Rewarding Learning

ADVANCED
General Certificate of Education
2022

Further Mathematics

Assessment Unit A2 2

assessing

Applied Mathematics

[AFM21]

TUESDAY 7 JUNE, AFTERNOON

MARK SCHEME

General Marking Instructions

GCE Advanced/Advanced Subsidiary Further Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters M, W and MW as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of following through their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from a candidate's inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

COVID-19 Context

Given the unprecedented circumstances presented by the COVID-19 public health crisis, senior examiners, under the instruction of CCEA awarding organisation, are required to train assistant examiners to apply the mark scheme in case of disrupted learning and lost teaching time. The interpretation and intended application of the mark scheme for this examination series will be communicated through the standardising meeting by the Chief or Principal Examiner and will be monitored through the supervision period. This paragraph will apply to examination series in 2021–2022 only.

AVAILABLE MARKS

1 **(i)** 3 m above PQ MW1

(ii)

Shape	PQ	RS	TU	PT	Total
Mass (m)	3ρ	2ρ	3ρ	6ρ	14ρ
Dist of CM from PT (d)	1.5	1	1.5	0	$\bar{y}$
md	4.5ρ	2ρ	4.5ρ	0	$14\rho\bar{y}$

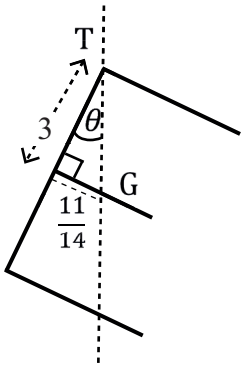
M1 W1
M1 W1
M1 W1

$\Rightarrow 14\rho\bar{y} = 11\rho$ M1

$\bar{y} = \frac{11}{14}$ W1

Hence centre of mass is $\frac{11}{14}$ m from PT.

(iii)

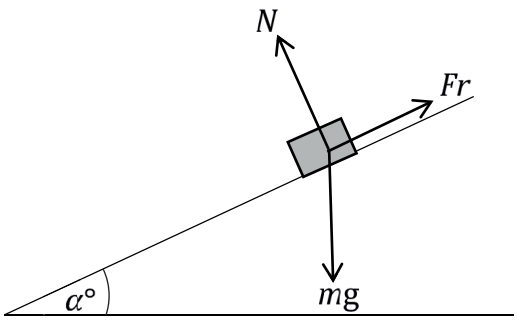


$\theta = \tan^{-1}\left(\frac{\frac{11}{14}}{3}\right)$ M1 W1

$= 14.7°$ (3 s.f.) W1 12

					AVAILABLE MARKS
2	**(a)**	**(i)**	$q^2 - 4pr = 0$	M1 W1	
		(ii)	$q^2 - 4pr > 0$	MW1	
	(b)	**(i)**	Auxiliary equation		
			$m^2 + 6m + 10 = 0$	M1 W1	
			$m = \dfrac{-6 \pm \sqrt{36 - 40}}{2}$	M1	
			$= -3 \pm \mathrm{i}$	W1	
			$\Rightarrow\ x = \mathrm{e}^{-3t}\,(A \cos t + B \sin t)$	MW2	
		(ii)	Using the model, the pointer will oscillate indefinitely about O	MW1	10

3 (i)



AVAILABLE MARKS

$Fr = \mu N$ MW1

$(\leftarrow)\quad N\sin\alpha - \mu N\cos\alpha = \dfrac{mu^2}{r}$ M2 W1

$(\uparrow)\quad N\cos\alpha + \mu N\sin\alpha = mg$ M1 W1

$\Rightarrow \dfrac{N\sin\alpha - \mu N\cos\alpha}{N\cos\alpha + \mu N\sin\alpha} = \dfrac{mu^2}{mrg}$ M1

$\Rightarrow \dfrac{\tan\alpha - \mu}{1 + \mu\tan\alpha} = \dfrac{u^2}{rg}$ M1

$\Rightarrow u^2 = \dfrac{rg(\tan\alpha - \mu)}{1 + \mu\tan\alpha}$ W1

(ii) If car is just about to slide up the slope, then μN acts down the slope. M1

$\Rightarrow v^2 = \dfrac{rg(\tan\alpha - (-\mu))}{1 + (-\mu)\tan\alpha}$ W1

$\Rightarrow v^2 = \dfrac{rg(\tan\alpha + \mu)}{1 - \mu\tan\alpha}$ W1

(iii) $u^2 = \dfrac{rg(1 - 0.6)}{1 + 0.6}$ M1

$= \dfrac{1}{4}rg$ W1

$v^2 = \dfrac{rg(1 + 0.6)}{1 - 0.6}$

$= 4rg$ MW1

$\Rightarrow v^2 = 16u^2$ M1

$\Rightarrow v = 4u$ W1 17

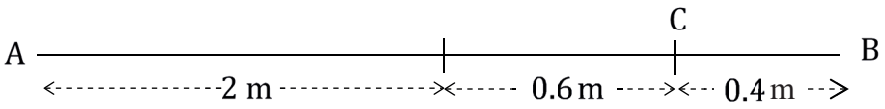
4 **(i)** $-T = m\ddot{x}$

$$-\frac{32x}{2} = 4\ddot{x}$$ M2 W1

$\Rightarrow \ddot{x} = -4x$ W1

i.e. SHM about the unextended position W1
$\Rightarrow \omega = 2$

(ii)



