

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname *StemUp Model Solution*

Forename(s)

Candidate signature

GCSE MATHEMATICS

H

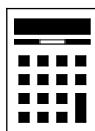
Higher Tier Paper 3 Calculator

Monday 12 November 2018 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use

Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
TOTAL	

Advice

In all calculations, show clearly how you work out your answer.



N 0 V 1 8 8 3 0 0 3 H 0 1

Answer **all** questions in the spaces provided

- 1** A shape is translated by the vector $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$
In which direction does the shape move?
Circle your answer.
- [1 mark]**

☒ up☐ down☐ left☐ right

- 2** What is 1.75 kilometres as a fraction of 700 metres?
Circle your answer.
- [1 mark]**

☒ $\frac{5}{2}$ ☐ $\frac{1}{4}$ ☐ $\frac{4}{1}$ ☐ $\frac{2}{5}$

- 3** The first 4 terms of a linear sequence are
3 11 19 27
Circle the expression for the n th term.
- [1 mark]**

☐ $8 - 5n$ ☐ $n + 8$ ☐ $8n + 3$ ☒ $8n - 5$ 

- 4 Work out the lowest common multiple (LCM) of 20, 30 and 40
Circle your answer.

[1 mark]

10

120

240

24 000

- 5 The length of a table is 110 cm to the nearest cm
Complete the error interval.

[2 marks]

109.5 cm $\leq$ length < 110.5 cm

Turn over for the next question

Turn over ►



6

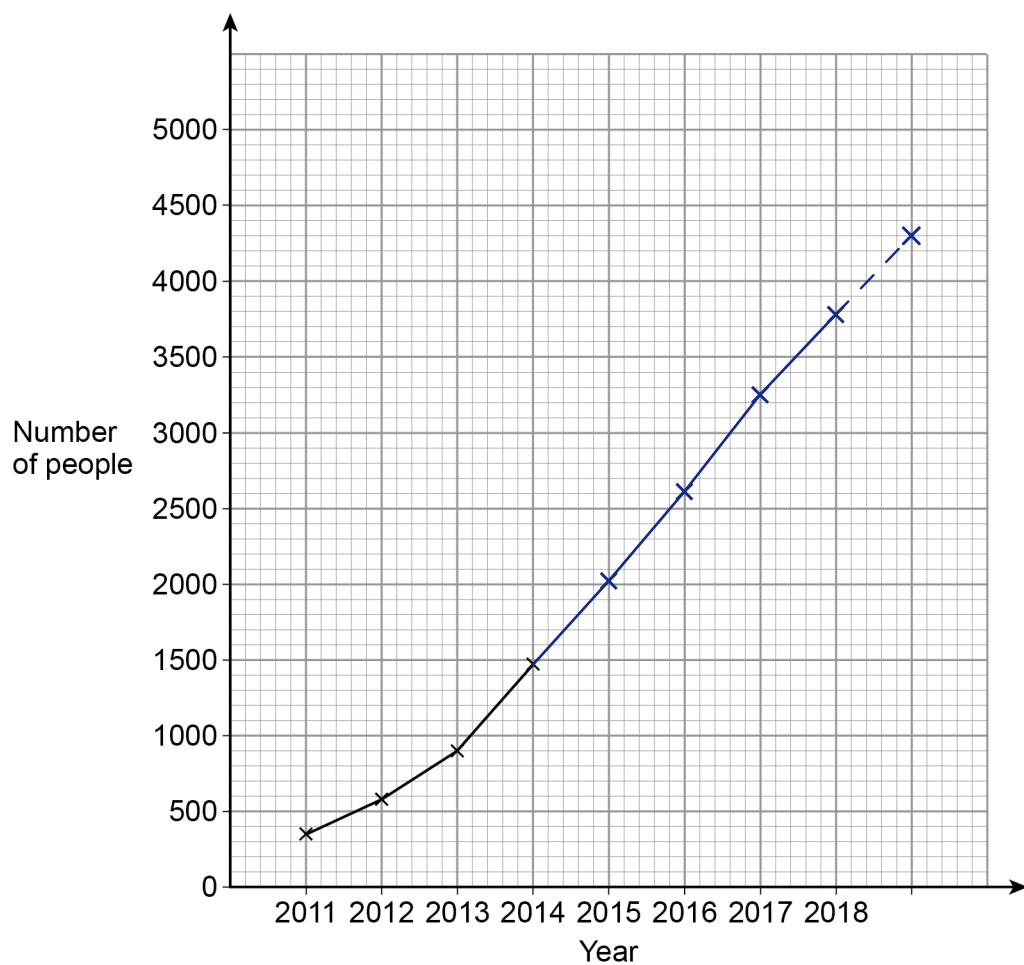
A music festival has taken place each year from 2011

