AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS MATHEMATICS

Paper 2

Wednesday 20 May 2020 Morning Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Identify the expression below that is equivalent to $e^{\frac{-2}{5}}$

Circle your answer.

[1 mark]

$\frac{1}{\sqrt[5]{e^2}}$ $-\sqrt{e^5}$ $-\sqrt[5]{e^2}$ $\frac{1}{\sqrt{e^5}}$

2 It is given that $y = \frac{1}{x}$ and $x < -1$

Determine which statement below fully describes the possible values of y.

Tick (✓) **one** box.

[1 mark]

$-\infty < y < -1$ ☐

$y > -1$ ☐

$-1 < y < 0$ ✓

$y < 0$ ☐

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3 It is given that

$$y = 3x^4 + \frac{2}{x} - \frac{x}{4} + 1$$

Find an expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

[3 marks]

$y = 3x^4 + 2x^{-1} - \frac{1}{4}x + 1$

$\frac{dy}{dx} = 12x^3 - 2x^{-2} - \frac{1}{4}$

$\frac{d^2y}{dx^2} = 36x^2 + 4x^{-3}$

$\frac{d^2y}{dx^2} = 36x^2 + \frac{4}{x^3}$

Turn over for the next question

Do not write outside the box

4 Find all the solutions of

$$9\sin^2 x - 6\sin x + \cos^2 x = 0$$

where $0° \leq x \leq 180°$

Give your solutions to the nearest degree.

Fully justify your answer.

[4 marks]

$\cos^2 x = 1 - \sin^2 x$

$9\sin^2 x - 6\sin x + 1 - \sin^2 x = 0$

$8\sin^2 x - 6\sin x + 1 = 0$

$(4\sin x - 1)(2\sin x - 1) = 0$

$\sin x = \frac{1}{4}$ or $\sin x = \frac{1}{2}$

$\sin x = \frac{1}{4}$: $x = 14.477...°$ or $x = 180° - 14.477...° = 165.52...°$

$\sin x = \frac{1}{2}$: $x = 30°$ or $x = 180° - 30° = 150°$

$\sin x > 0$ in both cases, so each gives one solution in $0° < x < 90°$

and one in $90° < x < 180°$, giving four solutions altogether.

$x = 14°,\ 30°,\ 150°,\ 166°$

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5 Joseph is expanding $(2 - 3x)^7$ in ascending powers of x.

He states that the coefficient of the fourth term is 15120

Joseph's teacher comments that his answer is almost correct.

Using a suitable calculation, explain the teacher's comment.

[4 marks]

The fourth term in ascending powers is the x^3 term:

$T_4 = {}^7C_3 \times 2^4 \times (-3x)^3$

$= 35 \times 16 \times (-27)x^3$

$= -15120x^3$

So the coefficient of the fourth term is -15120

Joseph has the right number but the wrong sign: he has used $(+3x)^3$ instead of $(-3x)^3$, so his coefficient should be negative, -15120.

Turn over for the next question

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6 A circle has equation

$$x^2 + y^2 + 10x - 4y - 71 = 0$$

6 (a) Find the centre of the circle.

[2 marks]

$x^2 + 10x + y^2 - 4y = 71$

$(x + 5)^2 - 25 + (y - 2)^2 - 4 = 71$

$(x + 5)^2 + (y - 2)^2 = 100$

Centre $(-5, 2)$

6 (b) Hence, find the equation of the tangent to the circle at the point (1, 10), giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

[4 marks]

Gradient of radius from $(-5, 2)$ to $(1, 10)$ $= \frac{10 - 2}{1 - (-5)} = \frac{8}{6} = \frac{4}{3}$

Tangent is perpendicular to the radius, so gradient $= -\frac{3}{4}$

$y - 10 = -\frac{3}{4}(x - 1)$

$4y - 40 = -3x + 3$

$3x + 4y - 43 = 0$

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7 The population of a country was 3.6 million in 1989.

It grew exponentially to reach 6 million in 2019.

Estimate the population of the country in 2049 if the exponential growth continues unchanged.