When $v = 0$, extension is 1 m
Hence amplitude is 1 m MW1
$v^2 = \omega^2 (r^2 - x^2)$ M1
At C, $x = 0.6$
$\Rightarrow v^2 = 4(1^2 - 0.6^2)$ W1
$\Rightarrow v = 1.6$ W1
Speed is 1.6 m s^{-1}

(iii) Time from B to C:
$x = r \cos \omega t$
$0.6 = 1 \cos 2t$ M1 W1
$t = \frac{1}{2} \cos^{-1} (0.6)$ W1
To return to C:
Time = Period $- 2 \times \frac{1}{2} \cos^{-1} (0.6)$ MW1
$= \frac{2\pi}{2} - 2 \times \frac{1}{2} \cos^{-1}(0.6)$ M1
Time = 2.21 seconds (3 s.f.) W1

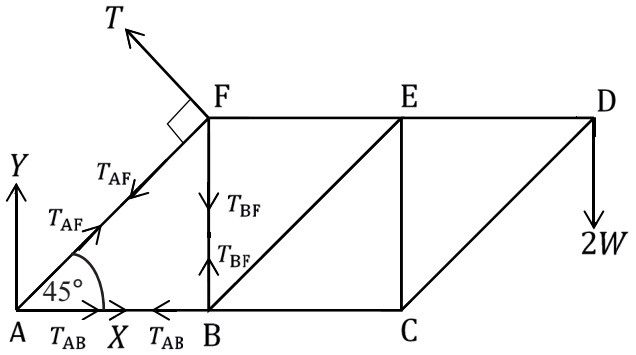
Alternative Solution
Time from C to centre of oscillation:
$x = r \sin \omega t$
$0.6 = 1 \sin 2t$ M1 W1

$t = \frac{1}{2} \sin^{-1}(0.6)$ W1

To return to C:
Time = $\frac{1}{2} \times$ Period $+ 2 \times \frac{1}{2} \sin^{-1}(0.6)$ MW1
$= \frac{1}{2} \times \frac{2\pi}{2} + 2 \times \frac{1}{2} \sin^{-1}(0.6)$ M1
Time = 2.21 seconds (3 s.f.) W1 15

AVAILABLE MARKS

5



(i) AF = $\sqrt{2}d$

M(A): $T \times \sqrt{2}d = 2W \times 3d$ M2 W2

$T = 3\sqrt{2}W$ W1

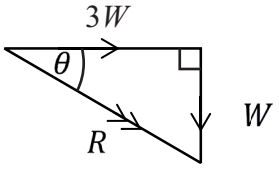
(ii) (↑) $Y + T\cos 45° = 2W$ M1 W1

$Y + 3W = 2W$

$Y = -W$ W1

(→) $X = T\cos 45°$ M1

$X = 3W$ W1



$R = \sqrt{9W^2 + W^2}$ M1

$R = W\sqrt{10}$ W1

$\theta = \tan^{-1}\left(\frac{1}{3}\right)$ M1

i.e. 18.4° below the horizontal W1

(iii) At A:

(↑) $Y + T_{AF}\cos 45° = 0$ M1

$T_{AF} = \sqrt{2}W$ W1

(→) $T_{AB} + T_{AF}\cos 45° + X = 0$ M1

$T_{AB} + W + 3W = 0$

$T_{AB} = -4W$ W1

At F:

