AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS MATHEMATICS

Paper 1

Wednesday 13 May 2020 Morning Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 At the point (1, 0) on the curve $y = \ln x$, which statement below is correct?

Tick (✓) **one** box.

[1 mark]

- The gradient is negative and decreasing ☐
- The gradient is negative and increasing ☐
- The gradient is positive and decreasing ☑
- The gradient is positive and increasing ☐

2 Given that $f(x) = 10$ when $x = 4$, which statement below must be correct?

Tick (✓) **one** box.

[1 mark]

- $f(2x) = 5$ when $x = 4$ ☐
- $f(2x) = 10$ when $x = 2$ ☑
- $f(2x) = 10$ when $x = 8$ ☐
- $f(2x) = 20$ when $x = 4$ ☐

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3 Jia has to solve the equation

$$2 - 2\sin^2\theta = \cos\theta$$

where $-180° \leq \theta \leq 180°$

Jia's working is as follows:

$$2 - 2(1 - \cos^2\theta) = \cos\theta$$
$$2 - 2 + 2\cos^2\theta = \cos\theta$$
$$2\cos^2\theta = \cos\theta$$
$$2\cos\theta = 1$$
$$\cos\theta = 0.5$$
$$\theta = 60°$$

Jia's teacher tells her that her solution is incomplete.

3 (a) Explain the **two** errors that Jia has made.

[2 marks]

Error 1: Jia divided both sides by cos θ, but cos θ could be 0. She should have factorised instead, so she has lost the solutions of cos θ = 0.

Error 2: cos θ = 0.5 has a second solution in the range, θ = −60°, which she has not given.

3 (b) Write down all the values of θ that satisfy the equation

$$2 - 2\sin^2\theta = \cos\theta$$

where $-180° \leq \theta \leq 180°$

[2 marks]

$2\cos^2\theta - \cos\theta = 0 \Rightarrow \cos\theta(2\cos\theta - 1) = 0$

$\cos\theta = 0 \Rightarrow \theta = \pm 90°$ $\quad\quad$ $\cos\theta = 0.5 \Rightarrow \theta = \pm 60°$

$\theta = -90°, -60°, 60°, 90°$

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4 In the binomial expansion of $(\sqrt{3}+\sqrt{2})^4$ there are two irrational terms.

Find the difference between these two terms.

[3 marks]

$(\sqrt{3}+\sqrt{2})^4 = (\sqrt{3})^4 + 4(\sqrt{3})^3(\sqrt{2}) + 6(\sqrt{3})^2(\sqrt{2})^2$

$+ 4(\sqrt{3})(\sqrt{2})^3 + (\sqrt{2})^4$

$= 9 + 4 \times 3\sqrt{3} \times \sqrt{2} + 6 \times 3 \times 2 + 4 \times \sqrt{3} \times 2\sqrt{2} + 4$

$= 9 + 12\sqrt{6} + 36 + 8\sqrt{6} + 4$

The irrational terms are $12\sqrt{6}$ and $8\sqrt{6}$

Difference $= 12\sqrt{6} - 8\sqrt{6}$

$= 4\sqrt{6}$

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5 Differentiate from first principles

$$y = 4x^2 + x$$

[4 marks]

$f(x) = 4x^2 + x$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h \to 0} \frac{4(x+h)^2 + (x+h) - (4x^2 + x)}{h}$

$f'(x) = \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 + x + h - 4x^2 - x}{h}$

$f'(x) = \lim_{h \to 0} \frac{8xh + 4h^2 + h}{h}$

$f'(x) = \lim_{h \to 0} (8x + 4h + 1)$

As $h \to 0$, $4h \to 0$

so $\frac{dy}{dx} = 8x + 1$

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6 (a) It is given that

$$f(x) = x^3 - x^2 + x - 6$$

Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

[2 marks]

$f(2) = 2^3 - 2^2 + 2 - 6$

$= 8 - 4 + 2 - 6 = 0$

Since $f(2) = 0$, by the factor theorem $(x - 2)$ is a factor of $f(x)$

6 (b) Find the quadratic factor of $f(x)$.

