

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

GCSE MATHEMATICS

H

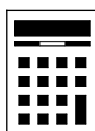
Higher Tier Paper 2 Calculator

Thursday 3 November 2022 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments
- the Formulae Sheet (enclosed).



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use

Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24	
TOTAL	



N 0 V 2 2 8 3 0 0 2 H 0 1

Answer **all** questions in the spaces provided.

1 Work out $\frac{4^6 - 11}{\sqrt{625} - 225}$

Circle your answer.

[1 mark]

-61.6

-20.425

204.25

3870.56

2 Work out $(3.1 \times 10^9)^2$

Circle your answer.

[1 mark] 6.2×10^{18} 6.2×10^{81} 9.61×10^{18} 9.61×10^{81}

3 The equation of a line is $y = 3x - 6$

Circle the coordinates of the y -intercept.**[1 mark]**

(0, -6)

(-6, 0)

(0, 3)

(3, 0)



4 $a \times b^4 = c$

Circle the correct expression for a .

[1 mark]

$\frac{c}{\sqrt[4]{b}}$

$\frac{c}{b^{-4}}$

$\left(\frac{c}{b}\right)^4$

$\frac{c}{b^4}$

5 Written as the product of prime factors,

$$12\,600 = 2^3 \times 3^2 \times 5^2 \times 7$$

and

$$14\,112 = 2^5 \times 3^2 \times 7^2$$

Work out the highest common factor (HCF) of 12 600 and 14 112

Give your answer as an integer.

[2 marks]

Common prime factors, lowest powers: $2^3, 3^2, 7$

$HCF = 2^3 \times 3^2 \times 7$

$= 8 \times 9 \times 7 = 504$

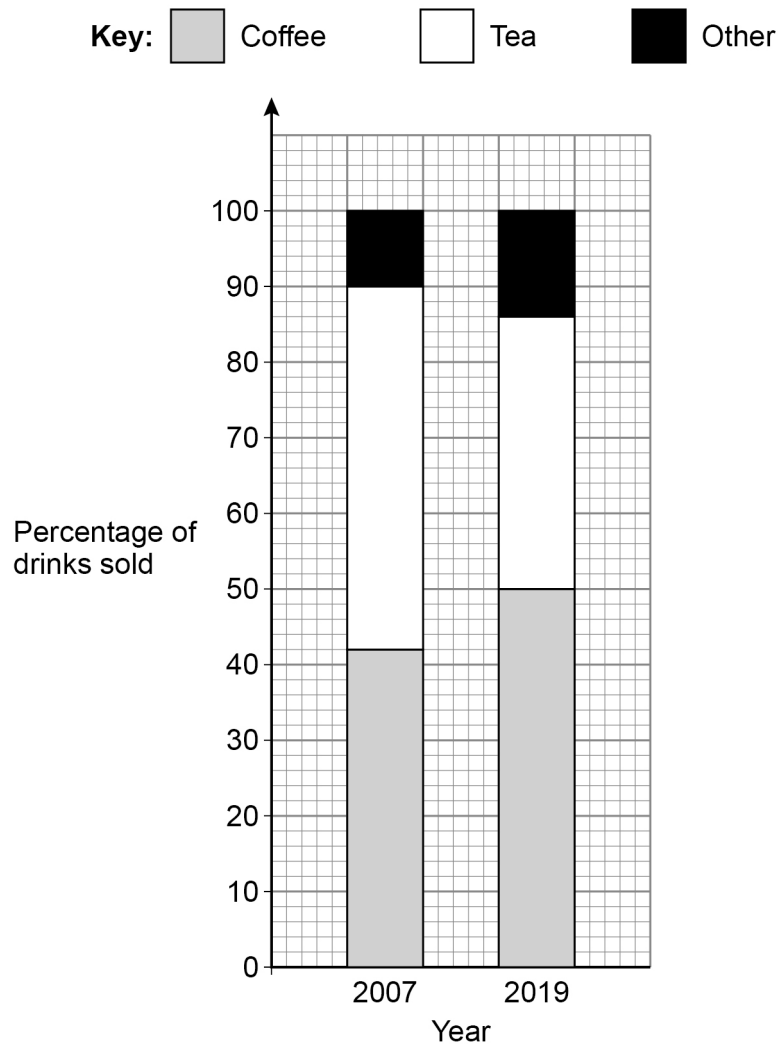
Answer 504

Turn over ►



6

The composite bar chart shows information about the **percentage** of drinks sold by a café in 2007 and 2019



- 6 (a) In 2007 the café sold a total of 24 000 drinks.

