New Specification

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2019

Further Mathematics

Assessment Unit AS 1

assessing

Pure Mathematics

[SFM11]

WEDNESDAY 12 JUNE, MORNING

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) FURTHER MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

			AVAILABLE MARKS
1	**(a)** **(i)** $\begin{pmatrix}3 & 7\\1 & 2\end{pmatrix}\begin{pmatrix}4 & -1\\-3 & 1\end{pmatrix} = \begin{pmatrix}-9 & 4\\-2 & 1\end{pmatrix}$	M1 W1	
	(ii) det **AB** $= -9 + 8 = -1$	M1 W1	
	$(\mathbf{AB})^{-1} = -1\begin{pmatrix}1 & -4\\2 & -9\end{pmatrix} = \begin{pmatrix}-1 & 4\\-2 & 9\end{pmatrix}$	MW1 W1	
	(iii) $\mathbf{A}^{-1} = \begin{pmatrix}-2 & 7\\1 & -3\end{pmatrix}$ $\quad$ $\mathbf{B}^{-1} = \begin{pmatrix}1 & 1\\3 & 4\end{pmatrix}$	MW1 MW1	
	$\mathbf{B}^{-1}\,\mathbf{A}^{-1} = \begin{pmatrix}1 & 1\\3 & 4\end{pmatrix}\begin{pmatrix}-2 & 7\\1 & -3\end{pmatrix} = \begin{pmatrix}-1 & 4\\-2 & 9\end{pmatrix}$	M1	
	$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\,\mathbf{A}^{-1}$	W1	
	(b) $\begin{pmatrix}X\\Y\end{pmatrix} = \begin{pmatrix}1 & 4\\2 & 3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x + 4y\\2x + 3y\end{pmatrix} = \begin{pmatrix}x + 4mx\\2x + 3mx\end{pmatrix}$	MW1 MW1 MW1	
	$Y = mX \quad \therefore \quad 2x + 3mx = m(x + 4mx)$	M1	
	$2x + 3mx = mx + 4m^2x$		
	$2m^2 - m - 1 = 0$	W1	
	$(2m + 1)(m - 1) = 0$		
	$m = -\frac{1}{2}$ or $m = 1$	W1	16
2	$\overrightarrow{AB} = \begin{pmatrix}5\\-2\\3\end{pmatrix}$ $\quad$ $\overrightarrow{AD} = \begin{pmatrix}2\\-3\\4\end{pmatrix}$ $\quad$ $\overrightarrow{AE} = \begin{pmatrix}3\\-2\\-3\end{pmatrix}$		
	Volume $= \left\lvert \overrightarrow{AE} \bullet \left(\overrightarrow{AB} \times \overrightarrow{AD}\right)\right\rvert$	M1	
	$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\5 & -2 & 3\\2 & -3 & 4\end{vmatrix} = (\mathbf{i} - 14\mathbf{j} - 11\mathbf{k})$	M1 W1 W1	
	$\overrightarrow{AE} \bullet \left(\overrightarrow{AB} \times \overrightarrow{AD}\right) = \begin{pmatrix}3\\-2\\-3\end{pmatrix} \bullet \begin{pmatrix}1\\-14\\-11\end{pmatrix} = 64$	M1 W1	6

3 **(a)** **(i)** $\det \mathbf{A} = 1(12 - 10) - 7(3 - 2) - 4(-5 + 4) = -1$ MW1

AVAILABLE MARKS

Minors $\begin{pmatrix} 2 & 1 & -1 \\ 1 & -1 & 2 \\ 2 & 2 & -3 \end{pmatrix}$ MW1

Cofactors $\begin{pmatrix} 2 & -1 & -1 \\ -1 & -1 & -2 \\ 2 & -2 & -3 \end{pmatrix}$ MW1

$\mathbf{A}^{-1} = \frac{1}{-1}\begin{pmatrix} 2 & -1 & 2 \\ -1 & -1 & -2 \\ -1 & -2 & -3 \end{pmatrix} = \begin{pmatrix} -2 & 1 & -2 \\ 1 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}$ W1