[1 mark]

$f(x) = (x - 2)(x^2 + x + 3)$

Quadratic factor: $x^2 + x + 3$

6 (c) Hence, show that there is only one real solution to $f(x) = 0$

[3 marks]

$f(x) = 0 \Rightarrow x - 2 = 0$ or $x^2 + x + 3 = 0$

For $x^2 + x + 3 = 0$: discriminant $b^2 - 4ac = 1^2 - 4 \times 1 \times 3$

$= 1 - 12 = -11 < 0$

so $x^2 + x + 3 = 0$ has no real solutions.

The only real solution comes from the factor $(x - 2)$,

so $x = 2$ is the only real solution to $f(x) = 0$

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6 (d) Find the exact value of x that solves

$$e^{3x} - e^{2x} + e^{x} - 6 = 0$$

[3 marks]

Let $y = e^x$, so $e^{3x} = y^3$ and $e^{2x} = y^2$

$y^3 - y^2 + y - 6 = 0$, i.e. $f(y) = 0$

From part (c), the only real solution is $y = 2$

$e^x = 2$

$x = \ln 2$

Turn over for the next question

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7 Curve C has equation $y = x^2$

C is translated by vector $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ to give curve C_1

Line L has equation $y = x$

L is stretched by scale factor 2 parallel to the x-axis to give line L_1

Find the exact distance between the two intersection points of C_1 and L_1

[6 marks]

C_1: $y = (x - 3)^2$

L_1: $y = \frac{1}{2}x$

$(x - 3)^2 = \frac{1}{2}x$

$x^2 - 6x + 9 = \frac{1}{2}x$

$2x^2 - 13x + 18 = 0$

$(2x - 9)(x - 2) = 0$

$x = 2$ or $x = \frac{9}{2}$

$x = 2$: $y = 1$ $\quad x = \frac{9}{2}$: $y = \frac{9}{4}$

Intersection points $(2, 1)$ and $(\frac{9}{2}, \frac{9}{4})$

$d^2 = (\frac{9}{2} - 2)^2 + (\frac{9}{4} - 1)^2 = (\frac{5}{2})^2 + (\frac{5}{4})^2$

$d^2 = \frac{25}{4} + \frac{25}{16} = \frac{125}{16}$

$d = \frac{\sqrt{125}}{4} = \frac{5\sqrt{5}}{4}$

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Turn over for the next question

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8 (a) Find the equation of the tangent to the curve $y = e^{4x}$ at the point (a, e^{4a}).

[3 marks]

$y = e^{4x} \Rightarrow \frac{dy}{dx} = 4e^{4x}$

At $x = a$, gradient $= 4e^{4a}$

$y - e^{4a} = 4e^{4a}(x - a)$

8 (b) Find the value of a for which this tangent passes through the origin.

[2 marks]

Through $(0, 0)$: $0 - e^{4a} = 4e^{4a}(0 - a)$

$-e^{4a} = -4a\, e^{4a}$

$e^{4a} \neq 0$, so $1 = 4a$

$a = \frac{1}{4}$

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8 (c) Hence, find the set of values of m for which the equation

$$e^{4x} = mx$$

has no real solutions.

[3 marks]

When $a = \frac{1}{4}$, the tangent touches the curve at $(\frac{1}{4}, e)$

and has gradient $4e^1 = 4e$, so the tangent is $y = 4ex$

Solutions of $e^{4x} = mx$ are intersections of $y = e^{4x}$ with $y = mx$,

a line through the origin.