[2 marks]

1989 to 2019 is 30 years: growth factor = $\frac{6}{3.6} = \frac{5}{3}$

2019 to 2049 is another 30 years, so the same factor applies

Population in 2049 = $6 \times \frac{6}{3.6}$

= 10 million

Turn over for the next question

Do not write outside the box

8 (a) Using $y = 2^{2x}$ as a substitution, show that

$$16^x - 2^{(2x+3)} - 9 = 0$$

can be written as

$$y^2 - 8y - 9 = 0$$

[2 marks]

$16^x = (2^4)^x = 2^{4x} = (2^{2x})^2 = y^2$

$2^{(2x+3)} = 2^{2x} \times 2^3 = 8y$

So $16^x - 2^{(2x+3)} - 9 = 0$ becomes

$y^2 - 8y - 9 = 0$

Do not write outside the box

8 (b) Hence, show that the equation

$$16^x - 2^{(2x+3)} - 9 = 0$$

has $x = \log_2 3$ as its only solution.

Fully justify your answer.

[4 marks]

$y^2 - 8y - 9 = 0 \Rightarrow (y - 9)(y + 1) = 0$

$y = 9$ or $y = -1$

$2^{2x} = 9$ or $2^{2x} = -1$

$2^{2x} > 0$ for all real x, so $2^{2x} = -1$ has no real solutions

$2^{2x} = 9 \Rightarrow 2x = \log_2 9$

$x = \frac{1}{2} \log_2 9 = \log_2 9^{\frac{1}{2}} = \log_2 3$ (since $9^{\frac{1}{2}} = 3$)

$\therefore$ $x = \log_2 3$ is the only solution

Turn over for the next question

Do not write outside the box

9 (a) (i) Find

$$\int (4x - x^3)\,\mathrm{d}x$$

[2 marks]

$\int (4x - x^3)\,dx = \frac{4x^2}{2} - \frac{x^4}{4} + c$

$= 2x^2 - \frac{x^4}{4} + c$

9 (a) (ii) Evaluate

$$\int_{-2}^{2} (4x - x^3)\,\mathrm{d}x$$

[1 mark]

$\int_{-2}^{2} (4x - x^3)\,dx = \left[2x^2 - \frac{x^4}{4}\right]_{-2}^{2}$

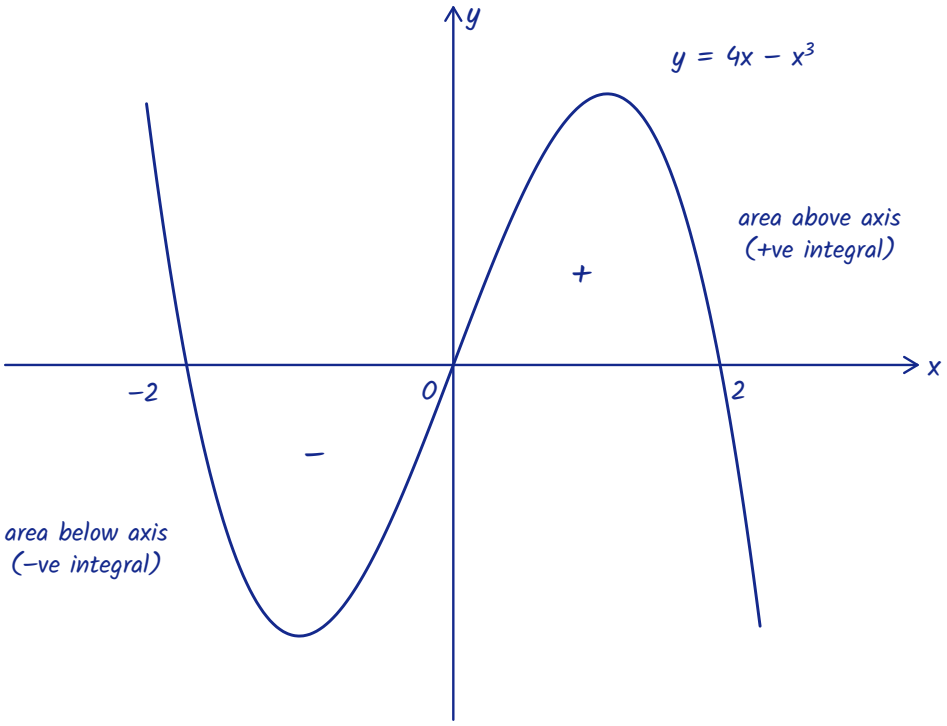
$= (8 - 4) - (8 - 4)$

$= 0$

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9 (b) Using a sketch, explain why the integral in part **(a)(ii)** does **not** give the area enclosed between the curve $y = 4x - x^3$ and the x-axis.