(↑) $T\cos 45° = T_{AF}\cos 45° + T_{BF}$ M1

$3W = W + T_{BF}$

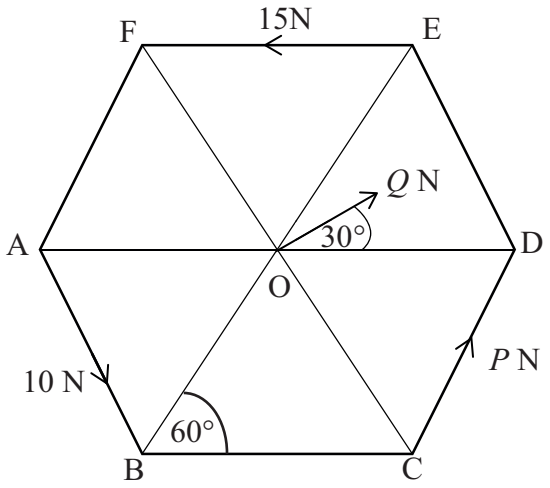
$T_{BF} = 2W$ W1

Force in AB is a thrust. MW1 21

Section A **75**

AVAILABLE MARKS

1



(i) (↑) $Q\cos 60° + P\cos 30° = 10\cos 30°$ M1 W2

(→) $10\cos 60° + P\cos 60° + Q\cos 30° = 15$ MW2

$\Rightarrow Q + \sqrt{3}P = 10\sqrt{3}$ M1

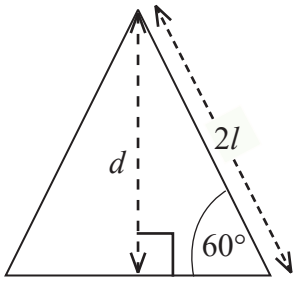
$\sqrt{3}Q + P = 20$

$\Rightarrow \sqrt{3}Q + 3P = 30$

$\sqrt{3}Q + P = 20$

$\Rightarrow P = 5,\ Q = 5\sqrt{3}$ W2

(ii)



$d = 2l\sin 60°$

$= \sqrt{3}l$

MW1

M(O): $10d + Pd + 15d$ M1 W1

$= 30d$

$\Rightarrow S = 30\sqrt{3}l$ Nm anticlockwise W2 13

2 **(i)** $\mathbf{v} = \int \mathbf{a}\, dt$

AVAILABLE MARKS

$= \int (5 \sin t\, \mathbf{i} + 2t\, \mathbf{j} + 5 \cos t\, \mathbf{k})\, dt$ M1

$= -5 \cos t\, \mathbf{i} + t^2\, \mathbf{j} + 5 \sin t\, \mathbf{k} + \mathbf{c}$ W3

$t = 0$, $\mathbf{v} = -5\mathbf{i} + 8\mathbf{j}$
$\Rightarrow -5\mathbf{i} + 8\mathbf{j} = -5\mathbf{i} + \mathbf{c}$ M1
$\mathbf{c} = 8\mathbf{j}$
$\mathbf{v} = -5 \cos t\, \mathbf{i} + (t^2 + 8)\mathbf{j} + 5 \sin t\, \mathbf{k}$ W1

(ii) Speed = 13
$\Rightarrow (-5 \cos t)^2 + (t^2 + 8)^2 + (-5 \sin t)^2 = 169$ M1 W1
$25 \cos^2 t + 25 \sin^2 t + (t^2 + 8)^2 = 169$
$(t^2 + 8)^2 = 144$
$t^2 + 8 = \pm 12$
$t^2 = -20, 4$
Hence time is 2 seconds. W1

(iii) $\mathbf{s} = \int \mathbf{v}\, dt$

$= -5 \sin t\, \mathbf{i} + \left(\frac{1}{3} t^3 + 8t\right)\mathbf{j} - 5 \cos t\, \mathbf{k} + \mathbf{d}$ M1 W3
$t = 0$, $\mathbf{s} = -\mathbf{i} - 3\mathbf{k}$ M1
$\Rightarrow -\mathbf{i} - 3\mathbf{k} = -5\, \mathbf{k} + \mathbf{d}$
$\mathbf{d} = -\mathbf{i} + 2\, \mathbf{k}$
$\mathbf{s} = (-5 \sin t - 1)\, \mathbf{i} + \left(\frac{1}{3} t^3 + 8t\right)\mathbf{j} + (-5 \cos t + 2)\mathbf{k}$ W1

(iv) If P crosses the x-axis, then both **j** and **k** components of **s** are zero at the same instant
$\frac{1}{3} t^3 + 8t = 0$ M1

$t\left(\frac{1}{3} t^2 + 8\right) = 0$

$t = 0$ MW1
The **k** component $(-5 \cos t + 2)$ is not zero at this time. M1
Hence the path of P never crosses the x-axis. W1 19

3 **(i)** A B

AVAILABLE MARKS

$m_A = m$ $m_B = km$

$u_A = u$ $u_B = 0$

$v_A = ?$ $v_B = ?$

$mu = mv_A + kmv_B$ ① M1 W1

$-eu = v_A - v_B$ ② M1 W1

$u = v_A + kv_B$ ① ($\div m$) M1

$-keu = kv_A - kv_B$ ② ($\times k$)

$\Rightarrow u(1 - ke) = (k + 1)v_A$

Hence

$v_A = \dfrac{u(1 - ke)}{(k + 1)}$ W1

$u = v_A + kv_B$ ① ($\div m$)

$-eu = v_A - v_B$ ②

$\Rightarrow u(1 + e) = (k + 1)v_B$

Hence

$v_B = \dfrac{u(1 + e)}{k + 1}$ MW1

(ii) $v_B = \dfrac{-eu(1 + e)}{k + 1}$ M1 W1

(iii) $k = 4$, $e = 0.5$

$v_A = \dfrac{u(1 - 2)}{5}$ $v_B = \dfrac{-0.5u(1 + 0.5)}{5}$

$= -0.2u$ $= -0.15u$ M1 W2

A and B both have negative velocities and are therefore travelling in the same direction.
A has a greater speed so B cannot catch up and there will not be a second collision. MW1 13