$m = 4e$: the line touches the curve (one solution)

$m > 4e$: the line is steeper and cuts the curve twice

$m < 0$: any negative gradient line cuts the curve once

$m = 0$: $e^{4x} = 0$ has no solutions, since $e^{4x} > 0$

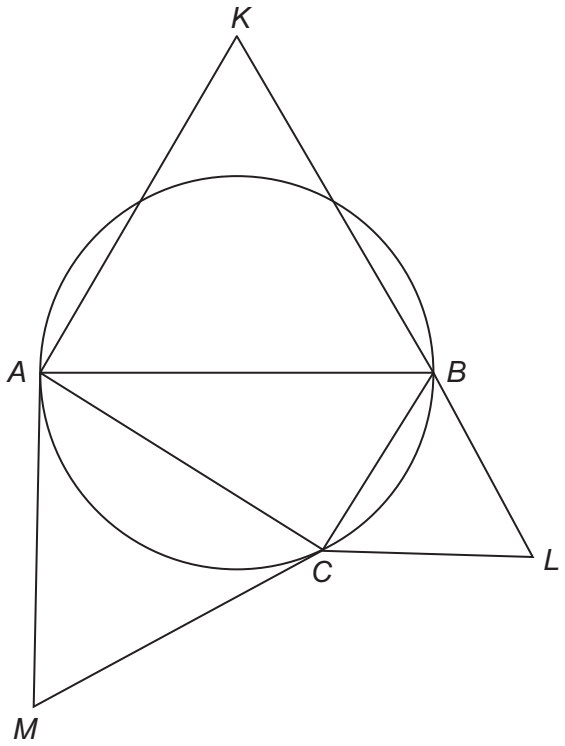
$0 < m < 4e$: the line lies below the curve, no intersection

No real solutions when $0 \leq m < 4e$

Turn over ▶

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9 The diagram below shows a circle and four triangles.



AB is a diameter of the circle. *C* is a point on the circumference of the circle.

Triangles *ABK*, *BCL* and *CAM* are equilateral.

Prove that the area of triangle *ABK* is equal to the sum of the areas of triangle *BCL* and triangle *CAM*.

[5 marks]

AB is a diameter and C is on the circumference, so angle ACB = 90°

(angle in a semicircle is a right angle).

By Pythagoras in triangle ABC: $AB^2 = AC^2 + BC^2$

Area of an equilateral triangle of side $s = \frac{1}{2} s^2 \sin 60°$

Area ABK = $\frac{1}{2} AB^2 \sin 60°$

Area BCL = $\frac{1}{2} BC^2 \sin 60°$

Area CAM = $\frac{1}{2} AC^2 \sin 60°$

Area BCL + Area CAM = $\frac{1}{2} \sin 60° (BC^2 + AC^2)$

$= \frac{1}{2} \sin 60° \times AB^2$

= Area ABK

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Hence the area of triangle ABK is equal to the sum of the areas of triangles BCL and CAM, as required.

Turn over for the next question

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10 Raj is investigating how the price, P pounds, of a brilliant-cut diamond ring is related to the weight, C carats, of the diamond.

He believes that they are connected by a formula

$$P = aC^n$$

where a and n are constants.

10 (a) Express $\ln P$ in terms of $\ln C$.

[2 marks]

$\ln P = \ln(aC^n) = \ln a + \ln C^n$

$\ln P = \ln a + n \ln C$

10 (b) Raj researches the price of three brilliant-cut diamond rings on a website with the following results.

