New Specification

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education

Further Mathematics

Assessment Unit AS 2
assessing
Applied Mathematics

[SFM21]

Assessment

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) FURTHER MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

Section A

1. MW1 Correct resultant
   M1 Attempting scalar product
   W1 Correctly
   W1 Correct answer

   Notes
   [A] If use vector product, then MW1 only i.e. max of 1/4
   [B] If use product of magnitudes of **F, s** then MW1 only i.e. max of 1/4

2. (i) MW1 Correct normal reaction
   MW1 Correct weight

   Notes
   [A] Any additional forces then lose 1 mark
   [B] No arrows, lose each mark

   (ii) M1 Attempting to resolve vertically
   W1 Correct equation
   W1 Correct answer

   (iii) M1 Attempting to resolve horizontally
   M1 Using $F = ma$ with correct formula for acceleration
   W1 Correct LHS
   W1 Correct RHS
   W1 Correct radius

   Notes
   [A] Any extra forces included e.g. friction, then can award
   M1 M1 W0 W1 W0 i.e. max of 3/5

   (iv) MW1 Correct statement

3. (a) M1 Attempting to integrate
   W1 Correct limits
   MW1 Correct integrand
   W1 Correct expression

   (b) MW1 KEs zero
   MW1 Remaining energies at P correct
   M1 Attempting GPE using correct formula
   W1 Correct value
   M1 Attempting EPE using correct formula
   W1 Correct value
   M1 Using conservation of energy (ft their values for energies at P, Q)
   W1 PQ correct

4. (i) MW1 Speed conversion correct
MW1 Force correct
M1 Correct answer

Notes
[A] No conversion of speed – can award MW0 MW1 M0 i.e. max of 1/3

(ii) M1 Attempting to resolve parallel to plane
W1 Correctly
M1 Trying to use Power = Force × Velocity
W1 Correct power

(iii) M1 Attempting Newton's 2nd law
W1 Correct $m \times a$
W1 Correct forces
W1 Correct propulsion force
MW1 Correct velocity

5. (i) MW1 Force diagram correct

Notes
[A] Weight must be given as 30g – not just W
[B] No arrows on forces then MW0

(ii) M1 Trying to use $Fr = \mu N$
MW1 Correct normal reaction
M1 Trying to use Work Done by Friction = $Fr \times$ displacement
W1 Correct answer

(iii) M1 Trying to use WD = Change in KE
M1 Attempting difference of forces
W1 LHS correct
MW1 Initial KE correct
MW1 Final KE correct
W1 Correct answer

Section B

1. M1 Trying to use dimensions
W1 Correct set-up
M1 Trying to find dimensions – evidence of one correct
W1 All dimensions correct
M1 Knowing to compare powers
W1 3 correct equations
M1 Trying to solve equations
W1 Correct answers

2.
M1 Correctly combining relative velocities
W1 Directions of vectors correct
W1 Angles correct
M1 Trying to use sine rule
W1 Correctly substituted values
MW1 Correct angle
W1 Correct bearing

3. (i)
M1 Trying to use Newton's law = weight on earth's surface
M1 Correct law of gravitation
W1 Correct answer

(ii)
W1 Correct change of units
M1 Using correct formula for period
W1 Correct ω
M1 Correct expression for central acceleration
MW1 Correct Newton's 2nd law
MW1 Correct r^3
W1 Correct r
W1 Correct height

4. (i)
MW1 Correct diagram

(ii)
MW1 Correct extension
M1 Attempting correct expression for tension
W1 Correct answer

(iii)
MW1 Correct tension
M1 Trying to use $Fr = \mu N$
W1 Correct
MW1 Correct component of weight
M1 Trying to equate forces along plane
W1 Correct answer

5. (i)
MW1 Correct vertical distance
MW1 Correct PE

(ii)
M1 Trying to use E = KE + PE
W1 Correct energy at C
MW1 Correct energy at A
M1 Trying to use conservation of energy
W1 Correct answer

(iii)
M1 Correct expression for acceleration
M1 Attempting Newton's 2nd law
W1 Correctly
W1 Correctly substitutes for mv^2 (allow ft of their answer from (ii))
W1 Correct N

