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General Certificate of Education

Further Mathematics *StemUp Model Solution*

SFM21

Assessment Unit AS 2

assessing

Applied Mathematics

[SFM21]

Assessment

Assessment Level of Control:

Tick the relevant box (✓)

Controlled Conditions	
Other	

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided and on the Section D Supplementary Answer Booklet (if applicable).
You must answer **all** questions from sections A and B **or** A and C **or** A and D **or** C and D.
You should spend equal time on each of the two sections.
Candidates taking Section D should use the Supplementary Answer Booklet provided and attach to your Answer Book using the treasury tag provided.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
The total mark for each section of this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

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SECTION A Mechanics 1

Answer all five questions in this section.

1 Two forces

$$\begin{pmatrix} 5 \\ 6 \\ -7 \end{pmatrix} \text{N} \quad \text{and} \quad \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} \text{N}$$

move a particle through a displacement

$$\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix} \text{m}$$

Find the work done by the resultant force. [4]

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2 **Fig. 1** below shows a car of mass 1200 kg travelling in a horizontal circle of radius r metres as it negotiates a bend in the road.

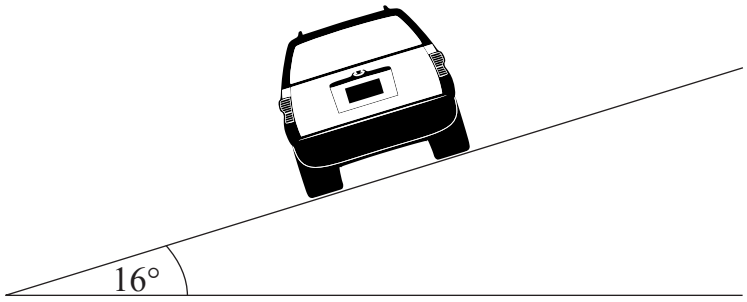


Fig. 1

The road is banked at an angle of 16° to the horizontal.

The car moves at a constant speed of 14 ms^{-1} and has no tendency to slip on the road surface at this speed.

(i) Draw a diagram showing all the external forces acting on the car. [2]

(ii) Find the normal reaction between the car and the road. [3]

(iii) Find the radius r. [5]

(iv) State one modelling assumption you have used. [1]

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3 **(a)** A light elastic string of natural length l metres and modulus of elasticity λ newtons rests on a horizontal surface with one end attached to a fixed point.

Show that the work done by the tension when the string is extended by e metres from its natural length is

$$-\frac{\lambda e^2}{2l} \text{ joules}$$

[4]

(b) A light elastic rope, of natural length 1.7 m and modulus of elasticity 98 newtons, has one end attached to a point P and the other end to a package of mass 5 kg.

The package is held at P and released from rest.

It drops vertically until it comes to instantaneous rest at a point Q.

Take the gravitational potential energy to be zero at P.

Using the Principle of Conservation of Mechanical Energy, find the length PQ. [8]

4 A horse of mass 450 kg canters along a race track and experiences a constant resistance.
When the track is horizontal, the horse canters at a constant speed of 18 km h^{-1}
The horse works at a constant rate of 2 kW.

(i) Show that the magnitude of the resistance it experiences is 400 N. [3]

The horse now runs up a hill inclined at an angle α to the horizontal where $\sin \alpha = 0.1$
It reduces its speed to a constant value of 10.8 km h^{-1} to ascend the hill.
Assume the resistance it experiences remains at 400 N.

(ii) Find the rate at which the horse now works. [4]

The horse then descends a hill inclined at an angle β to the horizontal where $\sin \beta = 0.05$
At a particular instant, it is working at a rate of 2 kW and moving with an acceleration of 0.4 ms^{-2}
Assume the resistance it experiences is now 300 N.

(iii) Find the velocity of the horse at this instant. [5]

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5 In an airport, suitcases slide down a straight, rough ramp to a loading bay.

The coefficient of friction between the ramp and the suitcase is 0.3

The ramp is 4 m long and is inclined at 20° to the horizontal.

A 30 kg suitcase starts to slide down from the top of the ramp.

It has an initial speed of $0.8\,\text{ms}^{-1}$

Model the suitcase as a particle.

(i) Draw a diagram showing the external forces acting on the suitcase. [1]

(ii) Calculate the work done by friction as the suitcase slides down the full length of the ramp. [4]

(iii) Using the Work–Energy Principle, find the speed of the suitcase at the bottom of the ramp. [6]

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SECTION B Mechanics 2

Answer all five questions in this section.

1 A scientist is studying a production process.

She believes that the mass M of its output depends on the viscosity μ and density ρ of the raw material and the power H of its engine.

She assumes a product relationship between the quantities:

$$M = k\,\mu^x \rho^y H^z$$

where k is a dimensionless constant.

Use the Method of Dimensions to find x, y and z.

Note that the dimensions of viscosity are $ML^{-1}T^{-1}$
and the dimensions of density are ML^{-3} [8]

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2 A river flows from West to East at $9\,\text{kmh}^{-1}$ as shown in **Fig. 1** below.

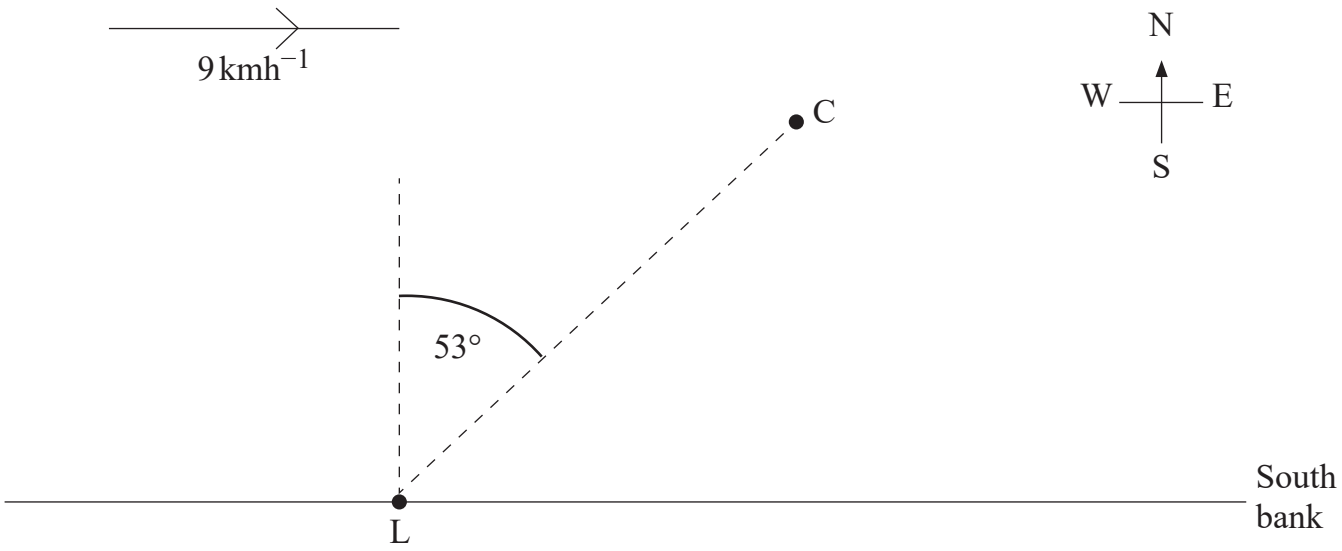


Fig. 1

A life-saving station L lies on the edge of the South bank.
A canoe C gets into difficulty in the river when it is on a bearing of 053° from L.
It then starts to drift with the current.
At this instant a helicopter, moving at 70kmh^{-1}, is dispatched from L to rescue the canoe.