4 **(i)** At maximum speed, $a = 0$ and therefore $F = 0$

AVAILABLE MARKS

$$0 = \frac{54\,000}{v} - kv^2 \qquad \text{M3}$$

$$0 = \frac{54\,000}{30} - k(30)^2 \qquad \text{W1}$$

$$\Rightarrow k = 2 \qquad \text{W1}$$

(ii) $\dfrac{54\,000}{v} - 2v^2 = 3000v\dfrac{\mathrm{d}v}{\mathrm{d}s}$ M2 W1

Separate the variables

$$\int_{18}^{24} \frac{3000v}{\frac{54\,000}{v} - 2v^2}\,\mathrm{d}v = \int_0^x \mathrm{d}s \qquad \text{M1 W1}$$

$$\int_{18}^{24} \frac{3000v^2}{54\,000 - 2v^3}\,\mathrm{d}v = \int_0^x \mathrm{d}s$$

$$[-500\ln(54\,000 - 2v^3)]_{18}^{24} = x \qquad \text{MW1 M1}$$

$x = 237\,\text{m}$ (3 s.f.) W1 13

5 (i)

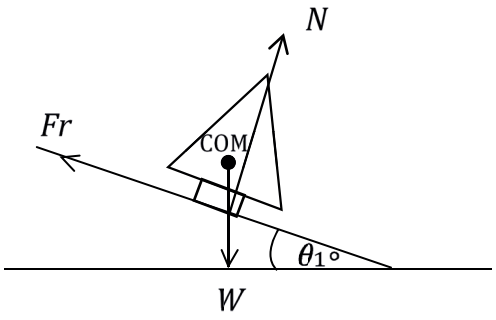
Shape	Cylinder	Cone	Total	
Mass (m)	$\pi a^2 (a)\rho$ $= \pi a^3 \rho$	$\frac{1}{3}\pi a(2a)^2 (4a)\rho$ $= \frac{16}{3}\pi a^3 \rho$	$\frac{19}{3}\pi a^3 \rho$	M1 W2
Dist of CM from base (d)	$\frac{1}{2}a$	$2a$	$\overline{y}$	MW2
md	$\frac{1}{2}\pi a^4 \rho$	$\frac{32}{3}\pi a^4 \rho$	$\frac{19}{3}\pi a^3 \rho\overline{y}$	M1 W1

$$\frac{1}{2}\pi a^4 \rho + \frac{32}{3}\pi a^4 \rho = \frac{19}{3}\pi a^3 \rho\overline{y} \quad \text{M1}$$

$$\frac{67a}{6} = \frac{19}{3}\overline{y}$$

$$\overline{y} = \frac{67a}{38} \quad \text{W1}$$

(ii)



About to slide

($\nearrow$) $N = W\cos\theta_1$ MW1

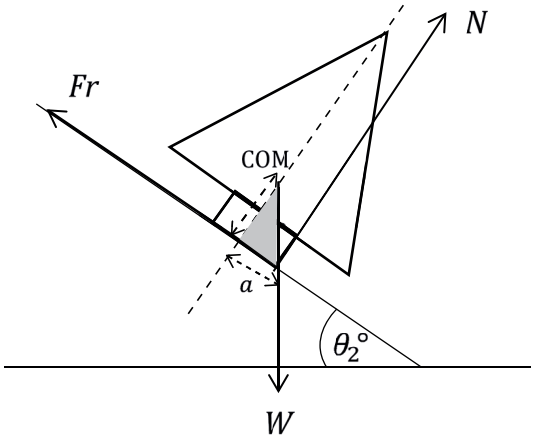
($\nwarrow$) $Fr = W\sin\theta_1$ MW1

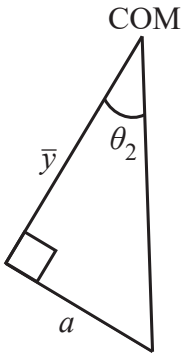
$Fr = 0.3N$

$W\sin\theta_1 = 0.3W\cos\theta_1$ MW1

$\tan\theta_1 = 0.3$

$\theta_1 = 16.7°$ (3 s.f.) W1





About to topple

$\tan\theta_2 = \frac{a}{\overline{y}}$ M1 W1

$= \frac{38}{67}$

$\theta_2 = 29.6°$ (3 s.f.) W1

Since the angle for sliding is less than the angle for toppling the ornament slides before it topples. MW1

17

Section B **75**

AVAILABLE MARKS

Section C Statistics

AVAILABLE MARKS

1 **(a)** The probability that the interval encloses the mean μ is 0.99 — MW1

(b) **(i)** $S^2 = \frac{n}{n-1}\sigma^2 = \frac{50}{49} \times 350^2 = 125\,000$ — M1 W1