(ii) $\mathbf{A}^2 + \mathbf{AB} = \mathbf{I}$

$\mathbf{A} + \mathbf{B} = \mathbf{A}^{-1}$ M1

$\mathbf{B} = \mathbf{A}^{-1} - \mathbf{A}$ MW1

$= \begin{pmatrix} -3 & -6 & 2 \\ 0 & -3 & 4 \\ 2 & 7 & 0 \end{pmatrix}$ W1

(b) $2x^2 + 5x - 6 = 0$ Roots α and β

$\therefore \alpha + \beta = -\frac{5}{2}$ $\alpha\beta = -3$ M1 W1

New roots α^2 and β^2

$\alpha^2 \beta^2 = 9$ W1

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ M1

$= \frac{25}{4} + 6$

$= \frac{49}{4}$ W1

Equation $x^2 - \frac{49}{4}x + 9 = 0$

$4x^2 - 49x + 36 = 0$ W1 13

			AVAILABLE MARKS
4	**(a)** **(i)** $(3 - 2i)(2 + i) = 6 + 3i - 4i - 2i^2 = 8 - i$	M1 W1	
	(ii) $\dfrac{3 - 2i}{2 - i} = \dfrac{(3 - 2i)(2 + i)}{(2 - i)(2 + i)} = \dfrac{8 - i}{5}$	M1 W1 W1	
	(iii) $(a + bi)^2 = 16i = (a^2 - b^2) + 2abi$	M1 W1	
	$\therefore\ a^2 - b^2 = 0 \qquad 2ab = 16$	MW1 MW1	
	$a = \pm b \Rightarrow a^2 = 8 \Rightarrow a = \pm 2\sqrt{2} = b$	W1 W1	
	$\sqrt{16i} = \pm\left(2\sqrt{2} + i\,2\sqrt{2}\right)$	W1	
	(b) $z = 1 - i$ is a root $\Rightarrow z = 1 + i$ is a root	MW1	
	Let 3rd root = k (real)	MW1	
	$\therefore\ (z - (1 - i))(z - (1 + i))(z - k) \equiv z^3 - 5z^2 + az + b = 0$	M1	
	i.e. $z^3 - (2 + k)\,z^2 + (2 + 2k)z - 2k \equiv z^3 - 5z^2 + az + b$	W1	
	Coeff. of z^2 $\quad -2 - k = -5 \quad \therefore \quad k = 3$ Coeff. of z $\quad a = 2 + 2k \quad \therefore \quad a = 8$ Number term $\quad b = -2k \quad \therefore \quad b = -6$ Roots are $1 - i$, $1 + i$, 3 $a = 8$ and $b = -6$	W3	19

5 **(a)** **(i)** $z_1 = -\sqrt{3} + 3\text{i} \quad |z_1| = \sqrt{(-\sqrt{3})^2 + 3^2} = 2\sqrt{3}$ M1 W1

(ii) $\arg z_1 = \tan^{-1}\left(\frac{3}{-\sqrt{3}}\right) = \tan^{-1}\left(-\sqrt{3}\right) = \frac{2\pi}{3}$ M1 W1

(iii) $|z_1 z_2| = |z_1|\,|z_2| = 2\sqrt{3} \,.\, \sqrt{3} = 6$ M1 W1

$\arg(z_1 z_2) = \arg z_1 + \arg z_2 = \frac{2\pi}{3} + \frac{\pi}{4} = \frac{11\pi}{12}$ M1 W1

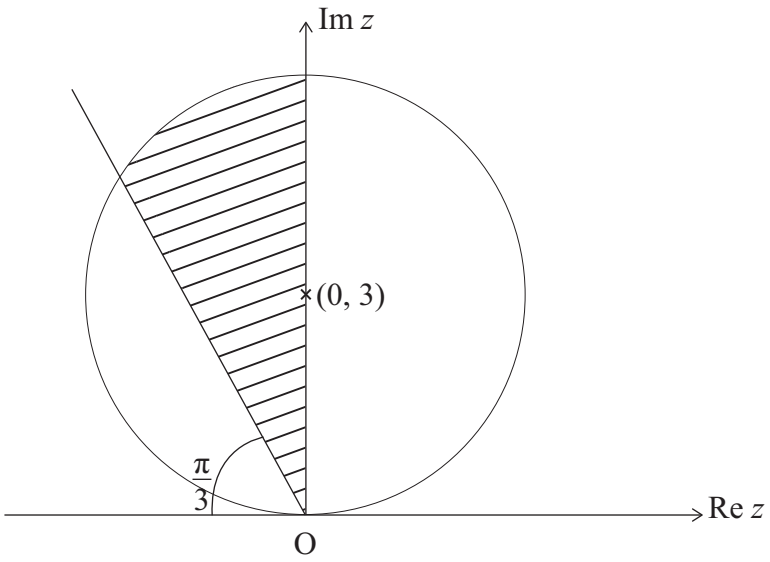
$z_1 z_2 = \left[6, \frac{11\pi}{12}\right]$ or $6\left(\cos\frac{11\pi}{12} + \text{i}\sin\frac{11\pi}{12}\right)$