The table shows the number of people who attended each year.

Year	2011	2012	2013	2014	2015	2016	2017	2018
Number of people	350	583	906	1471	2023	2612	3251	3780

The festival organisers draw a time series graph to represent the data.

The first four years have been plotted.



6 (a) Complete the graph.

[2 marks]

6 (b) Use the graph to estimate the number of people who will attend the festival in 2019

[2 marks]

Increases recently about 530 to 640 a year; continue the trend:

$3780 + 530 \approx 4300$ in 2019 (extended on the graph)

Answer 4300

Turn over for the next question



7

$$k = n^2 + 9n + 1$$

Mo says,

“ k will be a prime number for all integer values of n from 1 to 9”

Show that Mo is wrong.

You **must** show that your value of k is **not** prime.

[3 marks]

$$n = 1: k = 1 + 9 + 1 = 11 \quad (\text{prime})$$

$$n = 2: k = 4 + 18 + 1 = 23 \quad (\text{prime})$$

$$n = 3: k = 9 + 27 + 1 = 37 \quad (\text{prime})$$

$$n = 4: k = 16 + 36 + 1 = 53 \quad (\text{prime})$$

$$n = 5: k = 25 + 45 + 1 = 71 \quad (\text{prime})$$

$$n = 6: k = 36 + 54 + 1 = 91$$

$91 = 7 \times 13$, so 91 has factors 7 and 13 and is not prime.

So when $n = 6$, $k = 91$ is not prime. Mo is wrong.



8

Doug owes an amount of £600

He wants to pay off this amount in five months.

He says,

“Each month, I will pay back 20% of the amount I still owe.”

Show working to check if his method is correct.

[3 marks]

Paying back 20% leaves 80% owed, so multiply by 0.8 each month

After month 1: $600 \times 0.8 = £480$

After month 2: $480 \times 0.8 = £384$

After month 3: $384 \times 0.8 = £307.20$

After month 4: $307.20 \times 0.8 = £245.76$

After month 5: $245.76 \times 0.8 = £196.608 = £196.61$

$(600 \times 0.8^5 = 196.608)$

He still owes £196.61 after five months,

so his method is not correct.

Turn over for the next question

Turn over ►



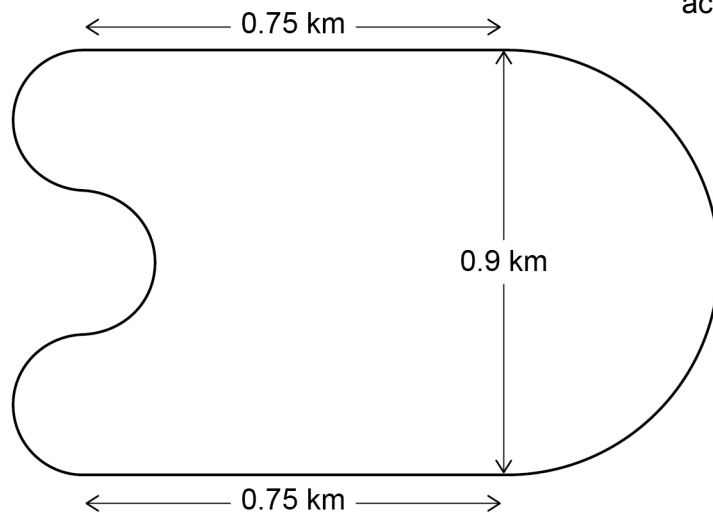
9

A motor racing circuit consists of

two parallel straight sections, each of length 0.75 km

a semicircle of diameter 0.9 km

three equal, smaller semicircles.