Find the bearing on which the helicopter should fly to intercept the canoe. [7]

3 A weather satellite is placed in a circular orbit lying in a plane through the equator.

The satellite orbits the earth with a period of 90 minutes.

The radius of the earth at the equator is 6378 km.

(i) Given that M kg is the mass of the earth and G is the universal gravitational constant, show that MG is approximately $3.9865 \times 10^{14}\,\text{m}^3\text{s}^{-2}$ [3]

(ii) Hence find the height of the weather satellite above the equator. [8]

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4 A small block B of mass m kg is resting in equilibrium on a rough slope as shown in **Fig. 2** below.

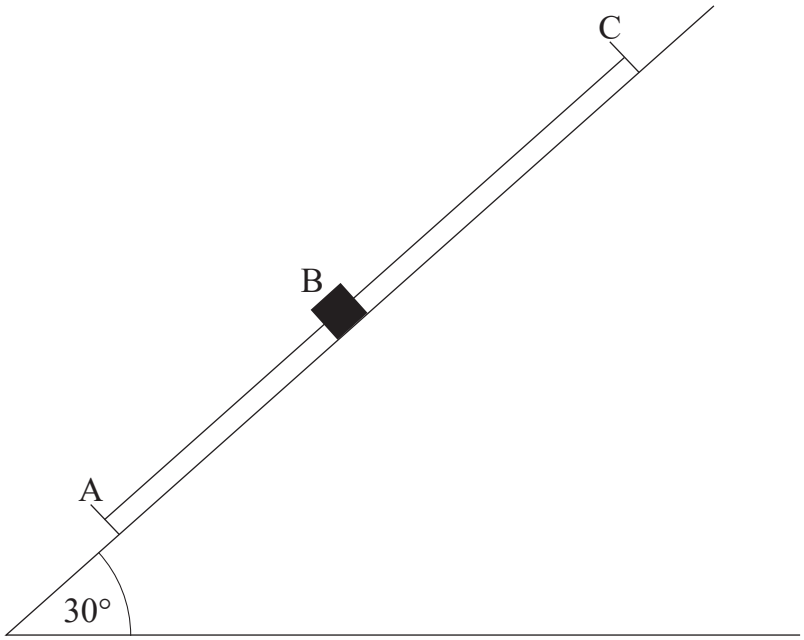


Fig. 2

The block is attached to two light elastic strings along the line of greatest slope.
The lower string joins the block to the fixed point A.
The upper string joins the block to the fixed point C.
AC = 0.9 m
The block can be modelled as a particle and is about to move up the slope.

Both strings are in tension.

(i) Copy the diagram showing all the forces acting on the block. [1]

The string from A to B has natural length 0.1 m and modulus of elasticity $k_1 mg$ N.

The string from B to C has natural length 0.4 m and modulus of elasticity $k_2 mg$ N.

The angle of inclination of the slope to the horizontal is 30°
The coefficient of friction between the block and the slope is $\frac{1}{\sqrt{3}}$

Take the extension in the string from A to B to be x m.

(ii) Show that the tension in the string from B to C is $\frac{1}{2} k_2 mg\,(2 - 5x)$ newtons. [3]

(iii) Show that the extension x is

$$\frac{2(k_2 - 1)}{5(4k_1 + k_2)}$$ [6]

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5 A small toy car of mass m kg moves around the outside of a fixed smooth cylindrical oil drum, as shown in **Fig. 3** below.

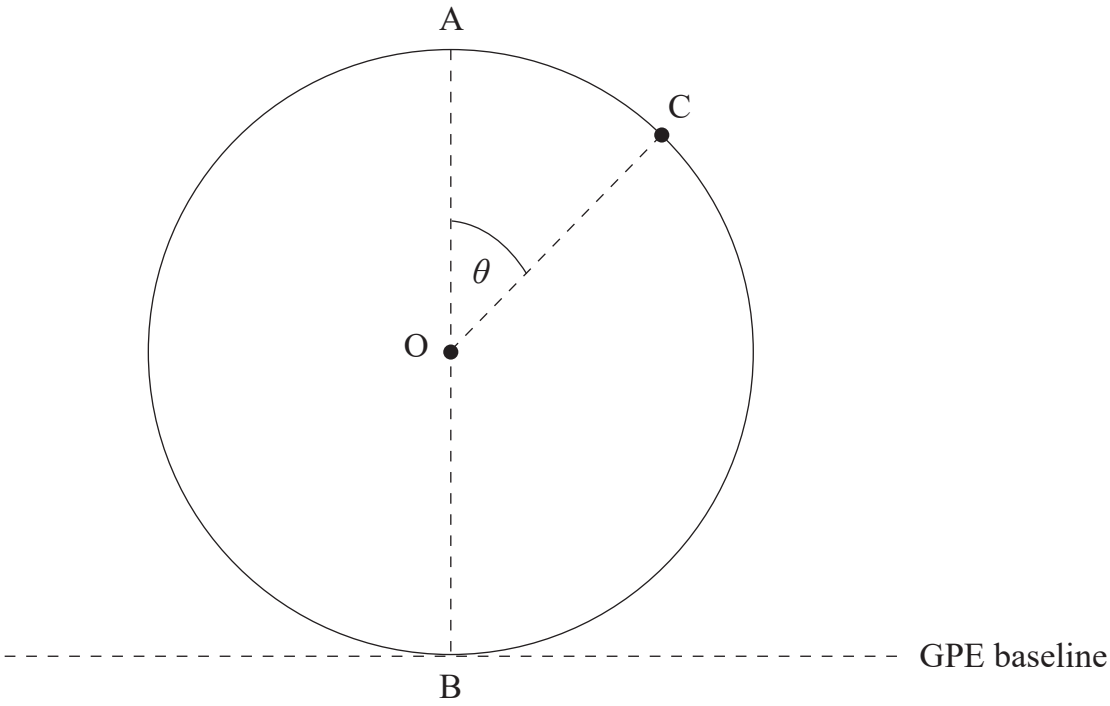


Fig. 3

The car C moves in a vertical circle of radius r metres and vertical diameter AOB where O is the centre of the circle and B is below A.
Initially, the car is at rest at point A and is displaced slightly so that it begins to move.
When the car is at point C, where the angle AOC = θ, it is moving with speed v ms^{-1}
Take the baseline for gravitational potential energy to be the horizontal line through B.
Model the car as a particle.