How many **more** teas than coffees were sold?

[2 marks]

$$\text{Coffee} = 42\% \quad \text{Tea} = 90 - 42 = 48\%$$

$$\text{Difference} = 48 - 42 = 6\%$$

$$6\% \text{ of } 24\,000 = \frac{6}{100} \times 24\,000 = 1440$$

Answer 1440



- 6 (b) Were more coffees sold at the café in 2019 than in 2007 ?

Tick a box.

☐

Yes

☐

No

☒

Cannot tell

Give a reason for your answer.

[1 mark]

The chart only shows percentages, not the numbers of drinks.

We do not know how many drinks were sold in 2019, so we cannot tell how many coffees were sold.

- 7 (a) k is a whole number between 40 and 50

The cube root of k is 3, to the nearest whole number.

Work out the **largest** possible value of k .

[2 marks]

$\sqrt[3]{k}$ rounds to 3 if $k < 3.5^3 = 42.875$

$\sqrt[3]{42} = 3.476... \rightarrow 3$ to the nearest whole number

$\sqrt[3]{43} = 3.503... \rightarrow 4$ to the nearest whole number

So the largest possible value is 42

Answer 42

- 7 (b) Fay tries to solve $x^2 = 100$

She says,

“The only possible value of x is 10”

Give a reason why she is **not** correct.

[1 mark]

x could also be -10 , because $(-10)^2 = 100$ as well,

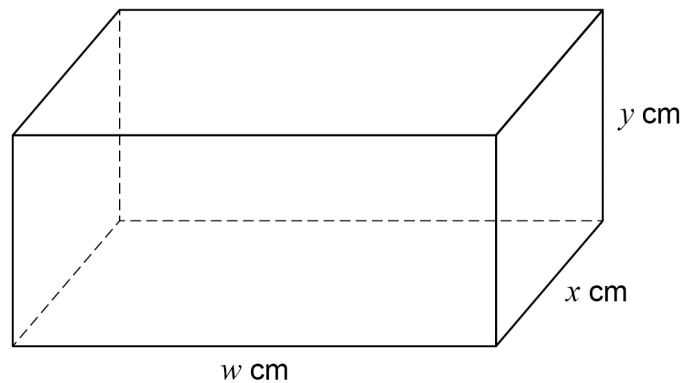
so there are two answers: 10 and -10

Turn over ►



8 (a) Here is a cuboid.

w , x and y are **different** whole numbers.



The total length of **all** the edges of the cuboid is 80 cm

The volume is **greater** than 200 cm^3

Work out one possible set of values for w , x and y .

[2 marks]

$$\text{Total edges: } 4w + 4x + 4y = 80$$

$$\text{so } w + x + y = 20$$

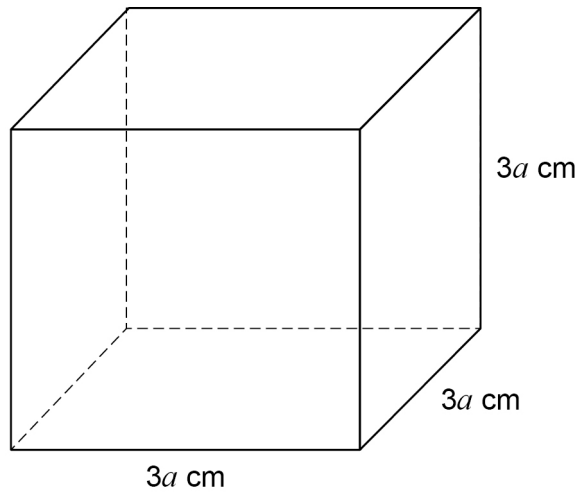
$$\text{Try } 9, 6, 5: 9 + 6 + 5 = 20 \text{ (all different)}$$

$$\text{Volume} = 9 \times 6 \times 5 = 270 \text{ cm}^3, \text{ which is greater than } 200$$

$$w = \underline{9} \quad x = \underline{6} \quad y = \underline{5}$$



- 8 (b) Here is a solid cube.



Circle the expression for the **total** surface area in cm^2

[1 mark]

$36a$

$54a$

$36a^2$

$54a^2$

9

The 47th triangular number is 1128

The 48th triangular number is 1176

Work out the 49th triangular number.

[1 mark]

Difference: $1176 - 1128 = 48$, so the next difference is 49

$$1176 + 49 = 1225$$

Answer 1225



- 10** The n th terms of two linear sequences, A and B, are added to give the n th term of a new sequence.