(iv)
M1 Knowing reaction = 0
W1 Correct angle

Section C

1. (i) M1 Attempting to use formula for b
MW1 Correctly substituted
W1 Correct b
MW1 Correct $\overline{x}$
MW1 Correct $\overline{y}$
MW1 Correct a
W1 Correct equation

(ii) M1 Attempting to find y
W1 Correct y
MW1 One valid comment

2. (i) MW1 Correct statement about randomly selecting clusters
MW1 Correct statement about random selection from each club
MW1 Correct statement referencing proportion selected
MW1 Correct statement about travelling costs or number of interview days
MW1 One valid comment

(ii) MW1 One valid technique
MW1 One valid reason

3. (i) M1 Correct $\lambda = 2$
M1 Using $1 - P(X \leq 2)$
MW1 Correct terms used (or evidence of correct value from tables/calculator)
W1 Correct answer

(ii) MW1 Correct substitution into correct formula
W1 Correct equation
M1 Attempting to use logs
W1 Correct answer

4. (i) M1 Trying to use combinations
W1 Correct answer

(ii) MW1 1st correct product
MW1 2nd correct product
MW1 3rd correct product
W1 Correct answer
MW1 Correct probability

(iii) MW1 Correct answer

(iv) M1 Treating 2 juniors as a single item
W1 Obtaining 4!
MW1 Correct answer

5. (i) M1 Trying to equate functions at $x = 0.5$
W1 Correct a

(ii) MW1 Correct integral (integrand and limits)
MW1 Correct integration
W1 Correct answer

(iii) MW1 Correct 0.125
M1 Correct integral
MW1 Correct limits
MW1 Correctly integrated
MW1 Correct substitutions
W1 Correct quadratic equation
M1 Attempting to solve quadratic
W1 2 correct solutions
W1 Correct answer selected
(if only correct answer stated then award final W2)

Alternative Solution
(iii) MW1 Knowing area of trapezium = 0.125
M1 Trying to find area of trapezium
MW1 Using $(M - 0.5)$
MW1 Using $(-4M + \frac{17}{4})$
MW1 Expanding double brackets
Final 4 marks as in original solution

Section D

1. (i) M1 Attempting closed tour of vertices
W1 Stating every vertex correctly

(ii) MW1 Edge from d (or g) identified
MW1 Correct edge

(iii) M1 Attempting tour of edges
W1 1st half correct
W1 All correct

2. (i) M1 Attempting to form auxiliary equation
W1 Correctly

(ii) MW1 Correctly solved quadratic
MW1 Forms expression correctly
M1 Attempts to equate 1st 2 terms
W1 Correctly
M1 Attempts to solve simultaneous equations
W1 Correct values
W1 Correct answer

(iii) M1 Checks modulus of value <1
W1 States tends to zero

3. (i) MW1 Correct answer

(ii) MW1 E1, E2, E4, E5, E7 correct
MW1 E3, E6, E8 correct
MW1 L8, L6, L5 correct
MW1 Other Ls correct

(iii) (a) M1 Correct answer
(b) M1 Tries to subtract early from late time
W1 Correct answer

(iv) MW1 Correct critical path
MW1 Correct answer

4. M1 Attempts set up for 1st 3 columns
W1 Correct
MW1 Columns 4, 5, 6 correct
MW5 1 mark for each of columns 7 – 11 correct
MW1 States column 9 = column 11

5. (i) MW1 Compares 2 suitable products
M1 States not commutative

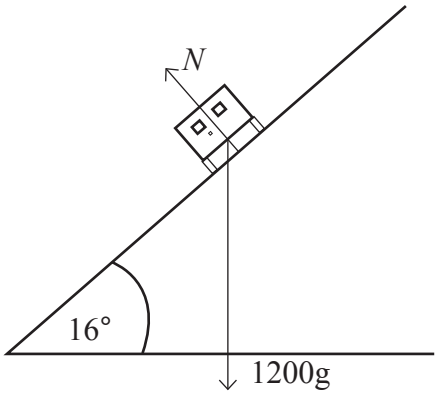
(ii) MW1 Period of identity and two order 2 elements correct
MW1 Other order 2 elements correct
MW1 Order 4 elements correct

