New Specification

Rewarding Learning

ADVANCED

General Certificate of Education

Further Mathematics

AFM11

Assessment Unit A2 1

assessing

Pure Mathematics

[AFM11]

Assessment

Assessment Level of Control:

Tick the relevant box (✓)

Controlled Conditions	
Other	

TIME

2 hours 15 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all twelve** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 150
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all twelve questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Show that

$$\sum_{r=1}^{2n} (2r-1)^2 \equiv \frac{2n}{3}(16n^2-1) \qquad n \geqslant 1 \qquad [6]$$

2 **(a)** Using the substitution $u = 2x + 1$ show that

$$\int \frac{\mathrm{d}x}{\sqrt{4x^2+4x+3}} = \frac{1}{2}\ln\left(2x+1+\sqrt{4x^2+4x+3}\right) + c \qquad [6]$$

(b) Evaluate the improper integral

$$\int_0^1 \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x \qquad [4]$$

3 **(i)** Express

$$\mathrm{f}(x) = \frac{2x^2+x+7}{(x-1)(x^2+4)}$$

in partial fractions. [7]

(ii) Hence show that

$$\int \mathrm{f}(x)\,\mathrm{d}x = 2\ln(x-1) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c \qquad [3]$$

4 **(i)** Given that

$$I_n = \int (\ln x)^n \, dx$$

show that for $n \geqslant 1$

$$I_n = x\,(\ln x)^n - n\,I_{n-1}$$ [6]

(ii) Evaluate

$$\int_1^2 (\ln x)^2 \, dx$$ [4]

5 Using mathematical induction, prove that

$$\sum_{r=1}^{n} 3^{r-1} = \frac{3^n - 1}{2} \qquad n \geqslant 1$$ [6]

6 The rate at which a bank's assets grow is modelled by

$$t\,\frac{dA}{dt} - A = t$$

where A represents the assets of the bank in £ millions and t is the number of years since the setting up of the bank.

(i) Given that the assets of the bank are £300 million after 20 years, find a formula expressing A in terms of t. [8]

(ii) Estimate the assets of the bank 50 years after set up. [2]

(iii) State possible limitations of the model. [2]

7 **(a)** **(i)** Given that

$$f(x) = \ln\left(2x + \sqrt{1 + 4x^2}\right)$$

show that

$$f'(x) = \frac{2}{\sqrt{1 + 4x^2}} \qquad [5]$$

(ii) Hence find the Maclaurin expansion of $f(x)$ up to and including the term in x^3 [4]

(b) Using small angle approximations, show that

$$\lim_{x \to 0} \left(\frac{1 - \cos 6x - 2x \sin 3x}{x^2}\right) = 12 \qquad [4]$$

8 **(a)** Using the exponential definitions of hyperbolic functions, prove that

$$\cosh(x + y) \equiv \cosh x \cosh y + \sinh x \sinh y \qquad [6]$$

(b) Solve the equation

$$4 \cosh x + 5 \sinh x = 2$$

leaving the answer in logarithmic form. [7]

9 **(i)** Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 20x - 6 \qquad [10]$$

(ii) Given that $\frac{dy}{dx} = 6$ and $y = 0$, when $x = 0$, find y in terms of x. [5]

10 (a) The polar equation of a curve C is

$$r = 3 \sin 2\theta$$

Find a cartesian equation for C. [6]

(b) **Fig. 1** below shows the graph of the finite region bounded by the half-lines $\theta = 0$ and $\theta = \frac{\pi}{4}$ and the curve with polar equation $r = a(1 + \tan\theta)$ where a is a positive constant.

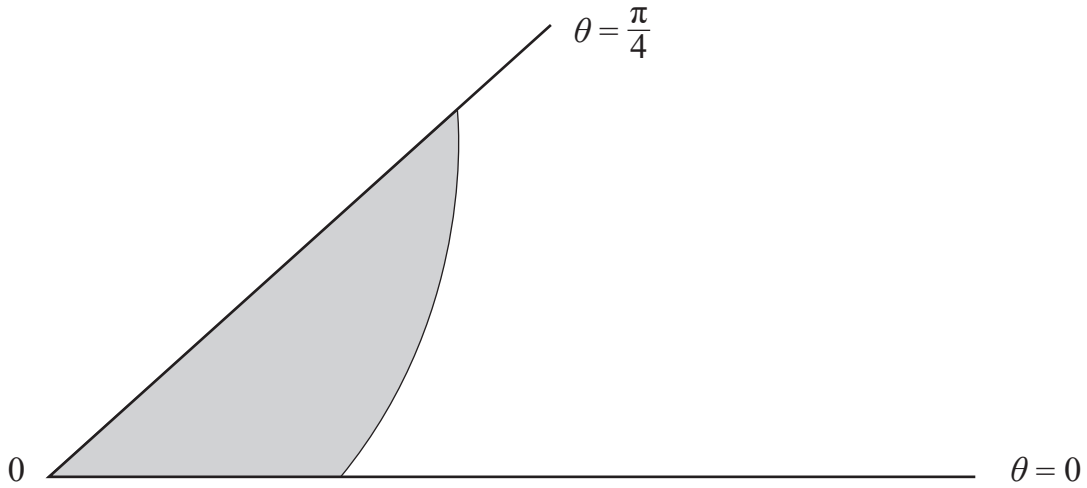


Fig. 1

Find, in terms of a, the area of the shaded region. [8]

11 (a) Given that n is a positive integer, show that

$$(1 + \mathrm{i})^{4n} - (1 - \mathrm{i})^{4n} = 0$$ [8]

(b) **(i)** Using De Moivre's theorem, express $\cos 4\theta$ in terms of $\sin\theta$ and $\cos\theta$ [4]

(ii) Using **(i)** show that

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$$ [2]

(iii) Hence using $\theta = \frac{\pi}{12}$, show that

$$\cos\frac{\pi}{12} = \frac{1}{2}\sqrt{2 + \sqrt{3}}$$ [7]

12 **(i)** By using exponential definitions, show that

$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \qquad -1 < x < 1$$ [6]

(ii) Using **(i)**, or otherwise, show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\tanh^{-1}(\sin x)\right) = \sec x$$

where $x \neq \frac{(2k+1)\pi}{2}, k \in \mathbb{Z}$ [6]

(iii) Hence, or otherwise, find

$$\int_0^{\frac{\pi}{6}} \sec^2 x \ \tanh^{-1}(\sin x)\,\mathrm{d}x$$ [8]

THIS IS THE END OF THE QUESTION PAPER