(ii) $\bar{x} + 2.576 \times \frac{S}{\sqrt{n}} = 361$ — M2 MW1

$\bar{x} + 2.576 \times \frac{\sqrt{125\,000}}{\sqrt{50}} = 361$

$\bar{x} = 232.2$ — MW1

95% CI $= 232.2 \pm 1.96 \times \frac{\sqrt{125\,000}}{\sqrt{50}}$ — M1 MW1

$= (134\ ,\ 330)$ (3 s.f.) — W1

(iii) $2 \times 1.96 \frac{\sqrt{125\,000}}{\sqrt{n}} \leqslant 150$ — M1 MW1

$\sqrt{n} \geqslant 9.24$

$n \geqslant 85.4$

i.e. smallest sample size is 86 — MW1

(iv) Distribution of sample mean $\overline{X}$ is (approximately) normal. — MW1

Sample size is greater than 30 $\therefore$ Central Limit Theorem applies. — MW1 — 15

2 **(i)** Let the difference between the before and after times, t_b and t_a, be modelled by the random variable D, where $D = T_b - T_a$

AVAILABLE MARKS

$H_0 : \mu_d = 0$ M1

$H_1 : \mu_d > 0$ MW1

Small sample $\therefore$ t-test (upper one-tailed) M1

$v = 10 - 1 = 9 \ \therefore \ T \sim t(9)$ MW1

where $T = \dfrac{\overline{D} - \mu d}{\frac{S_d}{\sqrt{n}}}$

Critical value at 10% with $v = 9$ is $t_c = 1.383$ MW1

Commuter	A	B	C	D	E	F	G	H	I	J
d	3	–3	–2	1	2	2	4	3	4	–2
d^2	9	9	4	1	4	4	16	9	16	4

M1

$\sum d^2 = 76$ W1

$\sum d = 12$ MW1

$\overline{d} = \dfrac{\sum d}{n} = \dfrac{12}{10} = 1.2$ MW1

$S_d^2 = \dfrac{n}{n-1}\left(\dfrac{\sum d^2}{n} - \overline{d}^2\right) = \dfrac{10}{9}\left(\dfrac{76}{10} - 1.2^2\right)$ M1

$S_d^2 = \dfrac{308}{45}$ W1

If H_0 is true then $\mu_d = 0$, and the test value is given by

$t_{test} = \dfrac{\overline{d}}{\frac{S_d}{\sqrt{n}}} = \dfrac{1.2}{\frac{\sqrt{\frac{308}{45}}}{\sqrt{10}}}$ M1 W1

$t_{test} = 1.450$ W1

$t_{test} > t_c \ \therefore$ reject H_0 MW1

i.e. there is sufficient evidence, at the 10% level, to claim that the upgrade reduces commuting times. MW1

(ii) Time differences are Normally distributed. MW1 17

			AVAILABLE MARKS
3	**(i)** Let $V = S_1 + S_2 + S_3 + S_4$		
	where S_1, S_2, S_3 and S_4 are independent observations of S	M1	
	$E(V) = 4E(S) = 4 \times 3000 = 12\,000$	MW1	
	$Var(V) = 4Var(S) = 4 \times 100^2 = 40\,000$	MW1	
	$V \sim N(12\,000, 40\,000)$		
	$P(V > 11\,900) = 1 - P(V < 11\,900)$	M2	
	$= 1 - 0.309$		
	$= 0.691$	W1	
	(ii) Let $W = U - 4S$	M1	
	$E(W) = E(U) - 4E(S)$		
	$= 11\,900 - 4 \times 3000$	M1	
	$= -100$	W1	
	$Var(W) = Var(U) + 4^2Var(S)$	M1	
	$= 150^2 + 16 \times 100^2$	MW1	
	$= 182\,500$	W1	
	$W \sim N(-100, 182\,500)$		
	$P(W > 0) = 1 - P(W < 0)$	M2	
	$= 1 - 0.593$		
	$= 0.407$	W1	
	(iii) To calculate the variance $Var(W)$	MW1	16

4 H_0 : Sample drawn from a Normally distributed population with mean $\mu = 37.273$

H_1 : Sample not drawn from a Normally distributed population with mean $\mu = 37.273$

AVAILABLE MARKS

Small sample $\therefore$ t-test (two-tailed) M1

$v = 12 - 1 = 11 \ \therefore \ T \sim t(11)$ MW1

where $T = \dfrac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}}$

$\overline{x} = \dfrac{\Sigma x}{n} = \dfrac{516}{12} = 43$ MW1

$S^2 = \dfrac{\sum (x - \overline{x})^2}{n - 1} = \dfrac{1342}{11} = 122$ M1 W1

Test value is given by

$t_{\text{test}} = \dfrac{\overline{x} - 37.273}{\frac{S}{\sqrt{n}}} = \dfrac{43 - 37.273}{\frac{\sqrt{122}}{\sqrt{12}}}$ M1 MW1