(iii) MW2 1 mark for each subgroup

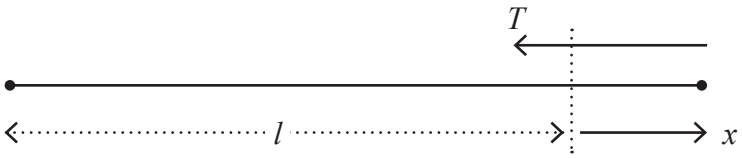
(iv) MW4 1 mark for each element and period correct

(v) MW1 Identify correct subgroup
M1 Correct reason

SECTION A: Mechanics 1

			AVAILABLE MARKS
1	Resultant $= \begin{pmatrix} 5 \\ 6 \\ -7 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ -9 \end{pmatrix}$	MW1	
	Work done $= \mathbf{F.r} = \begin{pmatrix} 8 \\ 5 \\ -9 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix}$	M1 W1	
	$= -18\,\text{J}$	W1	4
2	**(i)** 	MW2	
	(ii) $N\cos 16° = 1200 \times 9.8$	M1 W1	
	$N = 12\,233.9 = 12\,200\,\text{N}$ (3 s.f.)	W1	
	(iii) $N\sin\theta = \dfrac{mv^2}{r}$	M2	
	$N\sin 16° = \dfrac{1200 \times 14^2}{r}$	W2	
	$r = 69.7\,\text{m}$ (3 s.f.)	W1	
	(iv) Treat the car as a particle.	MW1	11

3 (a)

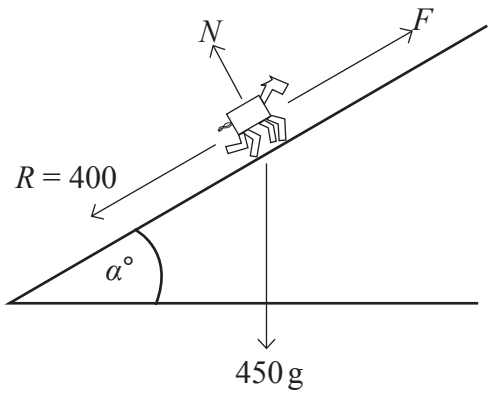


		AVAILABLE MARKS
Work done by tension $= \int_0^e -T\,dx$	M1 W1	
$= \int_0^e -\left(\frac{\lambda x}{l}\right) dx$	MW1	
$= -\frac{\lambda e^2}{2l}$ J	W1	
(b) KE at P = 0 and KE at Q = 0	MW1	
When at P: Gravitational PE = 0 Elastic PE = 0	MW1	
When at Q: Gravitational PE $= -mgh = -5 \times 9.8 \times (1.7 + e)$	M1 W1	
Elastic PE $= \frac{\lambda e^2}{2l} = \frac{98 \times e^2}{2 \times 1.7}$	M1 W1	
By Conservation of Mechanical Energy: Total energy at P = Total energy at Q	M1	
$0 = \frac{49e^2}{1.7} - 49(1.7 + e)$		
$e^2 - 1.7e - 2.89 = 0$ $e = 2.75$ m PQ = 4.45 m (3 s.f.)	W1	12

4 Running force = F
Resistance = R

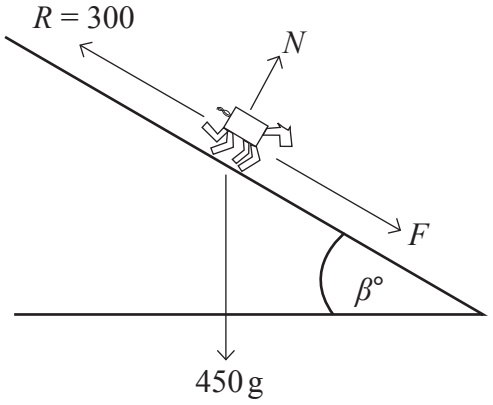
		AVAILABLE MARKS
(i) Speed = $18/3.6 = 5\,\text{m s}^{-1}$	MW1	
Running force $F = 2000/5 = 400\,\text{N}$	MW1	
Constant velocity $\Rightarrow R = F = 400\,\text{N}$	M1	