Not drawn
accurately

The length of a motor race must be greater than 305 km

What is the lowest number of **full** laps needed at this circuit?

You **must** show your working.

[5 marks]

$$\text{Large semicircle} = \pi \times 0.9 \div 2 = 0.45\pi = 1.4137... \text{ km}$$

$$\text{Small semicircle diameter} = 0.9 \div 3 = 0.3 \text{ km}$$

$$\text{Each small semicircle} = \pi \times 0.3 \div 2 = 0.15\pi = 0.4712... \text{ km}$$

$$\text{One lap} = 1.4137... + 3 \times 0.4712... + 2 \times 0.75$$

$$= 0.9\pi + 1.5 = 4.3274... \text{ km}$$

$$305 \div 4.3274... = 70.48...$$

$$70 \text{ laps} = 302.9 \text{ km (not more than 305 km)}$$

$$\text{so 71 full laps are needed (71 laps} = 307.2 \text{ km)}$$

Answer 71



10

Solve $8 > 3 - \frac{1}{2}x$

[2 marks]

Multiply both sides by 2: $16 > 6 - x$

$x > 6 - 16$

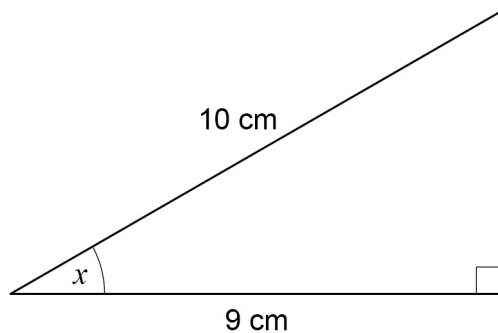
$x > -10$

Answer *$x > -10$*

11

Use trigonometry to work out the size of angle x .

[2 marks]

Not drawn
accurately

$\cos x = \text{adjacent} \div \text{hypotenuse} = 9 \div 10 = 0.9$

$x = \cos^{-1}(0.9)$

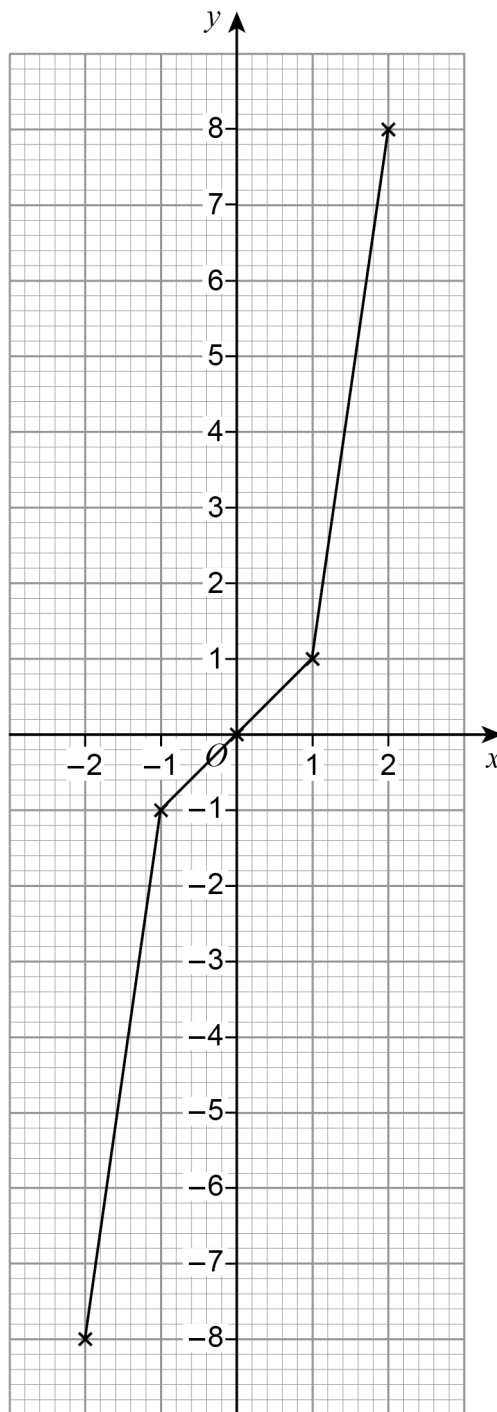
$x = 25.8419... = 25.8^\circ$

Answer *25.8* degrees

Turn over ►



- 12 Lewis wants to draw the graph $y = x^3$ for values of x from -2 to 2 . Here is his graph.