$t_{\text{test}} = 1.796$ W1

$t_c = t_{\text{test}}$ M1

$t_c = 1.796$

i.e. 5% in upper tail MW1

Two-tailed test $\therefore$ significance level = 10% MW1 11

5 **(i)** $a = 50 - 43.52 = 6.48$ MW1

(ii) Let X be the number of successful penalty kicks in a single session

$H_0 : X \sim B(4, 0.6)$ M1

$H_1 : X$ is not modelled by a B(4, 0.6) distribution MW1

x	O	E
0	3	1.28
1	10	7.68
2	12	17.28
3	23	17.28
4	2	6.48

Group $x = 0$ and $x = 1$ classes so that combined frequency is at least 5 M1

Test statistic: $\chi^2 = \sum \frac{(O-E)^2}{E}$

x	O	E	$\frac{(O-E)^2}{E}$	
0 or 1	13	8.96	1.8216	W1
2	12	17.28	1.6133	
3	23	17.28	1.8934	M1 W1
4	2	6.48	3.0973	
			$\chi^2 = 8.426$	W1

$v = 4 - 1 = 3$ MW1

Critical value: $\chi^2_{5\%}(3) = 7.815$ MW1

$\chi^2 > \chi^2_{5\%}(3) \therefore$ reject H_0 MW1

i.e. there is insufficient evidence, at the 5% level, that the results can be modelled by a B(4, 0.6) distribution. MW1

(iii) $\bar{x} = \frac{\sum fx}{\sum f} = \frac{111}{50} = 2.22$ M1 W1

$p = \frac{\bar{x}}{n} = \frac{2.22}{4} = 0.555$ MW1

(iv) Number of restrictions increased by 1 MW1 16

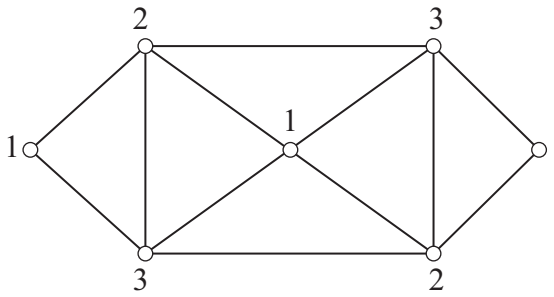
Section C 75

AVAILABLE MARKS

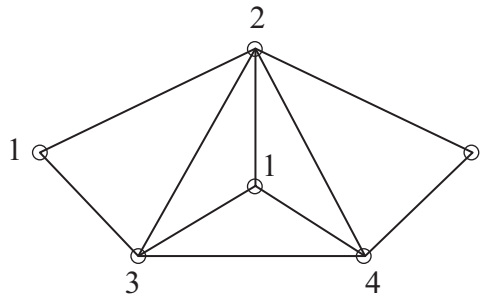
Section D Discrete and Decision Mathematics

AVAILABLE MARKS

1 **(a)** Method of vertex colouring M1



or equivalent W1



or equivalent W1

(b) $K_{5,7}$ is a bi-partite graph.
Either draw a graph or describe as follows:
Each of the 5-set of vertices is joined to each of the 7-set.
However, none of the 5-set is joined to another from that set, and the same for the 7-set. M1

So only 2 colours are needed – one for the 5-set and one for the 7-set. W1

(c) **(i)** Hall's theorem in this context means every set of k children wants at least k toys.

Consider P, Q, R, S i.e. 4 children. Together they only want toys A, B or C i.e. 3 toys. M1