(ii)



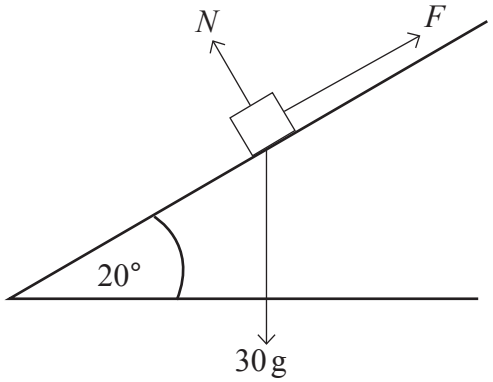
$F = R + 450\,\text{g}\sin\alpha$	M1 W1	
Speed $= 10.8/3.6 = 3\,\text{m s}^{-1}$		
Power $= 3(400 + 45 \times 9.8)$	M1	
Power $= 2523\,\text{W} \rightarrow 2520\,\text{W}$ (3 s.f.)	W1	

(iii)



$450 \times 0.4 = 450\,\text{g} \times 0.05 - 300 + F$	M1 W2	
$F = 259.5\,\text{N}$	W1	
$v = 2000/259.5 = 7.707\,\text{m s}^{-1}$		
$v = 7.71\,\text{m s}^{-1}$ (3 s.f.)	MW1	12

5 (i)



MW1

(ii) Friction = μN M1

$N = mg\cos\alpha = 30 \times 9.8 \times \cos 20°$ MW1

Work done by friction = force × displacement M1

$= -0.3 \times 30 \times 9.8 \times \cos 20° \times 4$

$= -331.524$

$= -332\,\text{J}$ (3 s.f.) W1

(iii) Work done by resultant force = change in kinetic energy M1

$mg\sin 20° \times 4 - 331.5 = \frac{1}{2}mv^2 - \frac{1}{2}m \times 0.8^2$ M1 W1 MW2

$30 \times 9.8 \times \sin 20° \times 4 - 331.5 = \frac{1}{2} \times 30 \times v^2 - \frac{1}{2} \times 30 \times 0.8^2$

$v = 2.31\,\text{m s}^{-1}$ (3 s.f.) W1 11

Section A **50**

AVAILABLE MARKS

SECTION B: Mechanics 2

AVAILABLE MARKS

1 $M = k\,\mu^x\,\rho^y\,H^z$

Taking dimensions — M1 W1

$M = (ML^{-1}T^{-1})^x(ML^{-3})^y(ML^2T^{-3})^z$ — M1 W1

Equating indices:

M: $1 = x + y + z$ (1) — M1

L: $0 = -x - 3y + 2z$ (2)

T: $0 = -x - 3z$ (3) — W1

Thus $x = -3z$

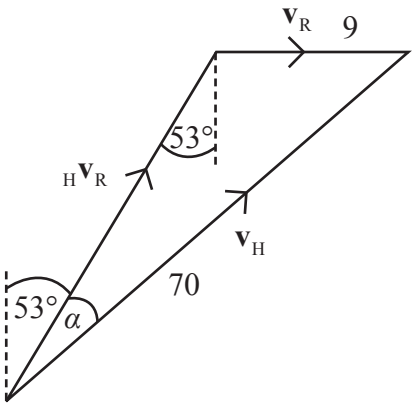
$3(1) + (2)$ gives $3 = 2x + 5z$ — M1

Thus $z = -3$ and $x = 9$

$y = 1 - x - z = -5$ — W1 — 8

$M = k\,\mu^9\,\rho^{-5}\,H^{-3}$

2



M1 W1 W1

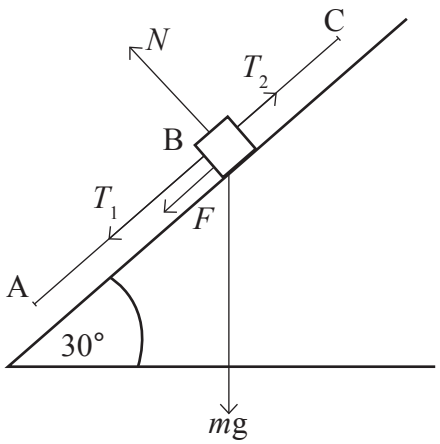
$\frac{\sin\alpha}{9} = \frac{\sin 143°}{70}$ — M1 W1