(iv) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$ M1 W1

$\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2 = \frac{2\pi}{3} - \frac{\pi}{4} = \frac{5\pi}{12}$ M1 W1

$\frac{z_1}{z_2} = \left[2, \frac{5\pi}{12}\right]$ or $2\left(\cos\frac{5\pi}{12} + \text{i}\sin\frac{5\pi}{12}\right)$

(b)



M1
W1
W1
M1 W1
MW1

18

AVAILABLE MARKS

		AVAILABLE MARKS
6 **(i)** $\begin{vmatrix} a & 4 & 1 \\ 1 & 2 & -1 \\ 1 & 1 & 1 \end{vmatrix} = a\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} - 4\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$	M1 MW2	
$= 3a - 9$	W1	
(ii) No unique solution if $a = 3$ i.e. det = 0	M1 W1	
(iii) $3x + 4y + z = 6$ (1) $x + 2y - z = 0$ (2) $x + y + z = 3$ (3)		
(2) – (3) $\Rightarrow y - 2z = -3$ (1) – 3 × (3) $\Rightarrow y - 2z = -3$	M1 W1	
$y - 2z = -3$ $\quad x + y + z = 3 \Rightarrow x = 6 - 3z$	W1	
$\therefore$ General Solution $\quad x = 6 - 3z \;\; ; \;\; y = 2z - 3 \;\; ; \;\; z = z$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$	W1	10
Other solutions $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -\frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ 0 \\ \frac{3}{2} \end{pmatrix} + \lambda\begin{pmatrix} -\frac{3}{2} \\ 1 \\ \frac{1}{2} \end{pmatrix}$		

7 (a) (i) $3\mathbf{i} + \mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = 2\mathbf{i} + 5\mathbf{j} + \mu(\mathbf{i} - \mathbf{j} + \mathbf{k})$ M1

Equate $3 + \lambda = 2 + \mu$ $1 + 2\lambda = 5 - \mu$ $-1 + 3\lambda = \mu$ $\Rightarrow 4 + 3\lambda = 7 \Rightarrow \lambda = 1 \quad \mu = 2$ MW2 M1 W1

$-1 + 3\lambda = -1 + 3 = 2 \quad \mu = 2 \quad \therefore$ lines intersect W1

(ii) Intersection point (4, 3, 2) MW1

(b) (i) The plane $\mathbf{r} \bullet (\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = -5$ can be written $x + 2y + z = -5$
The plane $\mathbf{r} \bullet (2\mathbf{i} - 3\mathbf{j} + \mathbf{k}) = 15$ can be written $2x - 3y + z = 15$ MW1

Solve $2x - 3y + z = 15$ M1
$x + 2y + z = -5$
$x - 5y = 20$ W1

$5y + 20 + 2y + z = -5$ M1

$7y = -25 - z$ W1

Let $y = \lambda \qquad x = 5\lambda + 20 \;;\; z = -7\lambda - 25$
$\mathbf{r} = (5\lambda + 20)\,\mathbf{i} + \lambda\mathbf{j} + (-7\lambda - 25)\mathbf{k}$

Line of intersection:
$\mathbf{r} = 20\mathbf{i} - 25\mathbf{k} + \lambda(5\mathbf{i} + \mathbf{j} - 7\mathbf{k})$ or $\mathbf{r} = \begin{pmatrix} 20 \\ 0 \\ -25 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 1 \\ -7 \end{pmatrix}$ W1

(ii) Vectors $(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ and $(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ are perpendicular to planes

Angle between planes ≡ angle between normals MW1

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = -3 = \sqrt{6}\ \sqrt{14}\ \cos\theta$$ M1 W1

$\cos\theta = \dfrac{-3}{\sqrt{6}\ \sqrt{14}} \therefore \theta = 109.1°$ W1

Acute angle = 70.9° W1 18

Total **100**

AVAILABLE MARKS