(i) Show that the gravitational potential energy of the car at C is

$$mgr(1 + \cos\theta) \text{ joules}$$ [2]

(ii) Hence find an expression for the kinetic energy of the car at C in terms of m, g, r and θ [5]

(iii) Show that the reaction of the drum on the car at C is given by

$$mg(3\cos\theta - 2) \text{ newtons}$$ [5]

The toy car leaves the surface of the drum at the point D.

(iv) Find the angle AOD. [2]

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SECTION C Statistics

Answer all five questions in this section.

1 A lorry company is investigating the deterioration of the tyres on its fleet of lorries over time. The company took a sample of its lorries and recorded the distance travelled (x, 1000s of miles) from the time new tyres were fitted and the corresponding tread depth (y, mm) of the tyres.

The summary statistics are shown below.

n	Σx	Σy	Σxy	Σx^2
8	100	39.1	382.6	1610

(i) Find the equation of the least squares regression line of tread depth on distance travelled. [7]

(ii) Estimate the tyre tread depth for a lorry that has travelled 30 000 miles from the time new tyres were fitted. Comment on the value. [3]

2 The Northern Ireland (NI) amateur hockey league consists of 20 hockey clubs located in various towns throughout Northern Ireland.
Each club holds a database of its season supporters, which has their names and contact details.
The organisers of the league would like to find out the views of the supporters regarding possible changes to fixture arrangements.
They wish to select a sample of supporters from the whole of NI and conduct face-to-face interviews at the respective club grounds for each selected supporter.

(i) Describe how the method of cluster sampling could be used for this investigation. Give one advantage and one disadvantage of using this technique. [5]

(ii) State a different sampling technique that could be used for this purpose and give one reason why it would be less suitable. [2]

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3 The number of major earthquakes that occur per year on the Earth follows a Poisson distribution.
Earthquakes with magnitude 8.0 or greater occur on the Earth at an average rate of two earthquakes per year.

(i) Find the probability that, in a single year, there will be more than two earthquakes of magnitude 8.0 or greater. [4]

Earthquakes with magnitude 9.0 or greater are less common.
The probability of no earthquakes of magnitude 9.0 or greater occurring in a single year on the Earth is 0.25

(ii) Find the average annual rate of earthquakes with magnitude 9.0 or greater. [4]

4 In a school there are six players on the senior tennis team and seven players on the junior tennis team.
A group of five players are randomly selected, from these 13 players, by the tennis coach to represent the school at a sporting event.

(i) Find the number of different ways of selecting the group of five players for the event. [2]

(ii) Find the probability that the group of five players consists of more senior players than junior players. [5]

The group of five players selected to represent the school at the event comprises three senior players and two junior players.
At the event, a photograph is taken of the group in a row of five seats.

Find the number of different ways of arranging the five players for the photograph if:

(iii) there are no restrictions on seating arrangements; [1]

(iv) the two junior players must be seated together. [3]

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5 The random variable X has the continuous probability density function f(x) defined by

$$f(x) = \begin{cases} 9x^2 & 0 \leqslant x \leqslant 0.5 \\ ax + \frac{17}{4} & 0.5 < x \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

where a is a constant.

The graph of f(x) is shown in **Fig. 1** below.

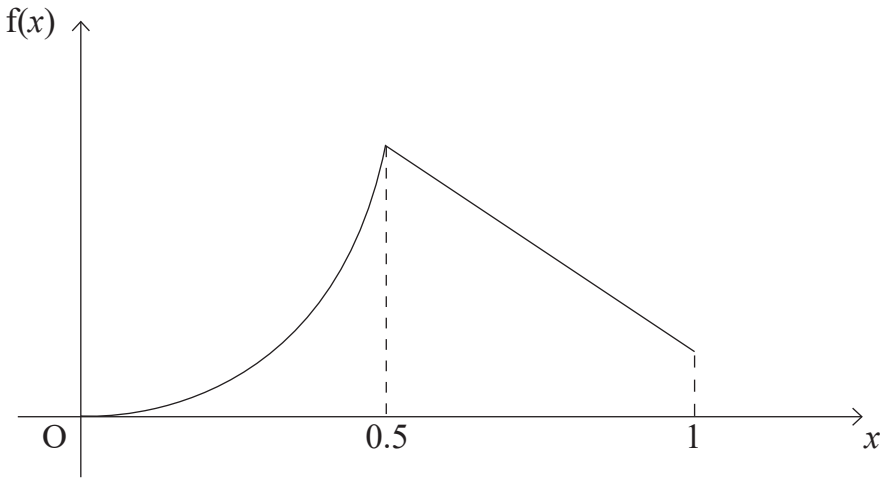


Fig. 1

(i) Show that $a = -4$ [2]

(ii) Show that $P(0 < X < 0.5) = 0.375$ [3]

(iii) Hence find the median value of X. [9]

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SECTION D Discrete and Decision Mathematics

Answer all five questions in this section.

Note: There is a Supplementary Answer Booklet for question **3(ii)**
Please attach to your answer booklet using the treasury tag provided.

1 **Fig. 1** below shows a graph G on nine vertices labelled a, b, ..., i.