$\alpha = \sin^{-1}\left(\frac{9 \times \sin 143°}{70}\right) = 4.4378°$ — MW1

Bearing = 057° (= N 57.4°E) — W1 — 7

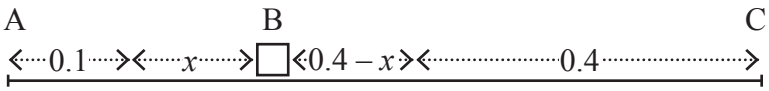
			AVAILABLE MARKS
3	**(i)** Let the mass of the satellite be m kg		
	On the earth's surface $mg = \frac{mMG}{R^2}$	M2	
	$MG = gR^2 = 9.8 \times 6\,378\,000^2 = 3.9865 \times 10^{14}\ m^3s^{-2}$	W1	
	(ii) Let r metres be the radius of the satellite's orbit.		
	Angular velocity of the satellite = ω		
	Period of the satellite in seconds = $90 \times 60 = 5400$ s	W1	
	Thus $5400 = \frac{2\pi}{\omega}$ where ω is the angular velocity of the earth	M1	
	So $\omega = \frac{\pi}{2700}$ radians per second	W1	
	Central acceleration of the satellite = $\omega^2 r$	M1	
	By Newton's 2nd Law		
	$m\omega^2 r = \frac{mMG}{r^2}$	MW1	
	$r^3 = \frac{MG}{\omega^2}$	MW1	
	$r = 6653$ km	W1	
	Height above the earth's surface = $6653 - 6378 = 275$ km	W1	11

4 **(i)**



MW1

(ii)



Extension in BC = $0.9 - 0.1 - 0.4 - x = 0.4 - x$ MW1

Tension in BC = $k_2 mg \dfrac{(0.4 - x)}{0.4} = \dfrac{1}{2} k_2 mg(2 - 5x)$ M1 W1

(iii) Tension in AB = $k_1 mg \dfrac{x}{0.1} = 10 k_1 mgx$ MW1

Friction = $\mu mg \cos 30° = \dfrac{1}{\sqrt{3}} mg \cos 30° = \dfrac{1}{2} mg$ M1 W1

Component of weight along plane = $mg \sin 30° = \dfrac{1}{2} mg$ MW1

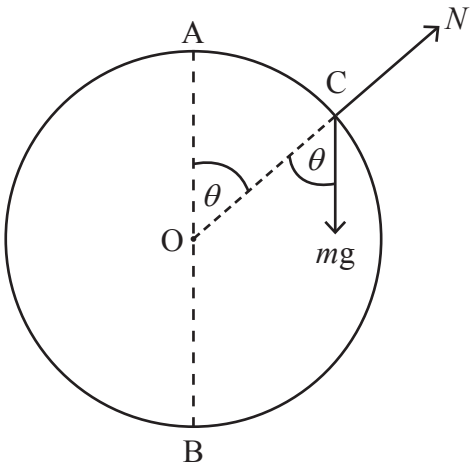
$10 k_1 mgx + \dfrac{1}{2} mg + \dfrac{1}{2} mg = \dfrac{1}{2} k_2 mg(2 - 5x)$ M1

$10 k_1 mgx + \dfrac{5}{2} k_2 mgx = k_2 mg - mg$

$10 k_1 x + \dfrac{5}{2} k_2 x = k_2 - 1$

$x = \dfrac{2(k_2 - 1)}{5(4k_1 + k_2)}$ W1 10

AVAILABLE MARKS

			AVAILABLE MARKS
5	**(i)** 		
	Vertical drop from C to B = $r + r\cos\theta$	MW1	
	Potential energy at C = $mgr(1 + \cos\theta)$	MW1	
(ii)	Energy of car at C = KE + $mgr(1 + \cos\theta)$	M1 W1	
	Energy of car at A = $0 + 2mgr$	MW1	
	KE = $mgr(1 - \cos\theta)$J	M1 W1	
(iii)	Expression for radial acceleration = $\frac{mv^2}{r}$	M1	
	Newton's 2nd Law		
	$\frac{mv^2}{r} = mg\cos\theta - N$	M1 W1	
	Using **(ii)** $2mg(1 - \cos\theta) = mg\cos\theta - N$	W1	
	$N = mg(3\cos\theta - 2)$ newtons	W1	
(iv)	Loss of contact from drum $\Rightarrow N = 0$	M1	
	$\cos\theta = \frac{2}{3}$		
	$\theta = 48.2°$ (3 s.f.)	W1	14
		Section B	**50**