The new sequence starts

8 13 18 23

The n th term of sequence A is $n + 1$

Work out the n th term of sequence B.

[4 marks]

8, 13, 18, 23: common difference 5

New sequence n th term = $5n + 3$ ($8 - 5 = 3$)

$B = (5n + 3) - (n + 1)$

$= 5n + 3 - n - 1$

$= 4n + 2$

Check $n = 1$: $A = 2$, $B = 6$, $2 + 6 = 8$ ✓

Answer $4n + 2$

- 11** A tank contains 40 litres of water.

- 11 (a)** Water leaks out of the tank at a rate of 1.2 litres per minute.

The leak is stopped after 20 minutes.

Show that, when the leak is stopped, the tank contains 16 litres of water.

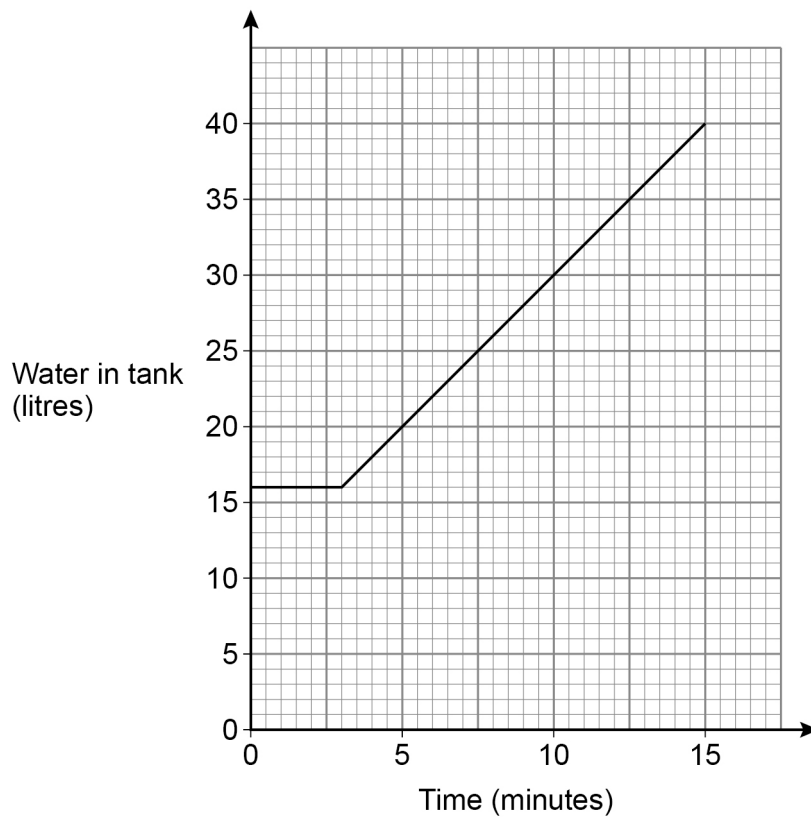
[1 mark]

Water leaked = $1.2 \times 20 = 24$ litres

Water left = $40 - 24 = 16$ litres



- 11 (b)** The tank is refilled with water from a tap.
The graph shows the amount of water in the tank **after** the leak is stopped.



Complete this report by writing a number in each answer space.

[3 marks]

Report

3 minutes after the leak is stopped, the tap starts to refill the tank.

The rate at which the tank refills is 2 litres per minute.

Line is flat until 3 minutes, so the tap starts after 3 minutes

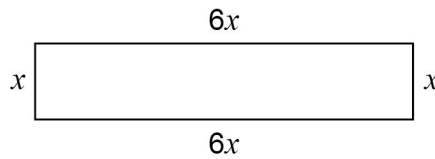
Rate = gradient = $(40 - 16) \div (15 - 3)$

= $24 \div 12 = 2$ litres per minute



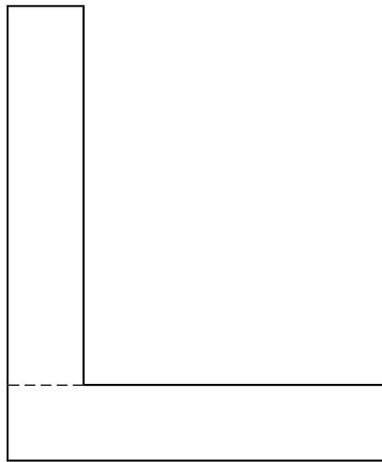
12

The length of this rectangle is 6 times the width.