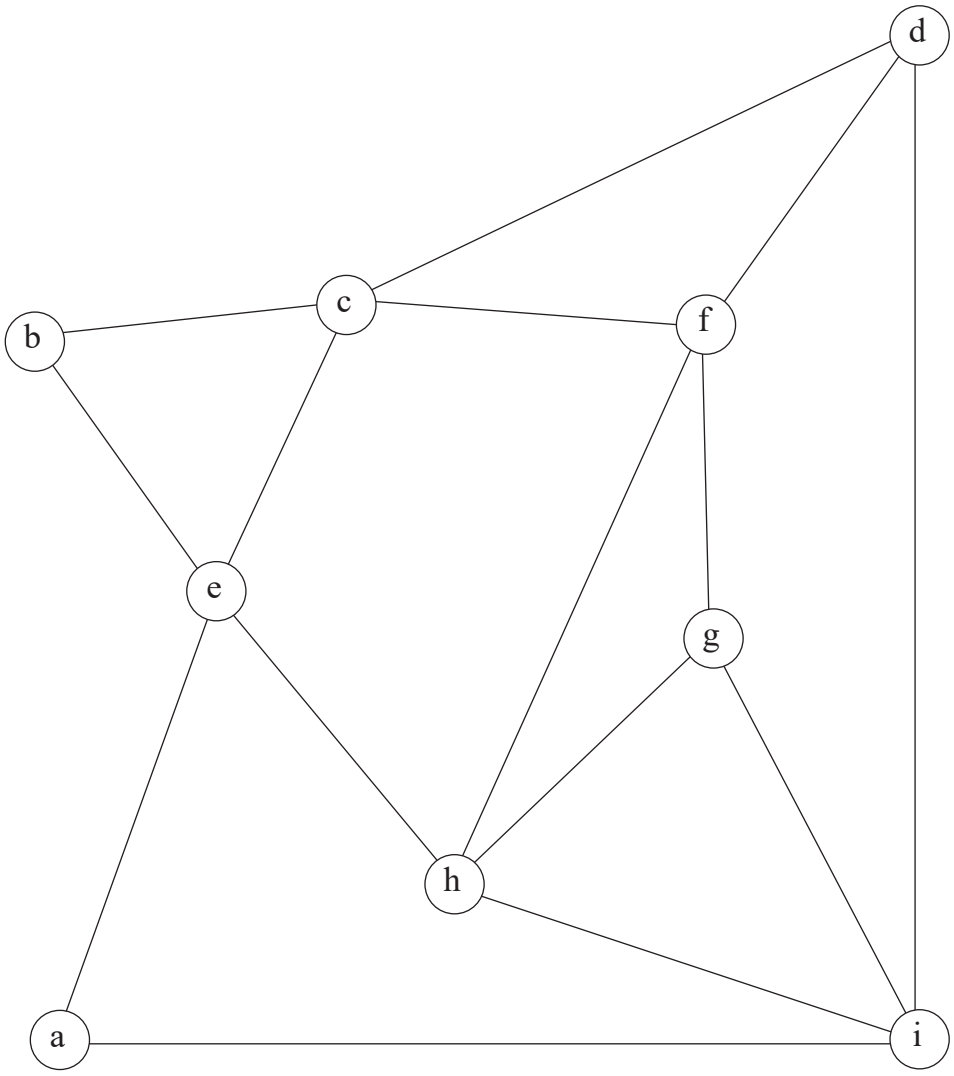


Fig. 1

(i) Write down a Hamiltonian cycle for this graph. [2]

An Eulerian circuit is possible in this graph if a single edge is added between two particular vertices to make a new graph G*

(ii) Write down these two vertices. [2]

(iii) Write down an Eulerian circuit for the new graph G* [3]

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2 The Lucas numbers are defined by the recurrence relationship

$$L_{n+2} = L_{n+1} + L_n \qquad n \geqslant 0$$

where $L_0 = 2$ and $L_1 = 1$

(i) Show that the auxiliary equation is

$$x^2 - x - 1 = 0$$ [2]

(ii) Hence show that

$$L_n = \alpha^n + \beta^n$$

where α and β are real numbers to be determined. [7]

(iii) Show that as n gets larger L_n approaches

$$2^{-n}\left(1 + \sqrt{5}\right)^n$$ [2]

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3 **Fig. 2** below shows the activity network used to model the set up and running of an event.
The activities involved are labelled A, B, ..., L and are represented by the edges.
The number on each edge represents the time, in days, required for one employee to complete that activity.

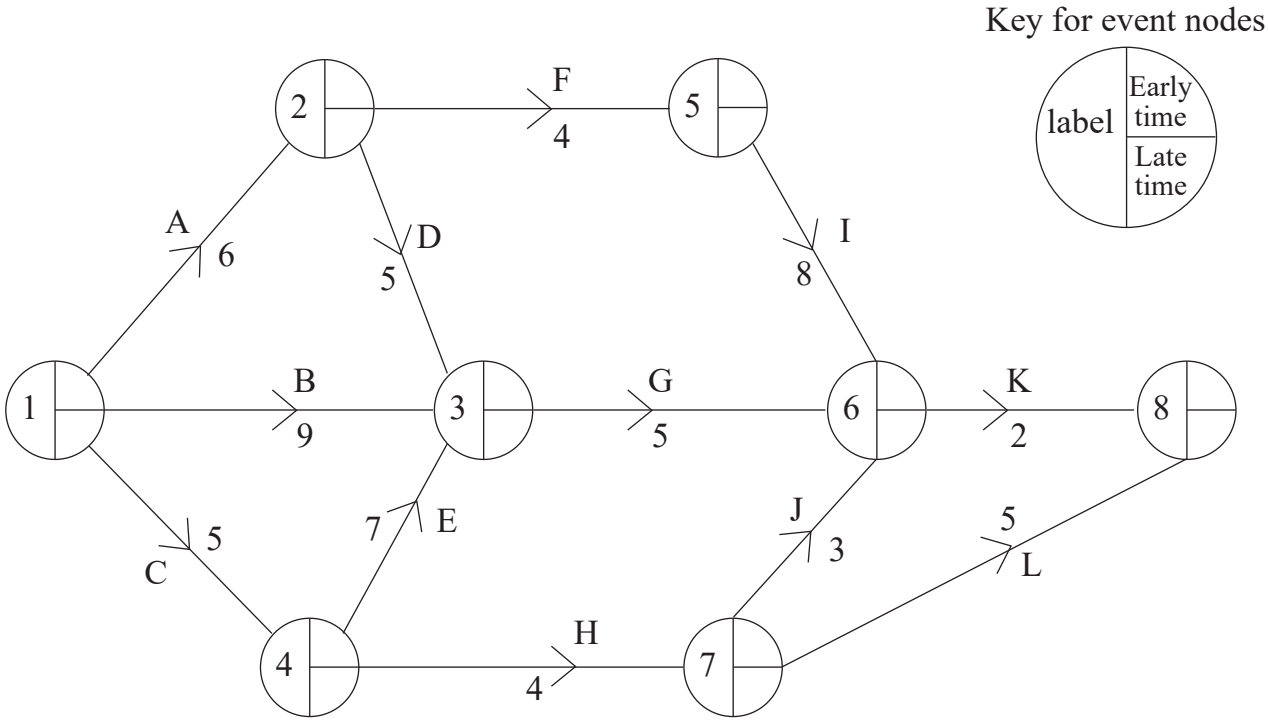


Fig. 2

(i) Explain how to determine the early time in node 3. [1]

(ii) Calculate the early time and late time for each event.
Write your answers in the spaces provided in the copy of **Fig. 2** in the Supplementary Answer Booklet. [4]

(iii) (a) What is meant by the float time for activity H? [1]

(b) Calculate the float time for activity H. [2]

(iv) Determine the critical activities and the length of the critical path. [2]

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4 Let p, q and r be propositional statements.

Use truth tables to prove

$$\sim[(\sim p \text{ or } r) \text{ and } q] \equiv (p \text{ and } \sim r) \text{ or } \sim q \qquad [9]$$

5 The Cayley table of the order 8 group G is given in **Fig. 3** below.

	1	a	a^2	a^3	b	ab	a^2b	a^3b
1	1	a	a^2	a^3	b	ab	a^2b	a^3b
a	a	a^2	a^3	1	ab	a^2b	a^3b	b
a^2	a^2	a^3	1	a	a^2b	a^3b	b	ab
a^3	a^3	1	a	a^2	a^3b	b	ab	a^2b
b	b	a^3b	a^2b	ab	1	a^3	a^2	a
ab	ab	b	a^3b	a^2b	a	1	a^3	a^2
a^2b	a^2b	ab	b	a^3b	a^2	a	1	a^3
a^3b	a^3b	a^2b	ab	b	a^3	a^2	a	1

Fig. 3

(i) Determine whether or not the group G is commutative, justifying your answer. [2]

(ii) Find the period of each element of G. [3]

(iii) List two subgroups of G of order 4, each of which contains the elements 1 and a^2 and two other elements. [2]

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A hexagon PQRSTUVW is illustrated in **Fig. 4** below.

It is formed by taking a regular hexagon and doubling the length of the horizontal sides.