SECTION C: Statistics

AVAILABLE MARKS

1 **(i)** $b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{\Sigma xy - \dfrac{(\Sigma x)(\Sigma y)}{n}}{\Sigma x^2 - \dfrac{(\Sigma x)^2}{n}}$ M1

$b = \dfrac{382.6 - \dfrac{100 \times 39.1}{8}}{1610 - \dfrac{100^2}{8}}$ MW1

$b = -\dfrac{2123}{7200} = -0.295$ (3 s.f.) W1

$\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{100}{8} = 12.5$ MW1

$\bar{y} = \dfrac{\Sigma y}{n} = \dfrac{39.1}{8} = 4.8875$ MW1

$a = \bar{y} - b\bar{x} = 4.8875 + \dfrac{2123}{7200}(12.5) = 8.57$ (3 s.f.) MW1

$y = 8.57 - 0.295x$ (3 s.f.) W1

(ii) When $x = 30$ $y = a + b(30)$ M1

$y = -0.273$ mm W1

Negative value for tread depth is not possible. Impossible value for y may arise from data that deviates at large distance from the linear relationship. MW1

OR

Negative value for tread depth is not possible. Values estimated by extrapolation can be unreliable. MW1 10

				AVAILABLE MARKS
2	**(i)**	Treat each hockey club as a cluster. Randomly select a pre-decided number of clusters (clubs).	MW1	
		Decide on the sample size and on the proportion of supporters to be selected from each chosen club.	MW1	
		Take a random sample of supporters from each club selected, in proportion to the relative size of the club.	MW1	
		Advantage: Interviews only need to be conducted at a sample of clubs. This reduces travelling costs and the number of days required for interviews, if, for example, all the interviews from a particular club can be scheduled for a single day.	MW1	
		Disadvantage: There will be similarities between supporters within a given club. Therefore, the range of views in the sample may not be representative of the full range of views in the population.	MW1	
	(ii)	Answer: Simple random sampling	MW1	
		Reason: The number of clubs represented in the sample is likely to be greater when compared with cluster sampling. Therefore, there will be a greater cost associated with visiting clubs to conduct interviews. There will also be a cost associated with constructing the complete sampling frame.	MW1	
		OR		
		Answer: Stratified sampling, where each club forms a separate stratum	MW1	
		Reason: Each club (stratum) would need to be visited to conduct interviews, increasing the cost compared with cluster sampling. There will also be a cost associated with constructing the complete sampling frame.	MW1	7
3	**(i)**	X is the random variable 'number of earthquakes of magnitude 8.0 or greater per year'		
		$X \sim \text{Po}(2)$	M1	
		$\text{P}(X > 2) = 1 - \text{P}(X \leqslant 2)$ $= 1 - [\text{P}(X = 0) + \text{P}(X = 1) + \text{P}(X = 2)]$	M1	
		$= 1 - \text{e}^{-2}\left(\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!}\right)$	MW1	
		$= 0.323$ (3 s.f.)	W1	
	(ii)	Y is the random variable 'number of earthquakes of magnitude 9.0 or greater per year'		
		$Y \sim \text{Po}(\lambda)$		
		$\text{P}(Y = 0) = \frac{\text{e}^{-\lambda}\lambda^0}{0!}$	MW1	
		$0.25 = \text{e}^{-\lambda}$	W1	
		$-\ln 0.25 = \lambda$	M1	
		$\lambda = 1.39$ (3 s.f.)	W1	8

4 **(i)** ${}^{13}C_5 = 1287$ M1 W1

(ii) (5S and 0J) or (4S and 1J) or (3S and 2J)
$= ({}^6C_5 \times {}^7C_0) + ({}^6C_4 \times {}^7C_1) + ({}^6C_3 \times {}^7C_2)$ MW3
$= 531$ W1