C	0.60	1.15	1.50
P	495	1200	1720

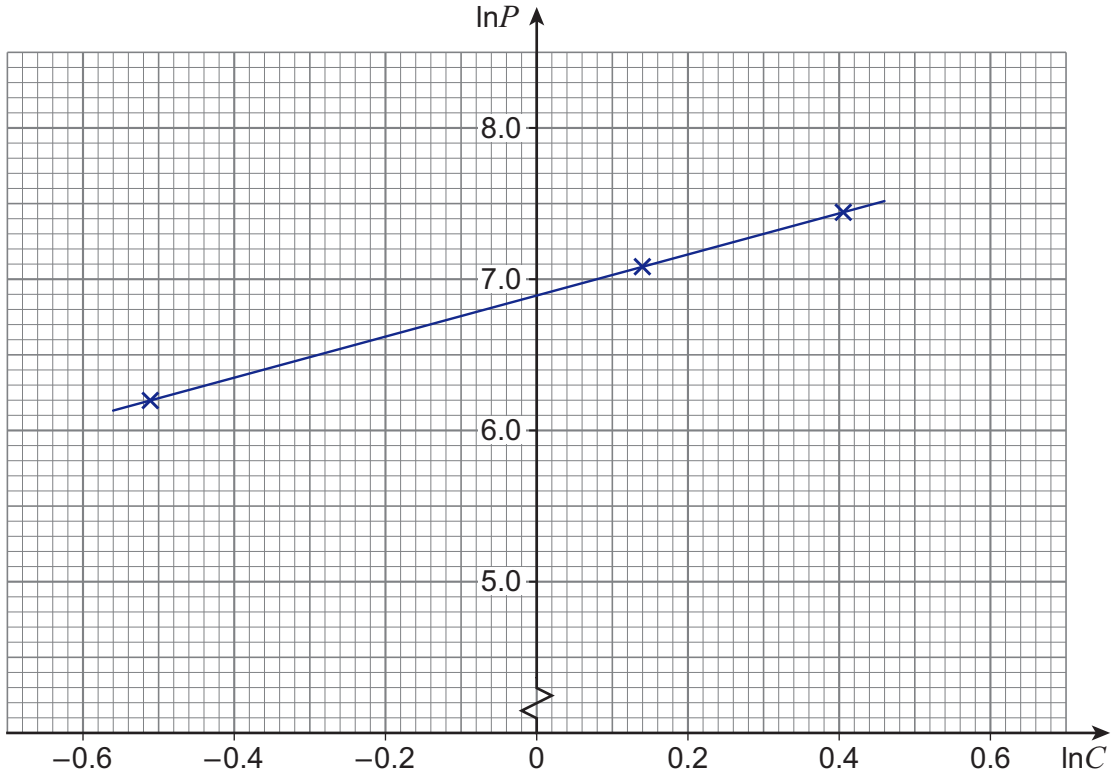
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10 (b) (i) Plot $\ln P$ against $\ln C$ for the three rings on the grid below.

[2 marks]

ln C: −0.511 0.140 0.405

ln P: 6.20 7.09 7.45



10 (b) (ii) Explain which feature of the plot suggests that Raj's belief may be correct.

[1 mark]

The three points lie (approximately) on a straight line.

Question 10 continues on the next page

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10 (b) (iii) Using the graph on page 15, estimate the value of a and the value of n.

[4 marks]

$\ln P = n \ln C + \ln a$ is a straight line, gradient n, intercept $\ln a$

Intercept on the $\ln P$ axis: $\ln a = 6.90$

$a = e^{6.90} = 992.27...$

Gradient: $n = \frac{7.45 - 6.20}{0.405 - (-0.511)} = \frac{1.25}{0.916} = 1.364...$

$a \approx 992, \quad n \approx 1.36$

10 (c) Explain the significance of a in this context.

[1 mark]

a is the price (£) of a ring with a 1 carat diamond,

since when $C = 1$, $P = a \times 1^n = a$

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10 (d) Raj wants to buy a ring with a brilliant-cut diamond of weight 2 carats.

Estimate the price of such a ring.

[2 marks]

$P = 992 \times 2^{1.36}$

$= 992 \times 2.5669... = 2546.4...$

Price $\approx$ £2550

Turn over for the next question

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Section B

Answer **all** questions in the spaces provided.

11 A go-kart and driver, of combined mass 55 kg, move forward in a straight line with a constant acceleration of $0.2\,\text{m}\,\text{s}^{-2}$

The total driving force is 14 N

Find the total resistance force acting on the go-kart and driver.

Circle your answer.

[1 mark]

0 N (3 N) 11 N 14 N

12 One of the following is an expression for the distance between the points represented by position vectors $5\mathbf{i} - 3\mathbf{j}$ and $18\mathbf{i} + 7\mathbf{j}$

Identify the correct expression.

Tick (✓) **one** box.