Therefore PQ = QR = RS = ... = WP, with R and V being the midpoints of QS and WU respectively.

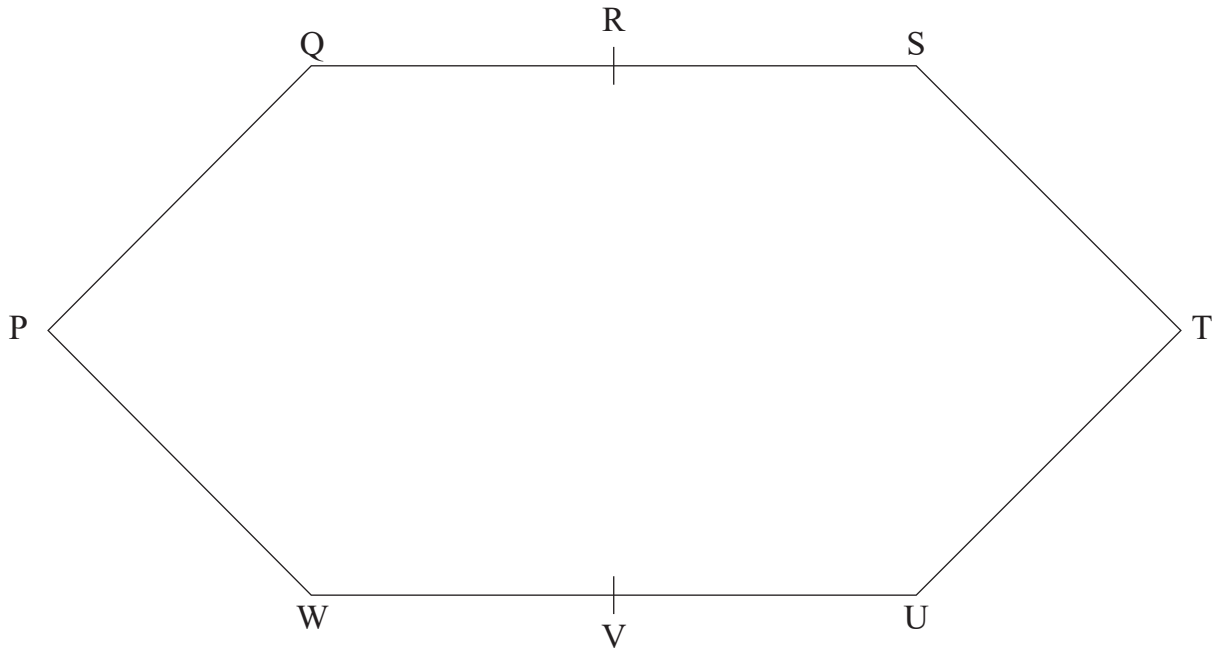


Fig. 4

(iv) List the elements of the symmetry group, H, of this hexagon and their corresponding periods. [4]

(v) State, with a reason, which of the two subgroups of order 4 of G, identified in part **(iii)**, is isomorphic to H. [2]

THIS IS THE END OF THE QUESTION PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

Section A Mechanics 1

A1 Resultant force $F = \begin{pmatrix} 5 \\ 6 \\ -7 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ -9 \end{pmatrix}$ N

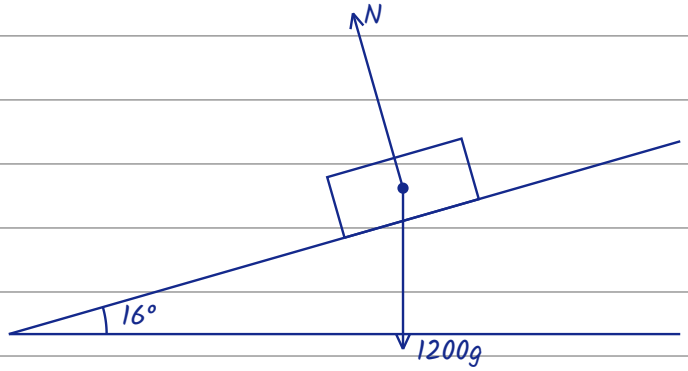
Work done $= F \cdot r = \begin{pmatrix} 8 \\ 5 \\ -9 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix}$

$= 8(4) + 5(-1) + (-9)(5)$

$= 32 - 5 - 45$

$= -18$ J

A2 (i)



Forces: normal reaction N (perpendicular to road) and weight 1200g.

(ii) Resolving vertically (no vertical acceleration):

$N \cos 16° = 1200g = 1200 \times 9.8 = 11\,760$

$N = \frac{11\,760}{\cos 16°} = 12\,233.9...$

Question number

$N = 12\ 200$ N (3 s.f.)

(iii) Resolving horizontally towards the centre, using $F = ma$

with acceleration $\frac{v^2}{r}$:

$$N \sin 16° = \frac{1200 \times 14^2}{r}$$

$$12\ 233.9 \times \sin 16° = \frac{235\ 200}{r}$$

$$r = \frac{235\ 200}{12\ 233.9 \times \sin 16°}$$

$r = 69.7$ m (3 s.f.)

(iv) The car is modelled as a particle.

A3 (a) When the extension is x, tension $T = \frac{\lambda x}{l}$, acting towards the fixed end, opposite to the direction of extension.

Work done by tension $= \int_0^e -T\,dx$

$$= \int_0^e -\frac{\lambda x}{l}\,dx$$

$$= \left[-\frac{\lambda x^2}{2l}\right]_0^e$$

$= -\frac{\lambda e^2}{2l}$ joules as required

(b) Let the extension at Q be e m, so PQ = 1.7 + e

Question number

At P: KE = 0, GPE = 0, EPE = 0 (rope slack)

At Q: KE = 0 (instantaneous rest)

GPE at Q $= -mgh = -5 \times 9.8 \times (1.7 + e) = -49(1.7 + e)$

EPE at Q $= \frac{\lambda e^2}{2l} = \frac{98e^2}{2 \times 1.7} = \frac{49e^2}{1.7}$

Conservation of mechanical energy:

Total energy at P = Total energy at Q

$0 = \frac{49e^2}{1.7} - 49(1.7 + e)$

$e^2 = 1.7(1.7 + e)$

$e^2 - 1.7e - 2.89 = 0$

$e = \frac{1.7 + \sqrt{1.7^2 + 4 \times 2.89}}{2}$ $(e > 0)$

$e = 2.7507...$

PQ $= 1.7 + 2.7507 = 4.45$ m (3 s.f.)