$P(S > J) = \frac{531}{1287} = \frac{59}{143} = 0.413$ (3 s.f.) MW1

(iii) $5! = 120$ MW1

(iv) Treating two juniors as a single item gives 4! arrangements M1 W1
The two juniors can be arranged in two different ways, so the total number of arrangements $= 2 \times 4!$
$= 48$ MW1

AVAILABLE MARKS

11

5 **(i)** $9(0.5)^2 = a(0.5) + \frac{17}{4}$ M1

$a = -4$ W1

(ii) $P(0 < X < 0.5) = \int_0^{0.5} 9x^2 \, dx$ MW1

$= 3[x^3]_0^{0.5}$ MW1
$= 3(0.5^3 - 0^3)$
$= 0.375$ W1

(iii) $0.5 - 0.375 = 0.125$ MW1

$\int_{0.5}^{M} -4x + \frac{17}{4} dx = 0.125$ M1 MW1

$\left[-2x^2 + \frac{17}{4}x\right]_{0.5}^{M} = 0.125$ MW1

$-2M^2 + \frac{17}{4}M - \left[-2(0.5)^2 + \frac{17}{4}(0.5)\right] = 0.125$ MW1

$2M^2 - \frac{17}{4}M + \frac{7}{4} = 0$

$8M^2 - 17M + 7 = 0$ W1

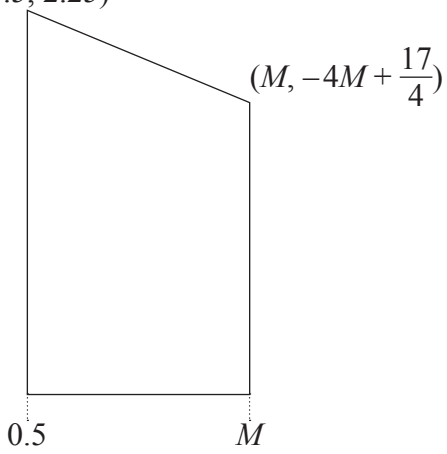
$M = \frac{17 \pm \sqrt{65}}{16}$ M1

$M_1 = 1.57$ (> 1 ∴ ignore) $M_2 = 0.559$ W1
Median = 0.559 (3 s.f.) W1

Alternative solution for part (iii) is given overleaf on page 20

Alternative solution for part (iii)

		AVAILABLE MARKS
$0.5 - 0.375 = 0.125$		
Area of trapezium = 0.125	MW1	
(0.5, 2.25)		



$0.125 = \frac{1}{2}\left(2.25 + \frac{17}{4} - 4M\right)\ (M - 0.5)$	M1 MW2	
$0.25 = 6.5M - 3.25 - 4M^2 + 2M$	MW1	
$8M^2 - 17M + 7 = 0$	W1	
$M = \frac{17 \pm \sqrt{65}}{16}$	M1	
$M_1 = 1.57$ (> 1 ∴ ignore) $\quad M_2 = 0.559$	W1	
Median = 0.559 (3 s.f.)	W1	14
Section C		**50**

SECTION D: Discrete and Decision Mathematics

AVAILABLE MARKS

1 (i) a e b c d f g h i a include all vertices M1
any correct cycle W1

(ii) edge from vertex d to vertex g MW2

(iii) e b c d i a e c f d g i h g f h e
16 edges M1
outside edges W1
inside edges W1
(last 2 marks for any correct circuit)
(need attempt at a full circuit for any marks) 7

2 (i) Substitute $L_n = kx^n$

obtaining $kx^{n+2} = kx^{n+1} + kx^n$ M1

$kx^n(x^2 - x - 1) = 0$
$x^2 - x - 1 = 0$ or $x = 0$ W1

(ii) $x^2 - x - 1 = 0$

$$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$ MW1

$$L_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$$ MW1

$L_0 = 2 = A + B$
and $L_1 = 1 = \frac{A}{2}(1+\sqrt{5}) + \frac{B}{2}(1-\sqrt{5})$ M1 W1

substituting $B = 2 - A$ and solving
$1 = \frac{A}{2}[(1+\sqrt{5}) - (1-\sqrt{5})] + 1 - \sqrt{5}$ M1