Not drawn
accurately

Two of these rectangles are joined, with no overlap, to make this L-shape.



Not drawn
accurately

The perimeter of the L-shape is 98.8 cm

Work out the value of the perimeter of **one** of the rectangles.

[4 marks]

$$\text{Perimeter of L-shape} = 6x + x + 5x + 6x + x + 6x + x = 26x$$

$$26x = 98.8$$

$$x = 98.8 \div 26 = 3.8$$

$$\text{Perimeter of one rectangle} = 6x + x + 6x + x = 14x$$

$$= 14 \times 3.8$$

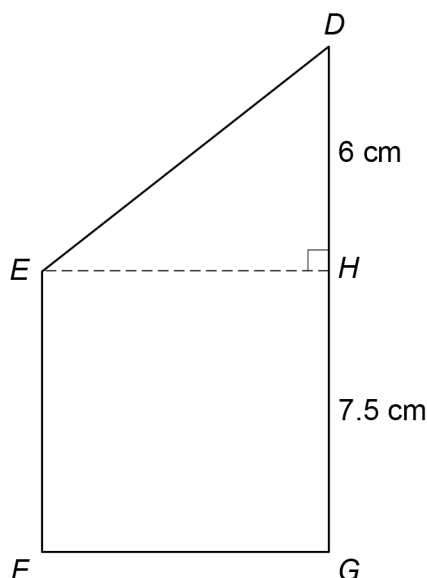
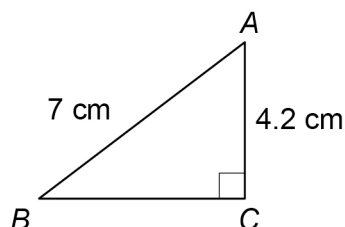
$$= 53.2 \text{ cm}$$

Answer 53.2 cm



13

Trapezium $DEFG$ is formed by joining
triangle DEH
to
rectangle $EFGH$.



Not drawn
accurately

ABC is similar to DEH .

Work out the area of $DEFG$.

[5 marks]

$$BC^2 = 7^2 - 4.2^2 = 49 - 17.64 = 31.36$$

$$BC = \sqrt{31.36} = 5.6 \text{ cm}$$

$$\text{Scale factor} = 6 \div 4.2, \text{ so } EH = 6 \div 4.2 \times 5.6 = 8 \text{ cm}$$

$$\text{Area of triangle } DEH = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$

$$\text{Area of rectangle } EFGH = 8 \times 7.5 = 60 \text{ cm}^2$$

$$\text{Area of } DEFG = 24 + 60$$

$$= 84 \text{ cm}^2$$

Answer 84 cm²

Turn over ►



14

Fred bought an apartment for £137 500

He made 8% profit when he sold the apartment.

He used all of this profit to pay 40% of the deposit on a house.

The deposit was one sixth of the price of the house.

Work out the price of the house.

[4 marks]

$$\text{Profit} = 8\% \text{ of } £137\,500 = 0.08 \times 137\,500 = £11\,000$$

£11 000 is 40% of the deposit

$$\text{Deposit} = 11\,000 \div 0.4 = £27\,500$$

Deposit is one sixth of the price:

$$\text{Price} = 27\,500 \times 6 = £165\,000$$

Answer £ 165 000

15

Circle the correct statement.

[1 mark]

$$1 \text{ m}^2 = 100 \text{ mm}^2$$

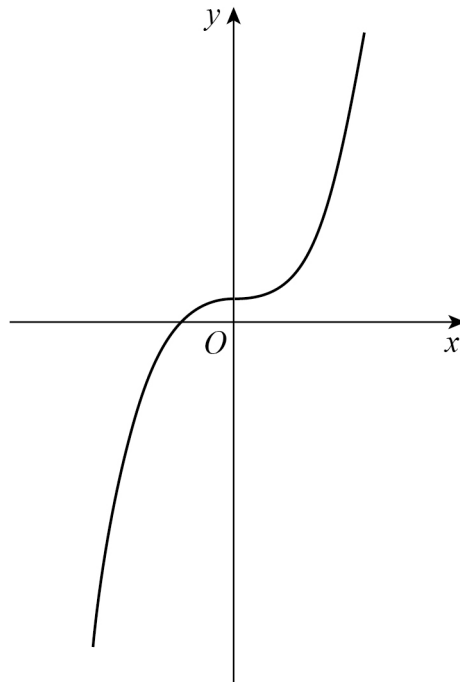
$$1 \text{ cm}^2 = 100 \text{ mm}^2$$

$$1 \text{ m}^2 = 100 \text{ cm}^2$$

$$1 \text{ km}^2 = 100 \text{ m}^2$$



- 16 Here is a sketch of a graph.