A4 (i) Speed $= 18$ km $h^{-1} = \frac{18}{3.6} = 5$ m s^{-1}

Driving force $F = \frac{P}{v} = \frac{2000}{5} = 400$ N

Constant speed, so resultant force is zero:

Resistance $R = F = 400$ N as required

(ii) Speed $= \frac{10.8}{3.6} = 3$ m s^{-1}

Constant speed, resolving parallel to the slope:

$F = 400 + 450g \sin \alpha$

$F = 400 + 450 \times 9.8 \times 0.1 = 841$ N

Question number

Power = $Fv = 841 \times 3 = 2523$ W

Rate of working = 2520 W (3 s.f.) (2.52 kW)

(iii) Newton's 2nd law down the slope:

$F + 450g \sin\beta - 300 = 450a$

$F + 450 \times 9.8 \times 0.05 - 300 = 450 \times 0.4$

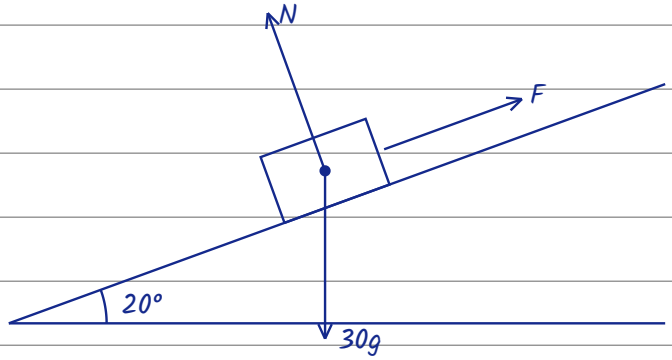
$F + 220.5 - 300 = 180$

$F = 259.5$ N

$v = \frac{P}{F} = \frac{2000}{259.5} = 7.707...$

$v = 7.71$ m s^{-1} (3 s.f.)

AS (i)



F is friction, acting up the slope (suitcase slides down).

(ii) Perpendicular to slope: $N = 30g \cos 20°$

Sliding, so $F = \mu N = 0.3 \times 30 \times 9.8 \times \cos 20°$

Work done by friction = $-F \times$ distance

$= -0.3 \times 30 \times 9.8 \times \cos 20° \times 4$

$= -331.5...$

$= -332$ J (3 s.f.)

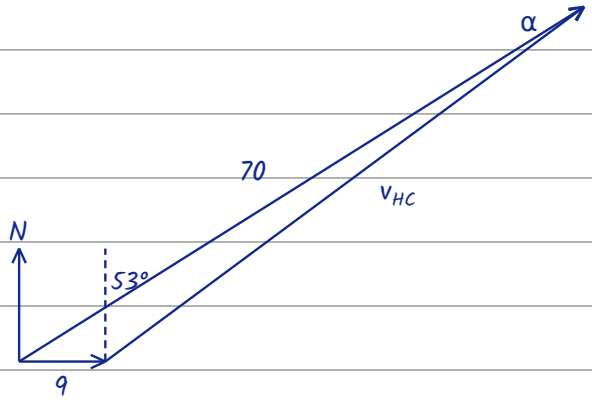
Question number	
(iii)	Work–Energy Principle:
	Work done by resultant force = change in KE
	$30g \sin 20° \times 4 - 331.52 = \frac{1}{2}(30)v^2 - \frac{1}{2}(30)(0.8)^2$
	$402.22 - 331.52 = 15v^2 - 9.6$
	$15v^2 = 80.30$
	$v^2 = 5.353$
	$v = 2.31 \text{ m s}^{-1}$ (3 s.f.)
	Section B Mechanics 2
B1	$M = k\mu^x \rho^y H^z$
	Dimensions: $[M] = M$, $[\mu] = ML^{-1}T^{-1}$, $[\rho] = ML^{-3}$
	$[H] = [\text{power}] = [\text{force} \times \text{velocity}] = MLT^{-2} \times LT^{-1} = ML^2T^{-3}$
	$M = (ML^{-1}T^{-1})^x (ML^{-3})^y (ML^2T^{-3})^z$
	Comparing powers:
	M: $1 = x + y + z$ (1)
	L: $0 = -x - 3y + 2z$ (2)
	T: $0 = -x - 3z$ (3)
	From (3): $x = -3z$
	3(1) + (2): $3 = 2x + 5z$
	$3 = -6z + 5z \Rightarrow z = -3$
	$x = -3(-3) = 9$
	$y = 1 - x - z = 1 - 9 + 3 = -5$
	$x = 9, \ y = -5, \ z = -3$
B2	Velocity of helicopter = velocity of canoe + velocity of

Question number

helicopter relative to canoe: $v_H = v_C + v_{HC}$

v_C = 9 km h^{-1} due East, $|v_H|$ = 70 km h^{-1},

and v_{HC} must be along the bearing 053° (line LC).



Angle between v_C and v_{HC} = 90° − 53° = 37°,

so the angle opposite the 70 side = 180° − 37° = 143°.

Let α = angle between v_H and line LC.

Sine rule: $\frac{\sin \alpha}{9} = \frac{\sin 143°}{70}$

$\sin \alpha = \frac{9 \sin 143°}{70} = 0.07738$

$\alpha = 4.44°$

Bearing = 053° + 4.44° = 057.4°

The helicopter should fly on a bearing of 057° (nearest degree).

B3 (i) Let m kg be the mass of an object on the Earth's surface.

Its weight equals the gravitational attraction of the Earth:

$mg = \frac{GMm}{R^2}$

$MG = gR^2 = 9.8 \times (6\ 378\ 000)^2$

$= 3.9865 \times 10^{14}\ m^3\ s^{-2}$ as required

Question number

(ii) Let r m be the radius of the orbit, ω its angular speed.

Period $T = 90 \times 60 = 5400$ s

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{5400} = \frac{\pi}{2700} \text{ rad s}^{-1}$$

Central acceleration $= \omega^2 r$

Newton's 2nd law towards the centre of the Earth:

$$m\omega^2 r = \frac{GMm}{r^2}$$

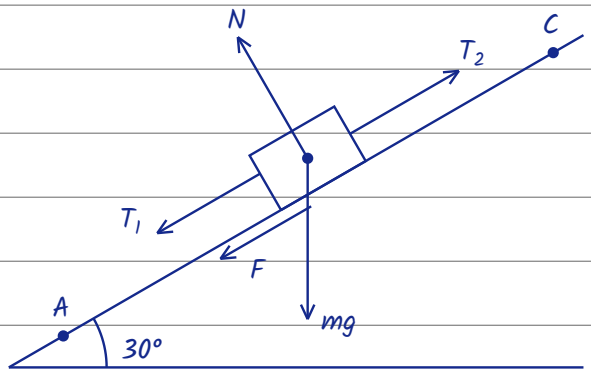
$$r^3 = \frac{MG}{\omega^2} = \frac{3.9865 \times 10^{14}}{(\pi/2700)^2}$$

$r^3 = 2.9446 \times 10^{20}$

$r = 6\,652\,848$ m $= 6653$ km

Height $= 6653 - 6378 = 275$ km (3 s.f.)

B4 (i)



T_1: tension in AB, T_2: tension in BC, F: friction acting down the slope (block about to move up), N: normal reaction.