Make **one** criticism of his graph.

[1 mark]

The graph should be a smooth curve, not straight lines

joining the points (he needs more points plotted).



- 13** The probability of Heads when a biased coin is thrown is 0.6
The coin is thrown 500 times.
Circle the expected number of Tails.

[1 mark]

20

200

250

300

- 14** The mean mass of a squad of 19 hockey players is 82 kg
A player of mass 93 kg joins the squad.
Work out the mean mass of the squad now.

[3 marks]

$$\text{Total mass of 19 players} = 19 \times 82 = 1558 \text{ kg}$$

$$\text{New total mass} = 1558 + 93 = 1651 \text{ kg}$$

$$\text{New mean} = 1651 \div 20 = 82.55 \text{ kg}$$

Answer 82.55 kg

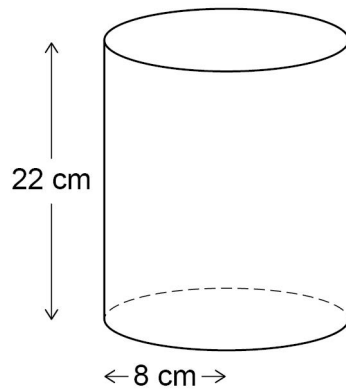
Turn over ►

- 15 A company makes two types of lampshade using fabric on wire frames.

Lampshade A

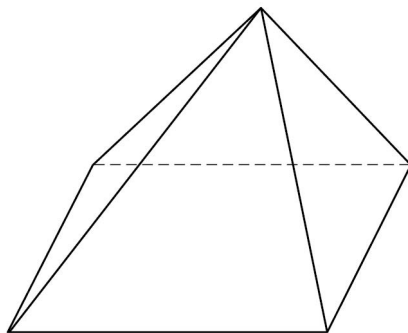
Fabric is used to make the curved surface of a cylinder.

The cylinder has radius 8 cm and height 22 cm

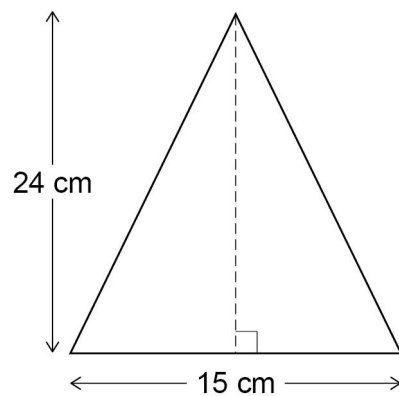


Lampshade B

Fabric is used to make the four triangular faces of a pyramid.



Each triangular face has base 15 cm and perpendicular height 24 cm



Not drawn
accurately



Cost of fabric	£400 per square metre
Other costs for A	£3.50 per lampshade
Other costs for B	£7.50 per lampshade

Work out the ratio cost of one lampshade A : cost of one lampshade B

Give your answer in the form $n : 1$

[5 marks]

$$\text{Lampshade A: curved surface area} = 2\pi rh = 2 \times \pi \times 8 \times 22$$

$$= 352\pi = 1105.84... \text{ cm}^2$$

$$1 \text{ m}^2 = 100^2 = 10\,000 \text{ cm}^2, \text{ so area} = 0.110584... \text{ m}^2$$

$$\text{Fabric cost} = 0.110584... \times 400 = £44.233...$$

$$\text{Cost of A} = 44.233... + 3.50 = £47.733...$$

$$\text{Lampshade B: area} = 4 \times \frac{1}{2} \times 15 \times 24 = 720 \text{ cm}^2$$

$$= 720 \div 10\,000 = 0.072 \text{ m}^2$$

$$\text{Fabric cost} = 0.072 \times 400 = £28.80$$

$$\text{Cost of B} = 28.80 + 7.50 = £36.30$$

$$A : B = 47.733... : 36.30$$

$$= 47.733... \div 36.30 : 1$$

$$= 1.31497... : 1 = 1.31 : 1$$

Answer 1.31 : 1



16

In a running club there are 50 females and 80 males.

