \frac{2}{n}$	MW1	11
			Section C	**50**

Section D Discrete and Decision Maths

AVAILABLE MARKS

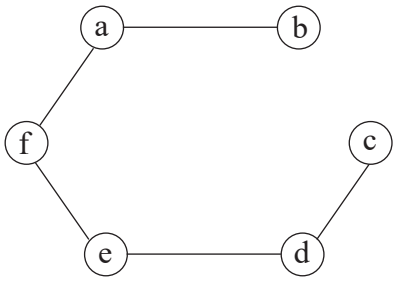
1 (a)



M1 W1

(b) (i) $12 - 5 = 7$ MW1

(ii)



M1 W1

(iii) No of edges in $K_6 = \frac{1}{2} \times 6 \times 5 = 15$ M1

$15 - 12 = 3$ W1

(c) d a e b d c e M1W1 9

or any alternative with edges adjacent

2 (i) Substitute $u_n = kx^n$ M1

obtaining $kx^{n+2} = \frac{5}{3}kx^{n+1} + \frac{2}{3}kx^n$

$kx^n(3x^2 - 5x - 2) = 0$

$x = 0$ or $3x^2 - 5x - 2 = 0$ W1

$(3x + 1)(x - 2) = 0$

$x = -\frac{1}{3}$ or 2 MW1

$u_n = A2^n + B\left(-\frac{1}{3}\right)^n$ M1

$n = 0 \quad 2 = A + B$

$n = 1 \quad \frac{5}{3} = 2A - \frac{1}{3}B$ thus $5 = 6A - B$ M1 W1

Solving $A = 1$ and $B = 1$

So $u_n = 2^n + \left(-\frac{1}{3}\right)^n$ W1

(ii) as n increases $u_n \to 2^n$ M1

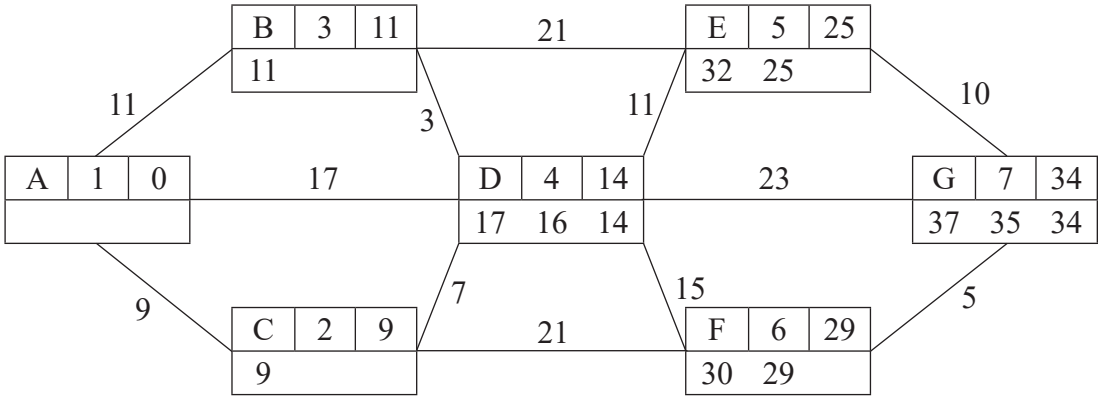
as $\left(-\frac{1}{3}\right)^n \to 0$ M1

2^t approaches 4096

when $t = 12$ W1 10

Check: $2^{12} + \left(-\frac{1}{3}\right)^{12} = 4096.000002$

3 (i)

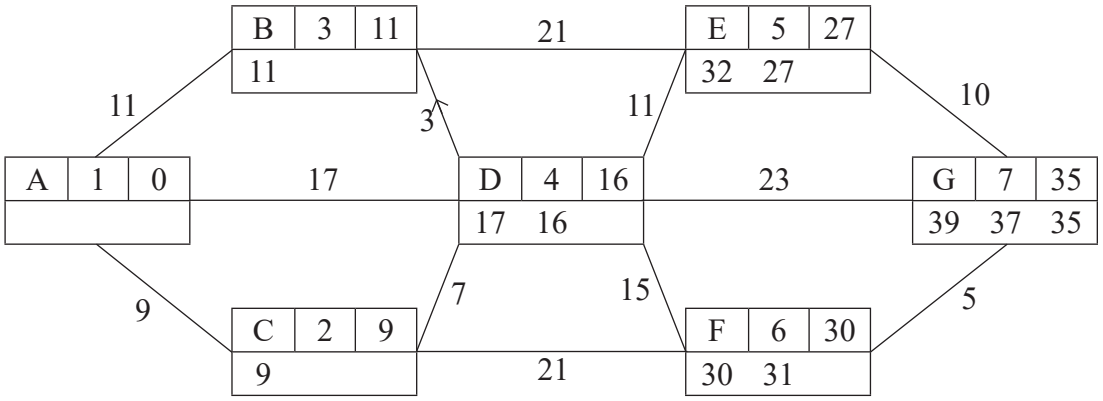


Boxes ABC M1
D, E, F, G W1 each (= W4)

(ii) A B D F G MW1
Time 34 minutes MW1

(iii) Time A → E 25 minutes MW1

(iv)



D MW1
E, F MW1
G MW1 11
Time 35 minutes

AVAILABLE MARKS

4 **(i)**

p	q	$p \Rightarrow q$	$\sim p$	$\sim q$	$\sim q \Rightarrow \sim p$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T
			a	b	c

a, b M1
c M1 W1

(ii)

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	LHS	$p \Rightarrow r$	expression
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T
a	a	a	b	c	e	d	f

a M1
b, c, d MW3
e MW1
f MW1
conclusion M1

10

AVAILABLE MARKS

5 **(i)** Identity permutation = u MW1

(ii)

$\circ$	p	q	r	s	t	u
p	q	u	t	r	s	p
q	u	p	s	t	r	q
r	s	t	u	p	q	r
s	t	r	q	u	p	s
t	r	s	p	q	u	t
u	p	q	r	s	t	u

MW4

(iii)

Element	p	q	r	s	t	u
Period	3	3	2	2	2	1

MW2

(iv) $\{p, q, r, s, t, u\}$ $\{u\}$ MW1
$\{p, q, u\}$ MW1
$\{r, u\}$ $\{s, u\}$ $\{t, u\}$ MW1

10

Section D **50**