$A = 1$, $B = 1$ W1

Thus $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$ W1

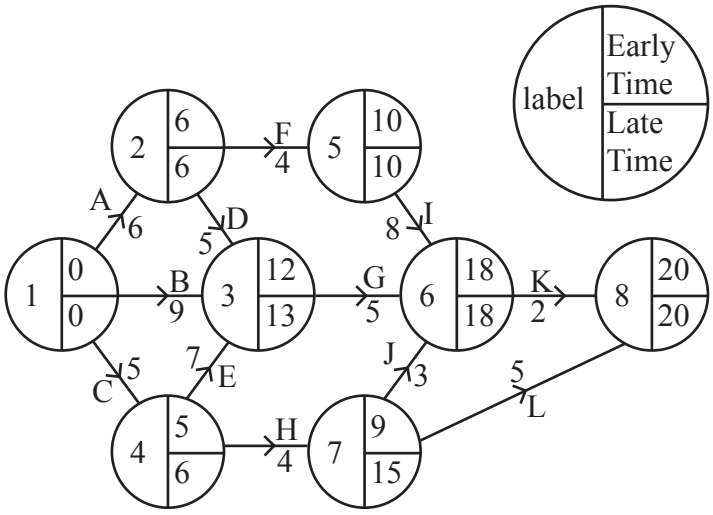
(iii) $\left|\frac{1-\sqrt{5}}{2}\right| = |-0.618| < 1$ M1

$\left(\frac{1-\sqrt{5}}{2}\right)^n \to 0$ as $n \to \infty$

$\therefore L_n \to 2^{-n}(1+\sqrt{5})^n$ W1 11

3 **(i)** Early time of a node is the greatest of the set of early times of preceding nodes plus the length of intervening activities
Thus early time of node 3 = max(6 + 5, 9, 5 + 7) = 12 MW1

(ii)



E_i = early time at node i
L_i = late time at node i
E_1 E_2 E_5 E_4 E_7 MW1
E_3 E_6 E_8 MW1
L_8 L_6 L_5 MW1
rest of L_is MW1

(iii) **(a)** Float time for activity H is the maximum number of days after the end of the preceding activity C by which event H must be started, to be finished in time M1

(b) Float time for H = 15 – 5 – 4 = 6 days M1 W1

(iv) Critical activities A F I K MW1
Length 20 days MW1

10

4

p	***q***	***r***	**~*p***	**~*q***	**~*r***	**~*p* or *r***	**(~*p* or *r*) and *q***	**~[(~*p* or *r*) and *q*]**	***p* and ~*r***	**(*p* and ~*r*) or ~*q***
T	T	T	F	F	F	T	T	F	F	F
T	T	F	F	F	T	F	F	T	T	T
T	F	T	F	T	F	T	F	T	F	T
T	F	F	F	T	T	F	F	T	T	T
F	T	T	T	F	F	T	T	F	F	F
F	T	F	T	F	T	T	T	F	F	F
F	F	T	T	T	F	T	F	T	F	T
F	F	F	T	T	T	T	F	T	F	T
								Column 9		Column 11

Set up for first 3 columns M1 W1
Columns 4, 5, 6 MW1
Columns 7, 8, 9, 10, 11 MW5
Stating column 9 = column 11 M1

9

AVAILABLE MARKS

5 **(i)** $ba = a^3b \neq ab$ MW1

so not commutative M1

AVAILABLE MARKS

(ii)

Element	1	a	a^2	a^3	b	ab	a^2b	a^3b
Period	1	4	2	4	2	2	2	2

MW3

(iii) $(1, a, a^2, a^3)$ and $(1, a^2, b, a^2b)$ MW2

(iv)

Element	Period
Identity	1
Rotation of 180° about centre	2
Reflection in line PT	2
Reflection in line RV	2

MW4

(v) Isomorphic to the following subgroup of G

Element	Period
1	1
a^2	2
b	2
a^2b	2

MW1

Reason – Identical periods or a correct isomorphism M1

13

Section D **50**

Total **100**