If a female is chosen at random, the probability she has blue eyes is 0.38

If a male is chosen at random, the probability he has blue eyes is 0.6

One person is chosen at random.

Show that the probability the person has blue eyes is **more than** 0.5

[4 marks]

$$\text{Females with blue eyes} = 0.38 \times 50 = 19$$

$$\text{Males with blue eyes} = 0.6 \times 80 = 48$$

$$\text{Total with blue eyes} = 19 + 48 = 67$$

$$\text{Total people} = 50 + 80 = 130$$

$$P(\text{blue eyes}) = \frac{67}{130} = 0.5153\dots$$

$$0.5153\dots > 0.5$$

so the probability is more than 0.5

17

$$w = \frac{3}{5\sqrt{x}}$$

Circle the expression for w^2

[1 mark]

$$\frac{6}{10x^2}$$

$$\frac{9}{25x^2}$$

$$\frac{6}{10x}$$

$$\frac{9}{25x}$$



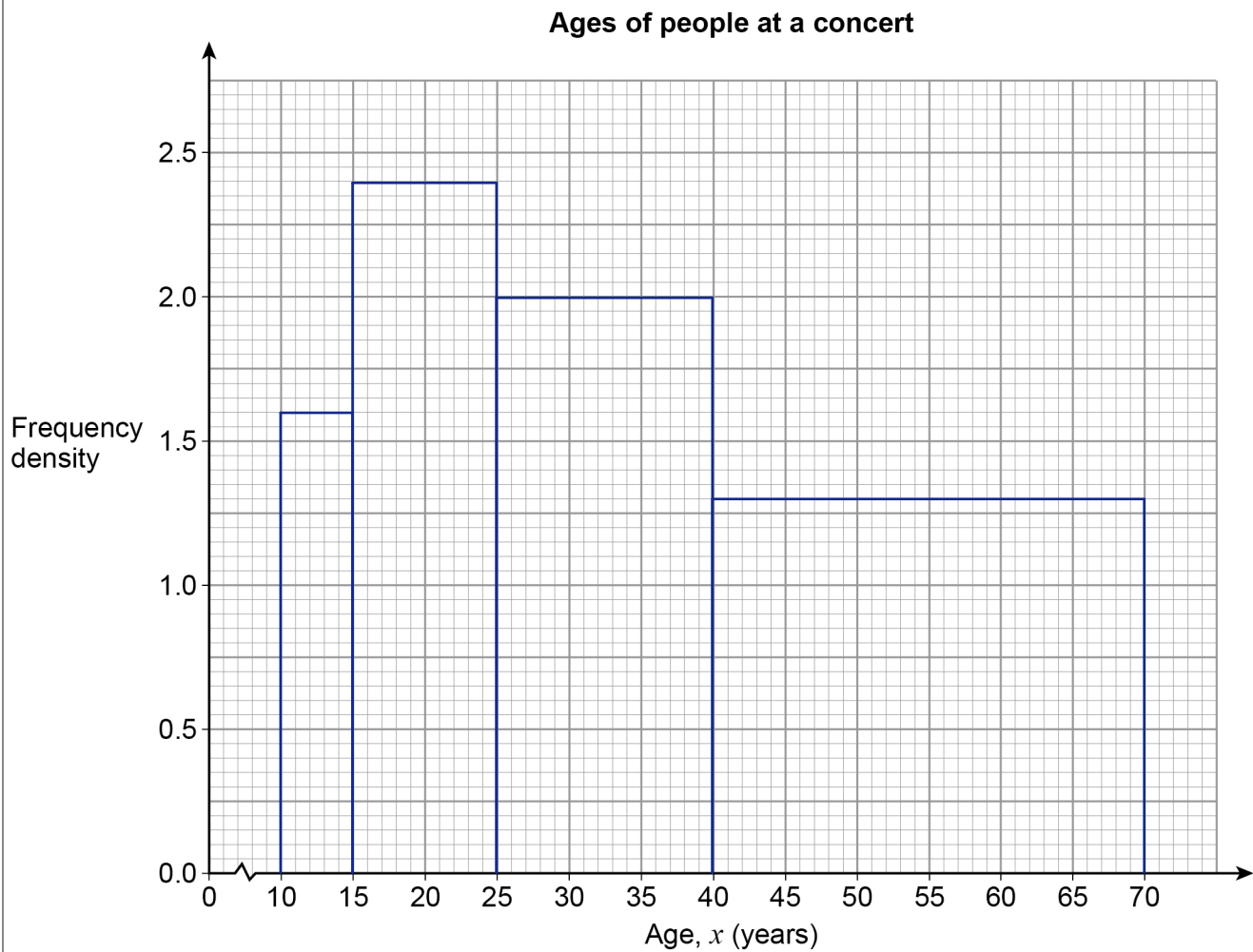
18

Here is some information about the ages of people at a concert.

Age, x (years)	Frequency	Frequency density
$10 \leq x < 15$	8	$8 \div 5 = 1.6$
$15 \leq x < 25$	24	$24 \div 10 = 2.4$
$25 \leq x < 40$	30	$30 \div 15 = 2$
$40 \leq x < 70$	39	$39 \div 30 = 1.3$

Draw a histogram to represent the information.

[3 marks]



Turn over ►



19

The length of a roll of ribbon is 30 metres, correct to the nearest half-metre.
A piece of length 5.8 metres, correct to the nearest 10 centimetres, is cut from the roll.
Work out the maximum possible length of ribbon left on the roll.

[3 marks]

$$\text{Roll: upper bound} = 30 + 0.25 = 30.25 \text{ m}$$

$$\text{Piece: lower bound} = 5.8 - 0.05 = 5.75 \text{ m}$$