Circle the possible equation of the graph.

[1 mark]

$$y = x^2 + 1$$

$$y = \frac{1}{x} + 1$$

$$y = x^3 + 1$$

$$y = 1 - x^2$$

- 17 A sequence of numbers is formed by the iterative process

$$u_{n+1} = \frac{20}{u_n + 3} \quad \text{where} \quad u_1 = 1$$

Work out u_3

Circle your answer.

[1 mark]

$$\frac{40}{11}$$

$$\frac{5}{2}$$

$$7$$

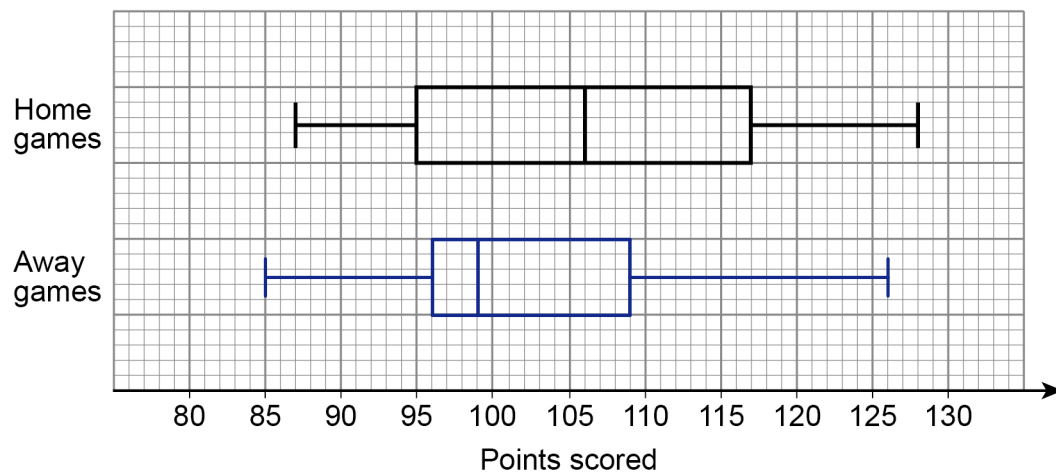
$$5$$



18

A basketball team plays 19 home games and 19 away games.

The box plot shows information about the points the team scored in **home** games.



Here are the points the team scored in the 19 **away** games.

85 89 93 95 96 96 98 98 98 99

100 103 105 107 109 110 114 119 126

18 (a) On the grid, draw a box plot for the away games.

[4 marks]

19 values, already in order:

Lowest = 85, Highest = 126

Median = 10th value = 99

Lower quartile = 5th value = 96

Upper quartile = 15th value = 109



- 18 (b)** On average, did the team score more points in home games or away games?
Use **one** statistical measure to support your decision.

[1 mark]

*Home, because the median for home games (106) is
higher than the median for away games (99).*

- 18 (c)** Was the number of points scored more consistent in home games or away games?
Use **one** statistical measure to support your decision.

[1 mark]

*Away, because the interquartile range for away games
(109 - 96 = 13) is lower than for home games
(117 - 95 = 22).*

- 19** Using the quadratic formula, or otherwise, solve $3x^2 + x - 5 = 0$

[2 marks]

$$a = 3, b = 1, c = -5$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 3 \times -5}}{2 \times 3}$$

$$x = \frac{-1 \pm \sqrt{61}}{6}$$

$$x = 1.135... \text{ or } x = -1.468...$$

$$x = 1.14 \text{ or } x = -1.47 \text{ (2 dp)}$$

Answer $x = 1.14 \text{ or } x = -1.47$

Turn over ►



20

A vending machine has a different item in each section.

It sells

7 drinks, 3 of which are juice

5 snacks, 2 of which are fruit bars

11 meals, 4 of which are salad.

One drink, one snack and one meal are chosen at random.

Show that the probability of getting a juice, a fruit bar and a salad is **more** than 5%

[3 marks]

$$P(\text{juice}) = \frac{3}{7} \quad P(\text{fruit bar}) = \frac{2}{5} \quad P(\text{salad}) = \frac{4}{11}$$

$$P(\text{all three}) = \frac{3}{7} \times \frac{2}{5} \times \frac{4}{11} = \frac{24}{385}$$

$$= 0.06233... = 6.23...\%$$

$$6.23\% > 5\%$$

so the probability is more than 5%



21 $f(x) = \frac{3x+9}{5}$ and $g(x) = 6x - 1$

21 (a) Show that $gf(2)$ is an integer.