[2 marks]



The area between x = −2 and x = 0 lies below the x-axis, so its integral is negative; the area between 0 and 2 is above the axis and positive. By symmetry these are equal in size and cancel, so the integral gives 0, not the total area enclosed.

9 (c) Find the area enclosed between the curve $y = 4x - x^3$ and the x-axis.

[2 marks]

$\text{Area} = 2 \times \int_0^2 (4x - x^3)\,dx = 2\left[2x^2 - \frac{x^4}{4}\right]_0^2$

$= 2 \times (8 - 4)$

$= 8$

Turn over ▶

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10 A curve has gradient function

$$\frac{dy}{dx} = 3x^2 - 12x + c$$

The curve has a turning point at $(-1, 1)$

10 (a) Find the coordinates of the other turning point of the curve.

Fully justify your answer.

[6 marks]

At a turning point $\frac{dy}{dx} = 0$

At $(-1, 1)$: $3(-1)^2 - 12(-1) + c = 0$

$3 + 12 + c = 0 \Rightarrow c = -15$

$\frac{dy}{dx} = 0$: $3x^2 - 12x - 15 = 0 \Rightarrow x^2 - 4x - 5 = 0$

$(x - 5)(x + 1) = 0 \Rightarrow x = -1$ or $x = 5$

$y = \int (3x^2 - 12x - 15)\,dx = x^3 - 6x^2 - 15x + k$

At $(-1, 1)$: $1 = -1 - 6 + 15 + k \Rightarrow k = -7$

$y = x^3 - 6x^2 - 15x - 7$

At $x = 5$: $y = 125 - 150 - 75 - 7 = -107$

Other turning point is $(5, -107)$

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10 (b) Find the set of values of x for which y is increasing.

[2 marks]

y is increasing when $\frac{dy}{dx} > 0$

$3x^2 - 12x - 15 > 0 \Rightarrow 3(x - 5)(x + 1) > 0$

Positive quadratic with roots -1 and 5, so it is positive outside the roots

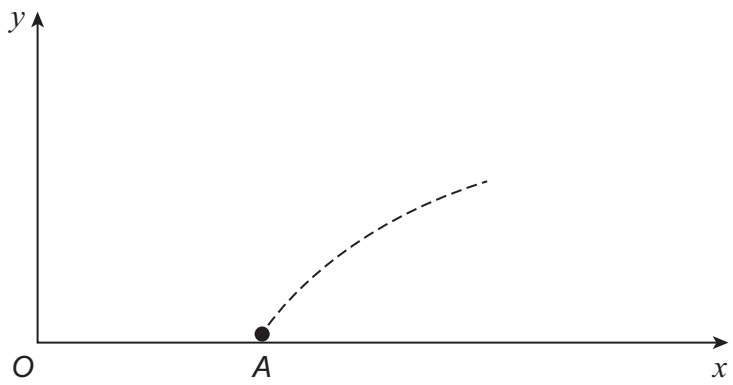
$x < -1$ or $x > 5$

Turn over for the next question

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11 A fire crew is tackling a grass fire on horizontal ground.

The crew directs a single jet of water which flows continuously from point *A*.



The path of the jet can be modelled by the equation

$$y = -0.0125x^2 + 0.5x - 2.55$$

where x metres is the horizontal distance of the jet from the fire truck at *O* and y metres is the height of the jet above the ground.

The coordinates of point *A* are $(a, 0)$

11 (a) (i) Find the value of a.

[3 marks]

At A, $y = 0$: $-0.0125x^2 + 0.5x - 2.55 = 0$

$\times (-80)$: $x^2 - 40x + 204 = 0$

$(x - 6)(x - 34) = 0 \Rightarrow x = 6$ or $x = 34$

A is where the jet leaves the ground (smaller root), so $a = 6$

11 (a) (ii) Find the horizontal distance **from *A*** to the point where the jet hits the ground.

[1 mark]

Jet hits the ground at $x = 34$: $34 - 6 = 28$ m

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11 (b) Calculate the maximum vertical height reached by the jet.