$$\text{Maximum left} = 30.25 - 5.75 = 24.5 \text{ m}$$

Answer 24.5 metres



20

Curve P has equation $y = 2(x - 1)^2 - 5$

Curve Q is a reflection in the y -axis of curve P.

Work out the equation of curve Q.

Give your answer in the form $y = ax^2 + bx + c$ where a , b and c are integers.

[3 marks]

Reflect in the y -axis: replace x with $-x$

$$y = 2(-x - 1)^2 - 5 = 2(x + 1)^2 - 5$$

$$y = 2(x^2 + x + x + 1) - 5$$

$$y = 2x^2 + 4x + 2 - 5$$

$$y = 2x^2 + 4x - 3$$

Answer $y = 2x^2 + 4x - 3$

Turn over for the next question

Turn over ►



21

Priya and Joe travel the same 16.8 km route.

Priya starts at 9.00 am and walks at a constant speed of 6 km/h

Joe starts at 9.30 am and runs at a constant speed.

Joe overtakes Priya at 10.20 am

At what time does Joe finish the route?

[5 marks]

$$9.00 \text{ am to } 10.20 \text{ am} = 1 \text{ h } 20 \text{ min} = 1.333... \text{ h}$$

$$\text{Priya's distance at } 10.20 \text{ am} = 6 \times 1.333... = 8 \text{ km}$$

$$\text{Joe runs } 8 \text{ km from } 9.30 \text{ am to } 10.20 \text{ am} = 50 \text{ min}$$

$$\text{Joe's speed} = 8 \div (50 \div 60) = 9.6 \text{ km/h}$$

$$\text{Joe's time for } 16.8 \text{ km} = 16.8 \div 9.6 = 1.75 \text{ h}$$

$$1.75 \text{ h} = 1 \text{ hour } 45 \text{ minutes}$$

$$9.30 \text{ am} + 1 \text{ h } 45 \text{ min} = 11.15 \text{ am}$$

Answer 11.15 am



- 22** An approximate solution to an equation is found using the iterative formula