[2 marks]

$$f(2) = \frac{3 \times 2 + 9}{5} = \frac{15}{5} = 3$$

$$gf(2) = g(3)$$

$$= 6 \times 3 - 1 = 17$$

17 is an integer, so $gf(2)$ is an integer

21 (b) Show that $f^{-1}(8)$ is **not** an integer.

[2 marks]

$$\text{Let } f(x) = 8: \frac{3x+9}{5} = 8$$

$$3x + 9 = 40$$

$$3x = 31$$

$$f^{-1}(8) = x = \frac{31}{3} = 10.333...$$

10.333... is not an integer, so $f^{-1}(8)$ is not an integer



22

Factorise fully $x^3 - 49x$ **[2 marks]**

$$x^3 - 49x = x(x^2 - 49)$$

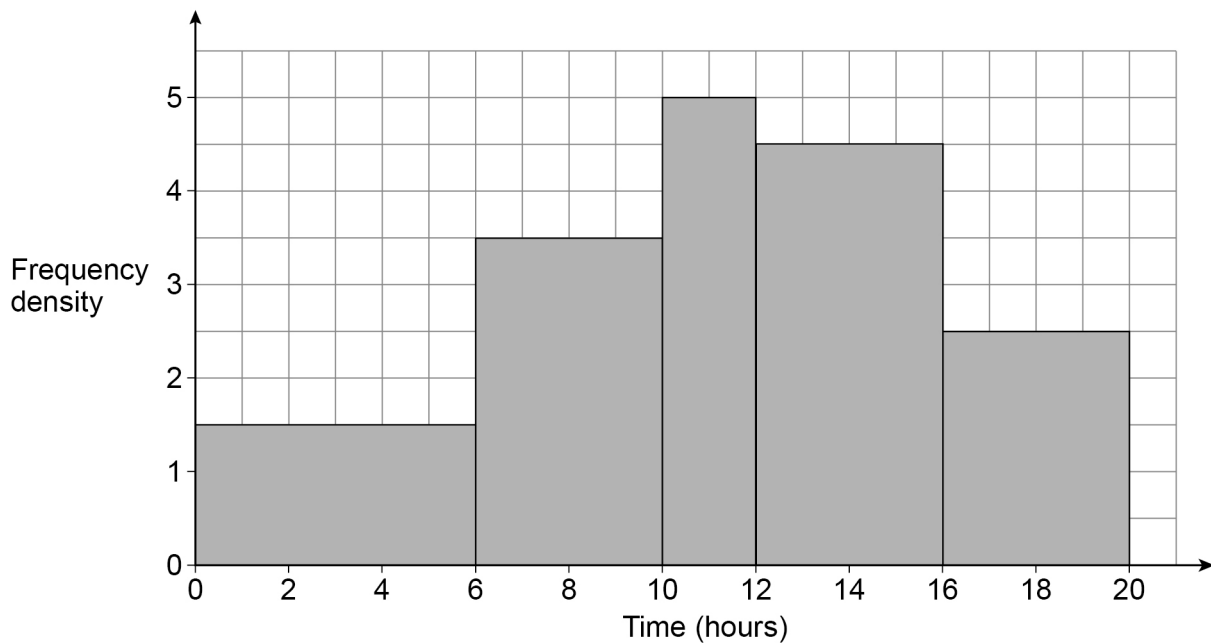
$$= x(x + 7)(x - 7) \quad (\text{difference of two squares})$$

Answer $x(x + 7)(x - 7)$

23

61 students recorded how many hours they spent revising for a test.

The histogram represents the results.



- 23 (a)** Work out an estimate of the mean time the 61 students spent revising.
You may use the table to help you.

[4 marks]

Time, x (hours)	Frequency	Midpoint	$f \times \text{midpoint}$
$0 \leq x < 6$	9	3	27
$6 \leq x < 10$	14	8	112
$10 \leq x < 12$	10	11	110
$12 \leq x < 16$	18	14	252
$16 \leq x < 20$	10	18	180

Frequency = frequency density $\times$ class width, eg $1.5 \times 6 = 9$

$$\Sigma fx = 27 + 112 + 110 + 252 + 180 = 681$$

$$\Sigma f = 9 + 14 + 10 + 18 + 10 = 61$$

$$\text{Mean} = 681 \div 61 = 11.1639\dots$$

$$= 11.2 \text{ hours (1 dp)}$$

Answer 11.2 hours

- 23 (b)** Give a reason why the answer to part (a) is an estimate.