[4 marks]

$\frac{dy}{dx} = -0.025x + 0.5$

At the maximum $\frac{dy}{dx} = 0$: $-0.025x + 0.5 = 0 \Rightarrow x = 20$

$y = -0.0125(20)^2 + 0.5(20) - 2.55$

$= -5 + 10 - 2.55 = 2.45$

Maximum height = 2.45 m

11 (c) A vertical wall is located 11 metres horizontally from *A* in the direction of the jet. The height of the wall is 2.3 metres.

Using the model, determine whether the jet passes over the wall, stating any necessary modelling assumption.

[3 marks]

Wall is at $x = 6 + 11 = 17$

$y = -0.0125(17)^2 + 0.5(17) - 2.55$

$= -3.6125 + 8.5 - 2.55 = 2.3375$ m

Assumption: the jet of water has no width (and the wall has no thickness).

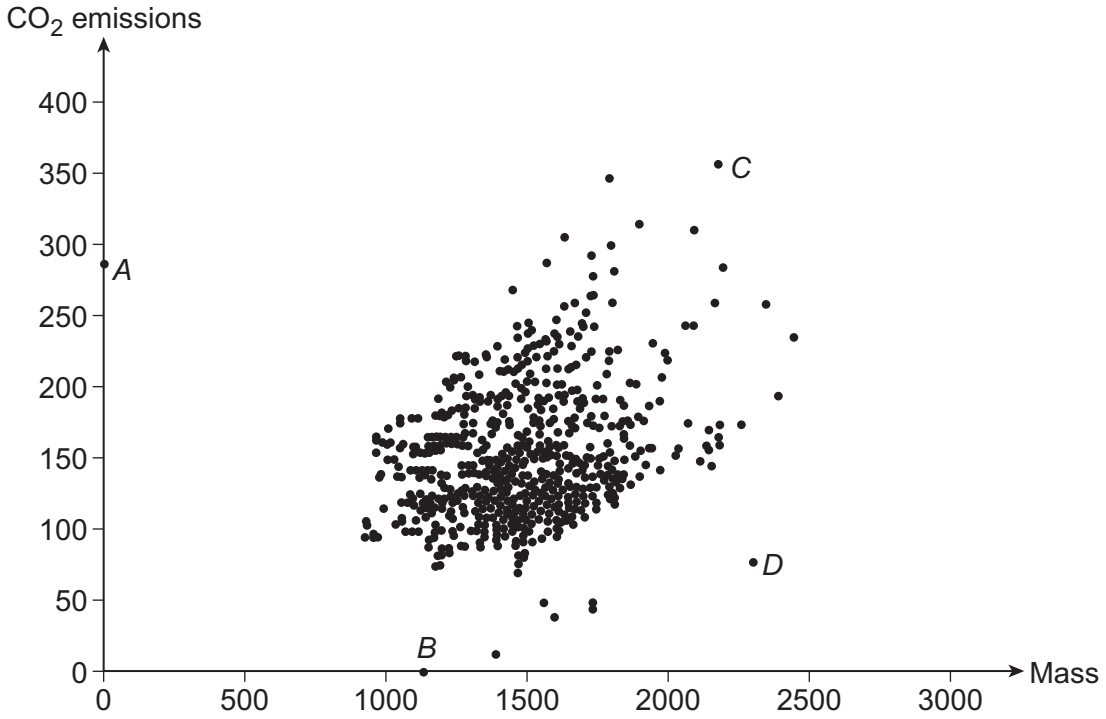
2.3375 > 2.3, so the jet passes over the wall.

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Section B

Answer **all** questions in the spaces provided.

12 A student plots the scatter diagram below showing the mass in kilograms against the CO_2 emissions in grams per kilogram for a sample of cars in the Large Data Set.



Their teacher tells them to remove an error to clean the data.

Identify the data point which should be removed.

Circle your answer below.

[1 mark]

(A) B C D

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13 The random variable X is such that $X \sim B\left(n, \frac{1}{3}\right)$

The standard deviation of X is 4

Find the value of n.

Circle your answer.

[1 mark]

9 12 18 72

$Var(X) = np(1-p) = 4^2 = 16$

$n \times \frac{1}{3} \times \frac{2}{3} = 16 \Rightarrow \frac{2n}{9} = 16 \Rightarrow n = 72$

Turn over for the next question

Do not write outside the box

14 A retail company has 5200 employees in 100 stores throughout the United Kingdom.

The company recently introduced a new reward scheme for its staff.