(ii) AB $= 0.1 + x$, so BC $= 0.9 - (0.1 + x) = 0.8 - x$

Extension of BC $= (0.8 - x) - 0.4 = 0.4 - x$

Question number

$T_2 = \dfrac{\lambda \times \text{extension}}{l} = \dfrac{k_2 mg(0.4 - x)}{0.4}$

$= k_2 mg(1 - 2.5x)$

$= \frac{1}{2} k_2 mg(2 - 5x)$ newtons as required

(iii)

$T_1 = \dfrac{k_1 mg\, x}{0.1} = 10 k_1 mgx$

Perpendicular to slope: $N = mg \cos 30°$

Limiting friction $F = \mu N = \frac{1}{\sqrt{3}} \times mg \times \frac{\sqrt{3}}{2} = \frac{1}{2} mg$

Component of weight down slope $= mg \sin 30° = \frac{1}{2} mg$

Resolving along the slope (equilibrium):

$T_2 = T_1 + F + mg \sin 30°$

$\frac{1}{2} k_2 mg(2 - 5x) = 10 k_1 mgx + \frac{1}{2} mg + \frac{1}{2} mg$

$k_2 - \frac{5}{2} k_2 x = 10 k_1 x + 1$

$k_2 - 1 = 10 k_1 x + \frac{5}{2} k_2 x$

Multiply by 2: $2(k_2 - 1) = x(20k_1 + 5k_2) = 5x(4k_1 + k_2)$

$x = \dfrac{2(k_2 - 1)}{5(4k_1 + k_2)}$ as required

BS (i)

Height of A above B $= 2r$. C is $r \cos\theta$ above O,

and O is r above B, so height of C above B $= r + r \cos\theta$.

GPE at C $= mg \times r(1 + \cos\theta) = mgr(1 + \cos\theta)$ joules as required

Question number

(ii) Energy at A = KE + GPE = 0 + mg(2r) = 2mgr

Energy at C = KE + mgr(1 + cos θ)

The drum is smooth, so mechanical energy is conserved:

KE + mgr(1 + cos θ) = 2mgr

KE = 2mgr − mgr − mgr cos θ

KE = mgr(1 − cos θ) joules

(iii) Acceleration towards O = $\frac{v^2}{r}$

Newton's 2nd law towards O (N acts away from O):

$mg\cos\theta - N = \frac{mv^2}{r}$

From (ii): $\frac{1}{2}mv^2 = mgr(1 - \cos\theta) \Rightarrow \frac{mv^2}{r} = 2mg(1 - \cos\theta)$

$mg\cos\theta - N = 2mg(1 - \cos\theta)$

$N = mg\cos\theta - 2mg + 2mg\cos\theta$

$N = mg(3\cos\theta - 2)$ newtons as required

(iv) The car leaves the drum when N = 0:

$mg(3\cos\theta - 2) = 0$

$\cos\theta = \frac{2}{3}$

Angle AOD = 48.2° (3 s.f.)

Section C Statistics

Question number

C1 (i)

$S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n} = 382.6 - \frac{100 \times 39.1}{8} = -106.15$

$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 1610 - \frac{100^2}{8} = 360$

$b = \frac{S_{xy}}{S_{xx}} = \frac{-106.15}{360} = -0.29486$

$\bar{x} = \frac{100}{8} = 12.5, \quad \bar{y} = \frac{39.1}{8} = 4.8875$

$a = \bar{y} - b\bar{x} = 4.8875 + 0.29486 \times 12.5 = 8.5732$

$y = 8.57 - 0.295x$ (3 s.f.)

(ii)

30 000 miles $\Rightarrow x = 30$

$y = 8.5732 - 0.29486 \times 30 = -0.273$ mm (3 s.f.)

A negative tread depth is impossible. $x = 30$ lies outside the data (extrapolation), so the estimate is unreliable; the linear relationship may not hold at large distances.

C2 (i)

Treat each of the 20 hockey clubs as a cluster.

Randomly select a pre-decided number of clubs (clusters), eg number the clubs 1 to 20 and use random numbers.

Decide on the total sample size and the proportion of supporters to be taken from each chosen club.

From each selected club's database, take a random sample of supporters in proportion to the size of the club.

Advantage: interviews only take place at the selected clubs, reducing travel costs and the number of interview days.

Disadvantage: supporters of the same club may hold similar views, so the sample may not represent the full range of

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views across all of NI.

(ii) Simple random sampling of all supporters in NI.

Less suitable: the sample would probably include supporters from many more clubs, so many more grounds must be visited, increasing the cost; a full sampling frame of all supporters would also need to be built.

C3 (i) X = number of magnitude 8.0+ earthquakes in a year, $X \sim Po(2)$

$P(X > 2) = 1 - P(X \leq 2)$

$= 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$

$= 1 - e^{-2}\left(\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!}\right)$

$= 1 - 5e^{-2} = 1 - 0.67668$

$= 0.323$ (3 s.f.)

(ii) Y = number of magnitude 9.0+ earthquakes in a year, $Y \sim Po(\lambda)$

$P(Y = 0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda}$

$e^{-\lambda} = 0.25$

$-\lambda = \ln 0.25$

$\lambda = -\ln 0.25 = \ln 4 = 1.39$ per year (3 s.f.)

C4 (i) Number of ways $= {}^{13}C_5 = 1287$

(ii) More seniors than juniors: (5S, 0J) or (4S, 1J) or (3S, 2J)

$= {}^6C_5 \times {}^7C_0 + {}^6C_4 \times {}^7C_1 + {}^6C_3 \times {}^7C_2$

$= 6 \times 1 + 15 \times 7 + 20 \times 21$

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$= 6 + 105 + 420 = 531$

$P(\text{more seniors}) = \frac{531}{1287} = \frac{59}{143} = 0.413$ (3 s.f.)

(iii) Number of arrangements $= 5! = 120$

(iv) Treat the two juniors as a single item: 4 items to arrange

in $4! = 24$ ways.

The two juniors can swap places within the pair: $2! = 2$ ways.

Number of arrangements $= 2 \times 4! = 48$

C5 (i) $f(x)$ is continuous at $x = 0.5$ (from the graph):

$9(0.5)^2 = a(0.5) + \frac{17}{4}$

$2.25 = 0.5a + 4.25$

$0.5a = -2 \Rightarrow a = -4$ as required

(ii) $P(0 < X < 0.5) = \int_0^{0.5} 9x^2 \, dx$

$= [3x^3]_0^{0.5}$

$= 3(0.5^3 - 0^3) = 3 \times 0.125$

$= 0.375$ as required

(iii) $0.375 < 0.5$, so the median M lies in $0.5 < x \leq 1$.

Need $P(0.5 < X < M) = 0.5 - 0.375 = 0.125$

$\int_{0.5}^{M} \left(-4x + \frac{17}{4}\right) dx = 0.125$

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$\left[-2x^2 + \frac{17}{4}x\right]_{0.5}^{M} = 0.125$

$-2M^2 + \frac{17}{4}M - \left(-2(0.5)^2 + \frac{17}{4}(0.5)\right) = 0.125$

$-2M^2 + \frac{17}{4}M - \frac{13}{8} = \frac{1}{8}$

$2M^2 - \frac{17}{4}M + \frac{7}{4} = 0$

$8M^2 - 17M + 7 = 0$

$M = \frac{17 \pm \sqrt{17^2 - 4(8)(7)}}{16} = \frac{17 \pm \sqrt{65}}{16}$

$M = 1.57$ (rejected, > 1) or $M = 0.5586...$

Median = 0.559 (3 s.f.)