$$x_{n+1} = \frac{(x_n)^3 - 2}{10} \quad \text{with } x_1 = -1$$

- 22 (a)** Work out the values of x_2 and x_3

[2 marks]

$$x_2 = ((-1)^3 - 2) \div 10 = (-1 - 2) \div 10 = -0.3$$

$$x_3 = ((-0.3)^3 - 2) \div 10 = (-0.027 - 2) \div 10 = -0.2027$$

$$x_2 = \underline{\hspace{10em} -0.3 \hspace{10em}}$$

$$x_3 = \underline{\hspace{10em} -0.2027 \hspace{10em}}$$

- 22 (b)** Work out the solution to 5 decimal places.

[1 mark]

$$x_4 = -0.2008328... \quad x_5 = -0.2008100...$$

$$x_6 = -0.20080975... \quad x_7 = -0.20080975...$$

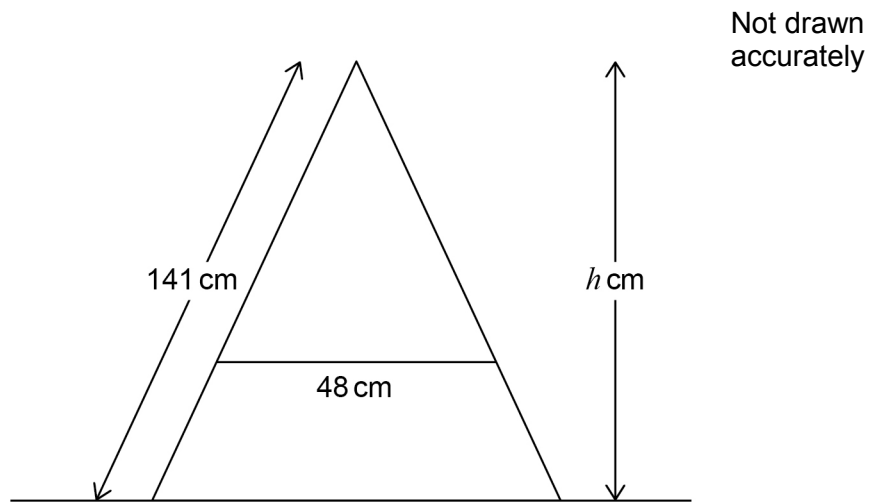
$$x_6 \text{ and } x_7 \text{ both round to } -0.20081$$

$$x = \underline{\hspace{10em} -0.20081 \hspace{10em}}$$



23

The diagram shows the side view of a step ladder with a horizontal strut of length 48 cm. The strut is one third of the way up the ladder. The symmetrical cross section of the ladder shows two similar triangles.



Work out the vertical height, h cm, of the ladder.

[5 marks]

The strut is one third of the way up, so the top triangle

is two thirds of the whole triangle (similar triangles)

Base of whole ladder = $48 \div 2 \times 3 = 72$ cm

Half the base = $72 \div 2 = 36$ cm

Right-angled triangle with hypotenuse 141 cm and base 36 cm

$h^2 = 141^2 - 36^2 = 19\,881 - 1296 = 18\,585$

$h = \sqrt{18\,585} = 136.326... = 136.3$ cm

Answer 136.3 cm



24

Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

Volume of a cone = $\frac{1}{3}\pi r^2 h$ where r is the radius and h is the perpendicular height

A sphere has radius $2x$ cm

A cone has

radius $3x$ cm

perpendicular height h cm

The sphere and the cone have the same volume.

Work out radius of cone : perpendicular height of cone

Give your answer in the form $a : b$ where a and b are integers.