The management team wanted to sample the staff to find out their opinions of the new scheme.

Three possible sampling methods were suggested:

Method A	Choose 100 people who work at the largest store
Method B	Choose one person at random from each of the 100 stores
Method C	List all employees in alphabetical order and assign each a number from 1 to 5200 Choose a random number between 1 and 52 Choose this person and every 52nd person on the list thereafter.

14 (a) Give one **disadvantage** of using Method A compared with using Method B.

[1 mark]

All 100 people work at the same (largest) store, so the sample is likely to be biased: it gives no opinions from staff at the other 99 stores.

14 (b) Give one **advantage** of using Method B compared with using Method C.

[1 mark]

Method B guarantees that responses are obtained from every one of the 100 stores, so staff from all stores are represented.

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14 (c) (i) Identify the method of sampling used in Method C.

[1 mark]

Systematic sampling

14 (c) (ii) Give a reason why Method C does **not** provide a random sample.

[1 mark]

Once the random starting number is chosen, the rest of the sample is fixed, so not every possible sample of 100 employees can be chosen: not all samples of size 100 are equally likely, so it is not random.

Turn over for the next question

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15 A random sample of ten CO_2 emissions was selected from the Large Data Set.

The emissions in grams per kilogram were:

13 45 45 0 49 77 49 49 49 78

15 (a) Find the standard deviation of the sample.

[1 mark]

$n = 10$, $\Sigma x = 454$, $\Sigma x^2 = 25836$, mean $= 45.4$

$\sigma = \sqrt{\left(\frac{25836}{10} - 45.4^2\right)} = 22.856\ldots = 22.9$ (3 s.f.)

15 (b) An environmentalist calculated the average CO_2 emissions for cars in the Large Data Set registered in 2002 and in 2016.

The averages are listed below.

Year of registration	2002	2016
Average CO_2 emission	171.2	120.4

The environmentalist claims that the average CO_2 emissions for 2002 and 2016 combined is 145.8

Determine whether this claim is correct.

Fully justify your answer.

[2 marks]

$(171.2 + 120.4) \div 2 = 145.8$, so the claim just averages the two means.

This is only valid if the number of cars in 2002 and 2016 is the same.

The Large Data Set has more cars registered in 2016 than in 2002, so

the combined mean is nearer 120.4 (below 145.8): the claim is incorrect.

Do not write outside the box

16 A mathematical puzzle is published every day in a newspaper.

Over a long period of time Paula is able to solve the puzzle correctly 60% of the time.

16 (a) For a randomly chosen 14-day period find the probability that:

16 (a) (i) Paula correctly solves exactly 8 puzzles

[1 mark]

$X \sim B(14, 0.6)$: $P(X = 8) = {}^{14}C_8(0.6)^8(0.4)^6 = 0.20659... = 0.207$

16 (a) (ii) Paula correctly solves at least 7 but not more than 11 puzzles.

[2 marks]

$P(7 \leq X \leq 11) = P(X \leq 11) - P(X \leq 6)$

$= 0.96020... - 0.15014...$

$= 0.81006... = 0.810$

16 (b) State one assumption that is necessary for the distribution used in part **(a)** to be valid.

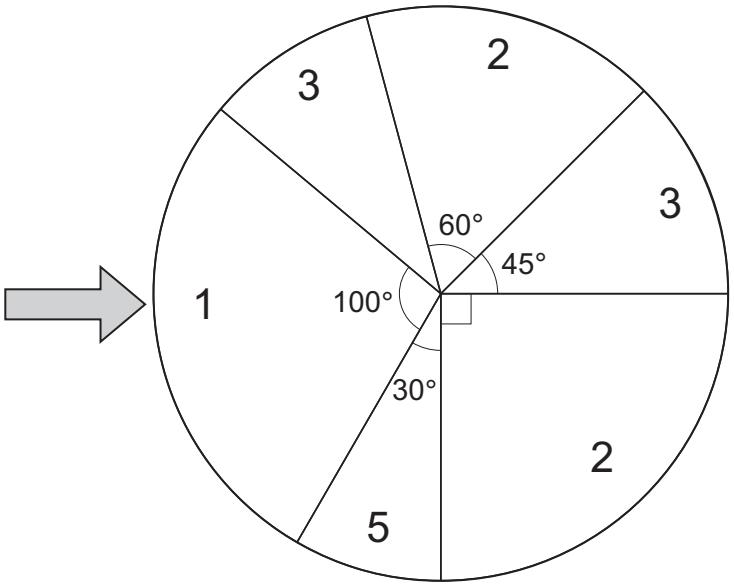
[1 mark]

Whether Paula solves one puzzle is independent of whether she solves any other puzzle.