Section D Discrete and Decision Mathematics

D1 (i) a e b c d f g h i a

(ii) Degrees: a 2, b 2, c 4, d 3, e 4, f 4, g 3, h 4, i 4

d and g are the only odd vertices, so add the edge dg.

The two vertices are d and g.

(iii) G^* has 16 edges, all vertices even. An Eulerian circuit is

e b c d i a e c f d g i h g f h e

D2 (i) Substitute $L_n = kx^n$ into $L_{n+2} = L_{n+1} + L_n$:

$kx^{n+2} = kx^{n+1} + kx^n$

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$kx^n(x^2 - x - 1) = 0$

For a non-trivial solution ($k \neq 0$, $x \neq 0$):

$x^2 - x - 1 = 0$ as required

(ii)

$$x = \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

General solution: $L_n = A\left(\frac{1 + \sqrt{5}}{2}\right)^n + B\left(\frac{1 - \sqrt{5}}{2}\right)^n$

$n = 0$: $L_0 = 2 = A + B$

$n = 1$: $L_1 = 1 = \frac{A(1 + \sqrt{5})}{2} + \frac{B(1 - \sqrt{5})}{2}$

So $2 = A(1 + \sqrt{5}) + B(1 - \sqrt{5})$

Substitute $B = 2 - A$:

$2 = A(1 + \sqrt{5}) + (2 - A)(1 - \sqrt{5})$

$2 = A + A\sqrt{5} + 2 - 2\sqrt{5} - A + A\sqrt{5}$

$0 = 2\sqrt{5}A - 2\sqrt{5} \Rightarrow A = 1, B = 1$

$$L_n = \left(\frac{1 + \sqrt{5}}{2}\right)^n + \left(\frac{1 - \sqrt{5}}{2}\right)^n$$

i.e. $L_n = \alpha^n + \beta^n$ with $\alpha = \frac{1 + \sqrt{5}}{2}$, $\beta = \frac{1 - \sqrt{5}}{2}$

(iii)

$$|\beta| = \left|\frac{1 - \sqrt{5}}{2}\right| = |-0.618...| < 1$$

so $\beta^n \to 0$ as $n \to \infty$

$\therefore L_n \to \left(\frac{1 + \sqrt{5}}{2}\right)^n = 2^{-n}(1 + \sqrt{5})^n$ as required

D3 (i)

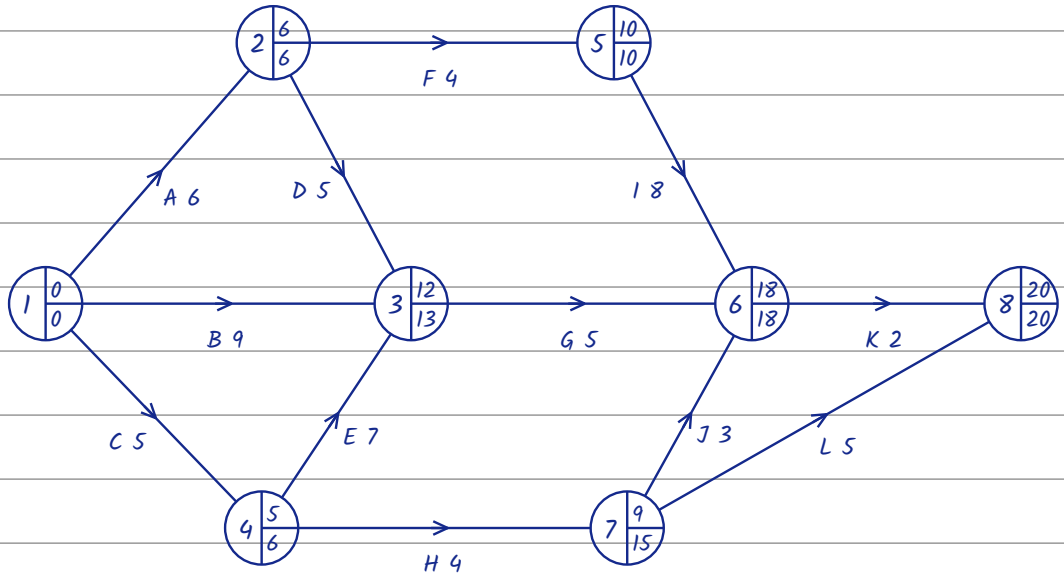
Early time at a node = the largest value of (early time of a

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preceding node + duration of the activity joining them).

Node 3: max(6 + 5, 0 + 9, 5 + 7) = max(11, 9, 12) = 12

(ii) Copy of Fig. 2 with early | late times (written in the Supplementary Answer Booklet):



Event	1	2	3	4	5	6	7	8
Early	0	6	12	5	10	18	9	20
Late	0	6	13	6	10	18	15	20

(iii)(a) The float time of H is the maximum amount of time by which activity H can be delayed (or extended) without delaying the completion of the whole project.

(b) Float = late time at end − early time at start − duration

Float for H = $L_7 - E_4 - 4 = 15 - 5 - 4 = 6$ days

(iv) Critical activities (zero float): A, F, I, K

Critical path 1 → 2 → 5 → 6 → 8, length 6 + 4 + 8 + 2 = 20 days

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D4

Columns: (7) ~p or r (8) (~p or r) and q (9) ~[(~p or r) and q]

(10) p and ~r (11) (p and ~r) or ~q

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
p	q	r	~p	~q	~r	(7)	(8)	(9)	(10)	(11)
T	T	T	F	F	F	T	T	F	F	F
T	T	F	F	F	T	F	F	T	T	T
T	F	T	F	T	F	T	F	T	F	T
T	F	F	F	T	T	F	F	T	T	T
F	T	T	T	F	F	T	T	F	F	F
F	T	F	T	F	T	T	T	F	F	F
F	F	T	T	T	F	T	F	T	F	T
F	F	F	T	T	T	T	F	T	F	T

Columns (9) and (11) are identical in every row, so

~[(~p or r) and q] ≡ (p and ~r) or ~q

D5 (i)

From the table: ab = ab (row a, column b)

ba = a^3b (row b, column a)

$a^3b \neq ab$, so $ab \neq ba$

∴ G is not commutative.

(ii)

Element	1	a	a^2	a^3	b	ab	a^2b	a^3b
Period	1	4	2	4	2	2	2	2

(eg a, a^2, $a^3 \neq 1$ and $a^4 = 1$, so a has period 4; $b^2 = 1$ so b has period 2)

(iii)

$\{1, a, a^2, a^3\}$ and $\{1, a^2, b, a^2b\}$

(iv)

Element	Period

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Identity	1
Rotation of 180° about the centre	2
Reflection in the line PT	2
Reflection in the line RV	2

(v) H is isomorphic to $\{1, a^2, b, a^2b\}$.

Reason: both have the identity plus three elements of period 2,

whereas $\{1, a, a^2, a^3\}$ contains elements of period 4 (a, a^3),

and H has no element of period 4.

eg $1 \leftrightarrow$ identity, $a^2 \leftrightarrow$ rotation 180°, $b \leftrightarrow$ reflection in PT,

$a^2b \leftrightarrow$ reflection in RV.

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