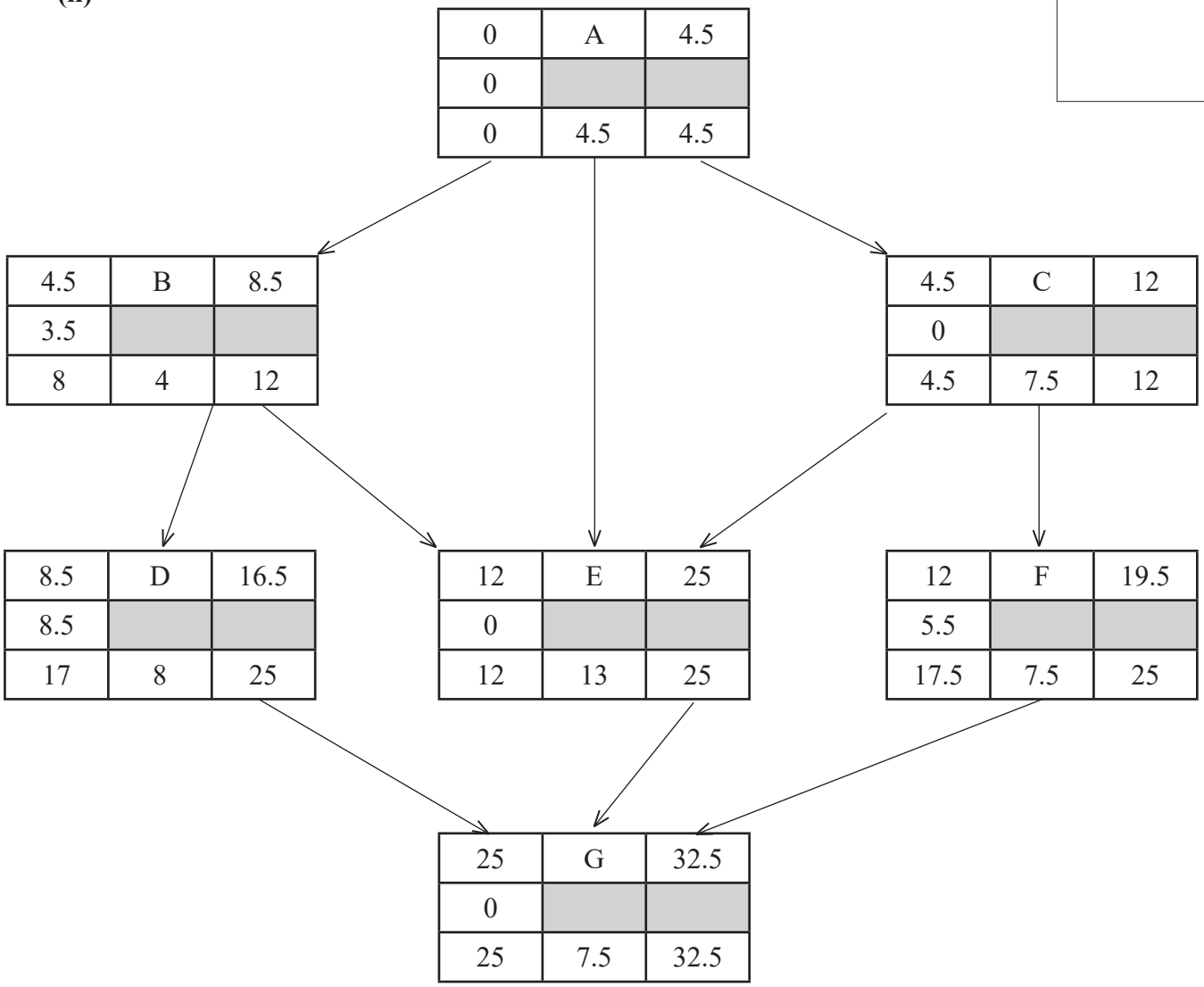
Thus by Hall's theorem, a complete matching is not possible. W1

(ii) Any correct matching using the wildcard H.
E.g. P-A Q-B R-C S-H T-F U-E V-G M1 W1 9

2 **(i)** 8, 13, 7.5, 7.5 MW1

AVAILABLE MARKS

(ii)



ES times MW1
EF times MW1
LS times MW1
LF times MW1

(iii) Critical Path = ACEG MW1

(iv) Float time for activity D = 8.5 days and is the longest time the activity can start after the earliest start time, for the project to remain on schedule. MW2

(v) Expected time = 4.5 + 7.5 + 13 + 7.5 = 32.5 days MW1
Project variance = (49 + 25 + 64 + 25)/36 = 163/36 = 4.528 M2 W1

$P(T < x) = 0.9$ M1

$P(Z < (x - 32.5)/\sqrt{4.528}) = 0.9$ MW1

$(x - 32.5)/\sqrt{4.528} = \Phi^{-1}(0.90) = 1.282$

$X = 32.5 + 1.282 \times \sqrt{4.528} = 35.2$ W1

15

3 **(i)** Introduce slack variables: M1

AVAILABLE MARKS

$$5x + 3y + u = 31.2$$
$$2x + 5y + v = 21.6$$
$$-4x - 7y + P = 0$$

MW1

(ii)

5	3	1	0	0	31.2
2	5	0	1	0	21.6
–4	–7	0	0	1	0

M1 W2

(iii) First iteration:
Identify pivot column – column (2) (as $-7 < -4$) M1
Identify pivot row – row (2) (as $21.6/5 < 31.2/3$) MW1
Divide pivot row by 5

5	3	1	0	0	31.2
0.4	1	0	0.2	0	4.32
–4	–7	0	0	1	0

MW1

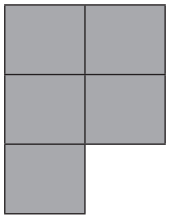
Reduce other entries in pivot column to zero by subtracting $k \times$ pivot row

3.8	0	1	–0.6	0	18.24
0.4	1	0	0.2	0	4.32
–1.2	0	0	1.4	1	30.24

MW1 9

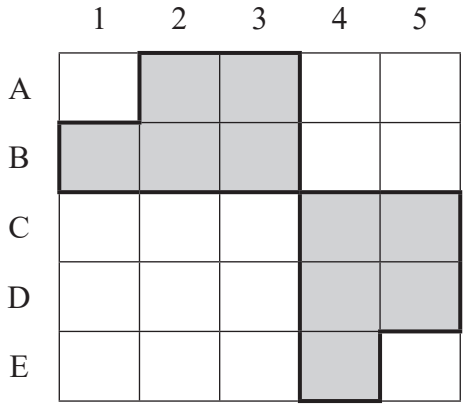
4 **(i)** 0 rooks 1 way
1 rook 5 ways
2 rooks 4 ways M1
So the rook polynomial is $1 + 5x + 4x^2$ W1