Turn over for the next question

Do not write outside the box

17 A game consists of spinning a circular wheel divided into numbered sectors as shown below.



On each spin the score, X, is the value shown in the sector that the arrow points to when the spinner stops.

The probability of the arrow pointing at a sector is proportional to the angle subtended at the centre by that sector.

17 (a) Show that $P(X = 1) = \frac{5}{18}$

[1 mark]

Sector 1 subtends 100° out of 360° at the centre

$P(X = 1) = \frac{100}{360} = \frac{5}{18}$

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17 (b) Complete the probability distribution for X in the table below.

x	1	2	3	5
$P(X = x)$	$\frac{5}{18}$	$\frac{5}{12}$	$\frac{2}{9}$	$\frac{1}{12}$

[2 marks]

Remaining sector 3 angle = $360° - (100° + 60° + 45° + 90° + 30°) = 35°$

$P(X = 2) = \frac{60 + 90}{360} = \frac{150}{360} = \frac{5}{12}$

$P(X = 3) = \frac{35 + 45}{360} = \frac{80}{360} = \frac{2}{9}$

$P(X = 5) = \frac{30}{360} = \frac{1}{12}$

Check: $\frac{5}{18} + \frac{5}{12} + \frac{2}{9} + \frac{1}{12} = \frac{20 + 30 + 16 + 6}{72} = 1$

Turn over for the next question

Do not write outside the box

18 (a) Bag A contains 7 blue discs, 4 red discs and 1 yellow disc.

Two discs are drawn at random from bag A **without replacement**.

Find the probability that exactly **one** of the discs is blue.

[2 marks]

$P(\text{exactly one blue}) = P(B, \text{not } B) + P(\text{not } B, B)$

$= \frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{7}{11}$

$= \frac{35}{132} + \frac{35}{132} = \frac{70}{132}$

$= \frac{35}{66}$

Do not write outside the box

18 (b) Bag A contains 7 blue discs, 4 red discs and 1 yellow disc.

Bag B contains 3 blue discs and 6 red discs.

A disc is drawn at random from Bag A and placed in Bag B.

A disc is then drawn at random from Bag B.

Find the probability that the disc drawn from Bag B is red.

[3 marks]

After one disc is added, Bag B has 10 discs.

Red from A, probability $\frac{4}{12}$: Bag B then has 7 red out of 10

Not red from A, probability $\frac{8}{12}$: Bag B then has 6 red out of 10

$P(\text{red from B}) = \frac{8}{12} \times \frac{6}{10} + \frac{4}{12} \times \frac{7}{10}$

$= \frac{48}{120} + \frac{28}{120} = \frac{76}{120}$

$= \frac{19}{30}$

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19 It is known from historical data that 15% of the residents of a town buy the local weekly newspaper, 'Local News'.

A new free weekly paper is introduced into the town.

The owners of 'Local News' are interested to know whether the introduction of the free newspaper has changed the proportion of residents who buy their paper.

In a random sample of 50 residents of the town taken after the free newspaper was introduced, it was found that 3 of them purchased 'Local News' regularly.

Investigate, at the 5% significance level, whether this sample provides evidence that the proportion of local residents who buy 'Local News' has changed.

[6 marks]

p = proportion of residents who buy Local News

H_0: $p = 0.15$

H_1: $p \neq 0.15$ (two-tailed test at 5%)

X = number in the sample who buy Local News

Under H_0, $X \sim B(50, 0.15)$

Expected value $50 \times 0.15 = 7.5$; 3 is below this, so use the lower tail

$P(X \leq 3) = 0.04604... = 0.046$

$0.046 > 0.025$ (2.5% in each tail)

So do not reject H_0

There is insufficient evidence at the 5% level to suggest that the proportion of residents who buy Local News has changed.

END OF QUESTIONS

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