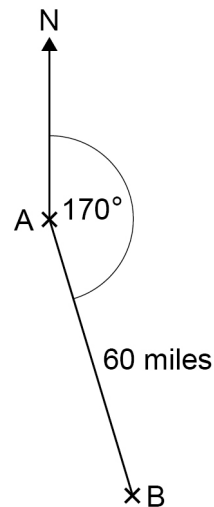
[1 mark]

The data is grouped, so we do not know the exact times;

the midpoints are used as estimates of each time.

Turn over ►

24

B is 60 miles from A on a bearing of 170° Not drawn
accuratelyA ship sails from A on a bearing of 247° It travels at a constant speed of 23 mph for $1\frac{1}{2}$ hours.

Is the ship now closer to B than it was when it left A?

You **must** show your working.**[5 marks]**

$$\text{Angle between AB and the ship's path} = 247^\circ - 170^\circ = 77^\circ$$

$$\text{Distance sailed} = 23 \times 1\frac{1}{2} = 34.5 \text{ miles}$$

Let S be the ship's position. Cosine rule in triangle ABS:

$$SB^2 = 34.5^2 + 60^2 - 2 \times 34.5 \times 60 \times \cos 77^\circ$$

$$= 1190.25 + 3600 - 931.297...$$

$$= 3858.95...$$

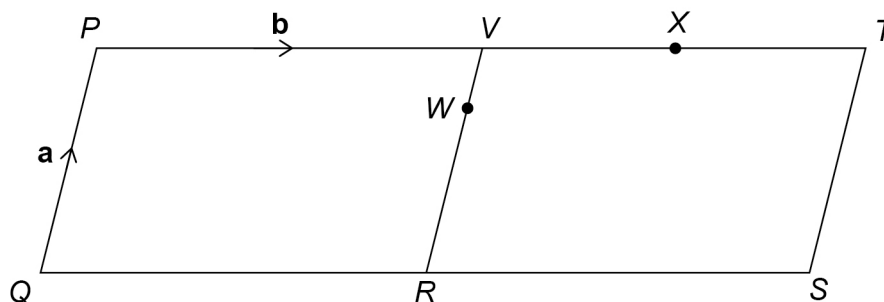
$$SB = \sqrt{3858.95...} = 62.12... = 62.1 \text{ miles}$$

$$62.1 \text{ miles} > 60 \text{ miles}$$

No, the ship is further away from B than when it left A.



25

Two congruent parallelograms, $PQRV$ and $VRST$, are joined.Not drawn
accurately

$$\overrightarrow{QP} = \mathbf{a} \quad \overrightarrow{PV} = \mathbf{b}$$

X is the midpoint of VT.

$$VW : WR = 1 : 2$$

Prove that Q, W and X lie on a straight line.

[3 marks]

$$\overrightarrow{VW} = \frac{1}{3} \overrightarrow{VR} = \frac{1}{3} (-\mathbf{a}) = -\frac{1}{3} \mathbf{a}$$

$$\overrightarrow{QW} = \overrightarrow{QP} + \overrightarrow{PV} + \overrightarrow{VW} = \mathbf{a} + \mathbf{b} - \frac{1}{3} \mathbf{a} = \frac{2}{3} \mathbf{a} + \mathbf{b}$$

$$\overrightarrow{QX} = \overrightarrow{QP} + \overrightarrow{PV} + \overrightarrow{VX} = \mathbf{a} + \mathbf{b} + \frac{1}{2} \mathbf{b} = \mathbf{a} + \frac{3}{2} \mathbf{b}$$

$$\overrightarrow{QX} = \frac{3}{2} \left(\frac{2}{3} \mathbf{a} + \mathbf{b} \right) = \frac{3}{2} \overrightarrow{QW}$$