(ii) Rook polynomial for



is $1 + 5x + 4x^2$ M1
Two shapes are non-interfering.
Combined rook polynomial is $(1 + 5x + 4x^2)^2$ M1
$= (1 + 5x + 4x^2)(1 + 5x + 4x^2)$
$= 1 + 5x + 4x^2 + 5x + 25x^2 + 20x^3 + 4x^2 + 20x^3 + 16x^4$ W2
$= 1 + 10x + 33x^2 + 40x^3 + 16x^4$
W1

(iii)



M1 W1

Rook polynomial for excluded positions
$= 1 + 10x + 33x^2 + 40x^3 + 16x^4$ MW1
Rook IE theorem M1
Number of ways = $5! - 10 \times 4! + 33 \times 3! - 40 \times 2! + 16$ M1 W1
= 14 ways W1 14

AVAILABLE MARKS

5 **(a)** **(i)** $P_G = \frac{1}{26}(x_1^{13} + 12x_{13}^1 + 13x_1^1 x_2^6)$ MW1

AVAILABLE MARKS

(ii) Replace x_i by 2 M1

Number of chains possible is $\frac{1}{26}(2^{13} + 12 \times 2^1 + 13 \times 2^1 \times 2^6)$ MW1

$= 380$ chains W1

(b) **(i)** $P_G = \frac{1}{24}(x_1^{12} + 3x_2^6 + 6x_1^2 x_2^5 + 6x_4^3 + 8x_3^4)$

Replace x_i by $P^i + G^i$ M1

$P_G = \frac{1}{24}[(P + G)^{12} + 3(P^2 + G^2)^6 + 6(P + G)^2 (P^2 + G^2)^5 + 6(P^4 + G^4)^3 + 8(P^3 + G^3)^4]$ M1 W1

(ii) Terms in P^4G^8: M1

$(P + G)^{12}$ gives $\binom{12}{4} P^4G^8 = 495\ P^4G^8$

$3(P^2 + G^2)^6$ gives $3\binom{6}{2} P^4G^8 = 45P^4G^8$

$6(P + G)^2 (P^2 + G^2)^5$ gives $6(P^2 + 2PG + G^2)(...5P^2G^8...10P^4G^6...) = 90P^4G^8$

$6(P^4 + G^4)^3$ gives $6\binom{3}{2} P^4G^8 = 18P^4G^8$

$8(P^3 + G^3)^4$ has no contribution MW4

Number of colourings is $\frac{1}{24}(495 + 45 + 90 + 18) = 27$ W1 13

		AVAILABLE MARKS
6	**(i)** LHS $= \binom{n}{r} = \frac{n!}{r!(n-r)!}$	
	RHS $= \frac{n!}{(n-r)!(n-n+r)!} = \frac{n!}{r!(n-r)!} =$ LHS M1W1	
	(ii) LHS $= \binom{8}{4} = 70$ M1	
	RHS $= \binom{4}{0}^2 + \binom{4}{1}^2 + \binom{4}{2}^2 + \binom{4}{3}^2 + \binom{4}{4}^2 = 1 + 16 + 36 + 16 + 1 = 70$ W1	
	(iii) Knowing to equate coefficients of x^n in both sides M1	
	Coefficient of x^n in $(1+x)^{2n} = \binom{2n}{n}$ M1	
	$(1+x)^n \times (1+x)^n =$	
	$\left[\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n}x^n\right] \times \left[\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n}x^n\right]$ M1	
	Coefficient of x^n in $(1+x)^n \times (1+x)^n$	
	$= \binom{n}{0} \times \binom{n}{n} + \binom{n}{1} \times \binom{n}{n-1} + \binom{n}{2} \times \binom{n}{n-2} + \ldots + \binom{n}{n} \times \binom{n}{0}$ MW1	
	Thus	
	$\binom{2n}{n} = \binom{n}{0} \times \binom{n}{n} + \binom{n}{1} \times \binom{n}{n-1} + \binom{n}{2} \times \binom{n}{n-2} + \ldots + \binom{n}{n} \times \binom{n}{0}$ W1	
	(iv) Using result from **(i)** e.g.	
	$\binom{n}{1} \times \binom{n}{n-1} = \binom{n}{1}^2$ MW1	
	Thus $\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \ldots + \binom{n}{n}^2$ W1	
	(v) Using $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n}x^n$	
	Differentiating $n(1+x)^{n-1} = 1 \times \binom{n}{1} + 2 \times \binom{n}{2}x + \ldots + n \times \binom{n}{n}x^{n-1}$ M1 W1	
	Setting $x = 1$ M1	
	Obtain $n \times 2^{n-1} = 1 \times \binom{n}{1} + 2 \times \binom{n}{2} + \ldots + n \times \binom{n}{n}$ W1	15
	Section D	**75**