[4 marks]

$$\text{Sphere: } V = \frac{4}{3}\pi(2x)^3 = \frac{4}{3}\pi \times 8x^3 = \frac{32}{3}\pi x^3$$

$$\text{Cone: } V = \frac{1}{3}\pi(3x)^2 h = \frac{1}{3}\pi \times 9x^2 h = 3\pi x^2 h$$

$$\text{Same volume: } \frac{32}{3}\pi x^3 = 3\pi x^2 h$$

$$32x^3 = 9x^2 h, \text{ so } 32x = 9h, h = \frac{32x}{9}$$

$$\text{radius : height} = 3x : \frac{32x}{9}$$

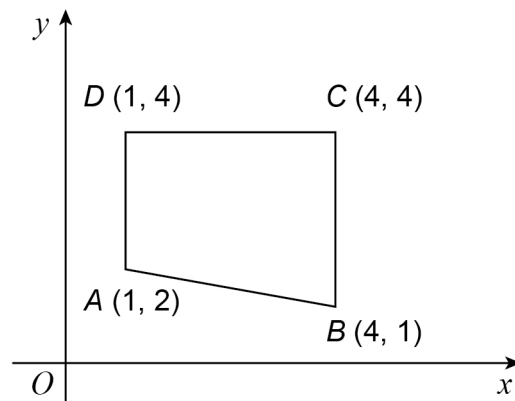
$$\text{Multiply by 9: } 27x : 32x = 27 : 32$$

Answer 27 : 32

Turn over ►



25

 $ABCD$ is a quadrilateral.Not drawn
accuratelyThe quadrilateral is reflected in the line $x = 4$

Which vertices are invariant?

Circle your answer.

[1 mark]

A and D

C and D

B and C

B and D



26

$$f(x) = \frac{2x+3}{x-4}$$

Work out $f^{-1}(x)$ **[4 marks]**

$$y = \frac{2x+3}{x-4}$$

$$y(x-4) = 2x+3$$

$$xy - 4y = 2x+3$$

$$xy - 2x = 4y+3$$

$$x(y-2) = 4y+3$$

$$x = \frac{4y+3}{y-2}$$

Answer $f^{-1}(x) = \frac{4x+3}{x-2}$

Turn over for the next question**Turn over ►**

- 27** The line $y = 3x + p$ and the circle $x^2 + y^2 = 53$ intersect at points A and B .
 p is a positive integer.

- 27 (a)** Show that the x -coordinates of points A and B satisfy the equation

$$10x^2 + 6px + p^2 - 53 = 0$$

[3 marks]

Substitute $y = 3x + p$ into $x^2 + y^2 = 53$:

$$x^2 + (3x + p)^2 = 53$$

$$(3x + p)^2 = 9x^2 + 3px + 3px + p^2 = 9x^2 + 6px + p^2$$

$$x^2 + 9x^2 + 6px + p^2 = 53$$

$$10x^2 + 6px + p^2 - 53 = 0$$



27 (b) The coordinates of A are (2, 7)

Work out the coordinates of B.

You **must** show your working.

[5 marks]

$$A(2, 7) \text{ is on the line: } 7 = 3 \times 2 + p, \text{ so } p = 1$$

$$10x^2 + 6 \times 1 \times x + 1^2 - 53 = 0$$

$$10x^2 + 6x - 52 = 0$$

$$5x^2 + 3x - 26 = 0$$

$$(5x + 13)(x - 2) = 0$$

$$x = 2 \text{ (this is A) or } x = -13 \div 5 = -2.6$$

$$y = 3 \times (-2.6) + 1 = -7.8 + 1 = -6.8$$

Answer (-2.6 , -6.8)

Turn over for the next question

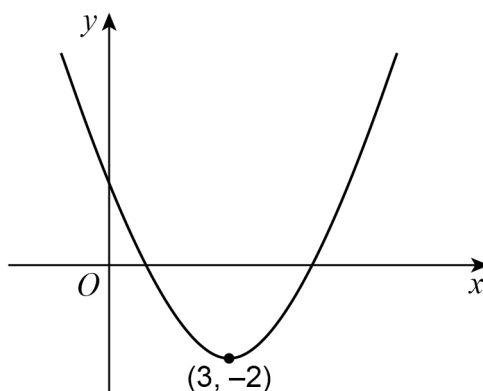


28

Here is a sketch of a quadratic curve.

The turning point is $(3, -2)$

Not drawn
accurately



Circle the correct statement about the gradient of the curve for $x < 3$

[1 mark]

gradient is positive

gradient is negative

gradient is zero

gradient could be any value

END OF QUESTIONS



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