So QX is a multiple of QW, so QX is parallel to QW,

and QX and QW share the common point Q,

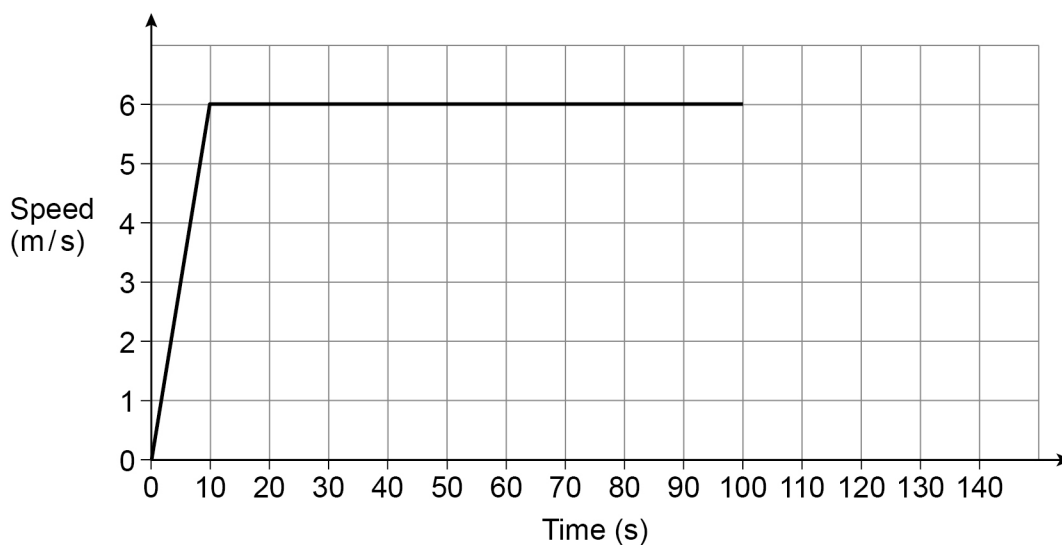
so Q, W and X lie on a straight line.



26

Helena ran an 800-metre race in 140 seconds.

The speed-time graph represents the first 100 seconds of her run.



Helena ran the last 40 seconds with constant deceleration.

Work out her speed as she finished the race.

[4 marks]

Distance = area under the speed-time graph

First 10 s: $\frac{1}{2} \times 10 \times 6 = 30 \text{ m}$

10 s to 100 s: $90 \times 6 = 540 \text{ m}$

Distance in first 100 s = $30 + 540 = 570 \text{ m}$

Remaining distance = $800 - 570 = 230 \text{ m}$

Last 40 s (trapezium): $\frac{1}{2} \times (6 + v) \times 40 = 230$

$20(6 + v) = 230 \rightarrow 6 + v = 11.5$

$v = 5.5 \text{ m/s}$

Answer 5.5 metres per second



27

In a class there are

 n boys

a total of 25 students.

Two of the students are chosen at random.

The probability that both students are boys is $\frac{7}{20}$ Work out the value of n .**[4 marks]**

$$P(\text{both boys}) = \frac{n}{25} \times \frac{n-1}{24} = \frac{7}{20}$$

$$\frac{n(n-1)}{600} = \frac{7}{20}$$

$$20n(n-1) = 4200$$

$$n(n-1) = 210$$

$$n^2 - n - 210 = 0$$

$$(n-15)(n+14) = 0$$

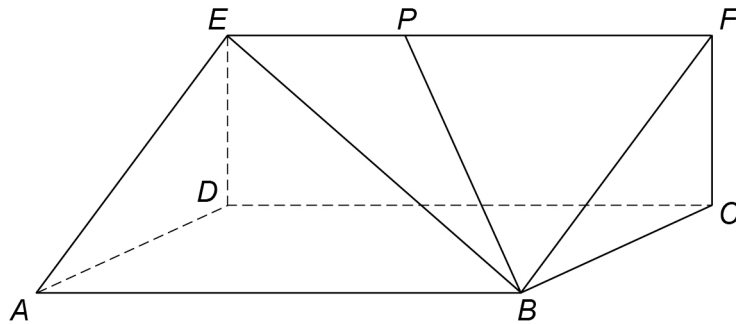
$$n = 15 \text{ or } n = -14$$

$$n \text{ must be positive, so } n = 15$$

$$n = \underline{\hspace{2cm}15\hspace{2cm}}$$



28

 $ABCDEF$ is a triangular prism. P is a point on EF . $EB = 29$ cmAngle $EBP = 35^\circ$ Angle $EPB = 114^\circ$ Work out the length of EP .**[2 marks]**

Angle EBP is opposite EP , angle EPB is opposite EB . Sine rule:

$$\frac{EP}{\sin 35^\circ} = \frac{29}{\sin 114^\circ}$$

$$EP = \frac{29 \times \sin 35^\circ}{\sin 114^\circ} = 18.2078...$$

$$EP = 18.2 \text{ cm (3 sf)}$$

Answer 18.2 cm

END OF QUESTIONS

There are no questions printed on this page

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outside the
box*

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ANSWER IN THE SPACES PROVIDED**



[illegible]