[1 mark]

$\sqrt{13^2 + 4^2}$ ☐

$\sqrt{13^2 + 10^2}$ ☑

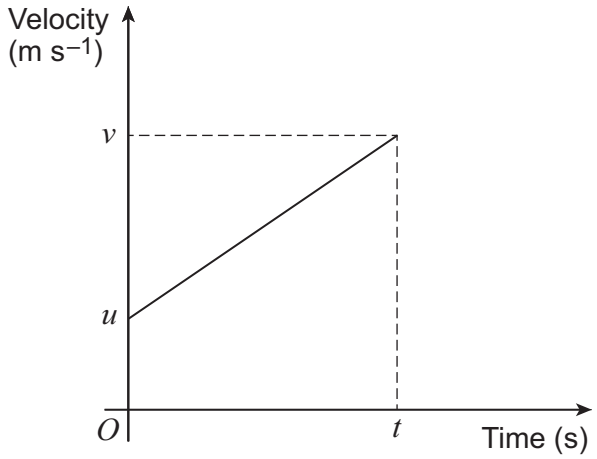
$\sqrt{23^2 + 4^2}$ ☐

$\sqrt{23^2 + 10^2}$ ☐

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13 An object is moving in a straight line, with constant acceleration $a\,\text{m}\,\text{s}^{-2}$, over a time period of t seconds.

It has an initial velocity u and final velocity v as shown in the graph below.



Use the graph to show that

$$v = u + at$$

[3 marks]

The gradient of a velocity-time graph is the acceleration.

Gradient of the line = $\frac{v-u}{t-0} = \frac{v-u}{t}$

$a = \frac{v-u}{t}$

$at = v - u$

$\therefore\ v = u + at$

Turn over for the next question

Do not write outside the box

14 A particle of mass 0.1 kg is initially stationary.

A single force **F** acts on this particle in a direction parallel to the vector $7\mathbf{i} + 24\mathbf{j}$

As a result, the particle accelerates in a straight line, reaching a speed of $4\,\text{m}\,\text{s}^{-1}$ after travelling a distance of 3.2 m

Find **F**.

[5 marks]

$u = 0, \quad v = 4, \quad s = 3.2$

$v^2 = u^2 + 2as: \quad 4^2 = 0^2 + 2 \times a \times 3.2$

$16 = 6.4a \quad \Rightarrow \quad a = 2.5 \text{ m s}^{-2}$

$F = ma = 0.1 \times 2.5 = 0.25 \text{ N} \quad \text{(magnitude of F)}$

$|7i + 24j| = \sqrt{(7^2 + 24^2)} = \sqrt{625} = 25$

$F = 0.25 \times \frac{7i + 24j}{25} = \frac{1}{100}(7i + 24j)$

$F = (0.07i + 0.24j) \text{ N}$

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Turn over for the next question

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15 A particle, P, is moving in a straight line with acceleration $a\,\text{m s}^{-2}$ at time t seconds, where

$$a = 4 - 3t^2$$

15 (a) Initially P is stationary.

Find an expression for the velocity of P in terms of t.

[2 marks]

$v = \int (4 - 3t^2)\,dt = 4t - t^3 + c$

$t = 0,\ v = 0 \quad \Rightarrow \quad c = 0$

$v = 4t - t^3$

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15 (b) When $t = 2$, the displacement of P from a fixed point, O, is 39 metres.

Find the time at which P passes through O, giving your answer to three significant figures.

Fully justify your answer.

[5 marks]

$s = \int (4t - t^3)\, dt = 2t^2 - \frac{1}{4}t^4 + k$

$t = 2,\ s = 39$: $\quad 39 = 8 - 4 + k \Rightarrow k = 35$

$s = 2t^2 - \frac{1}{4}t^4 + 35$

At O, $s = 0$: $\quad 2t^2 - \frac{1}{4}t^4 + 35 = 0$

$t^4 - 8t^2 - 140 = 0$

$t^2 = \frac{8 \pm \sqrt{(8^2 + 4 \times 140)}}{2} = \frac{8 \pm \sqrt{624}}{2}$

$t^2 = 16.489...$ or $t^2 = -8.489...$

t^2 cannot be negative, so $t^2 = 16.489...$

$t = 4.0607...$ ($t \geq 0$, time cannot be negative)

P passes through O when $t = 4.06$ s

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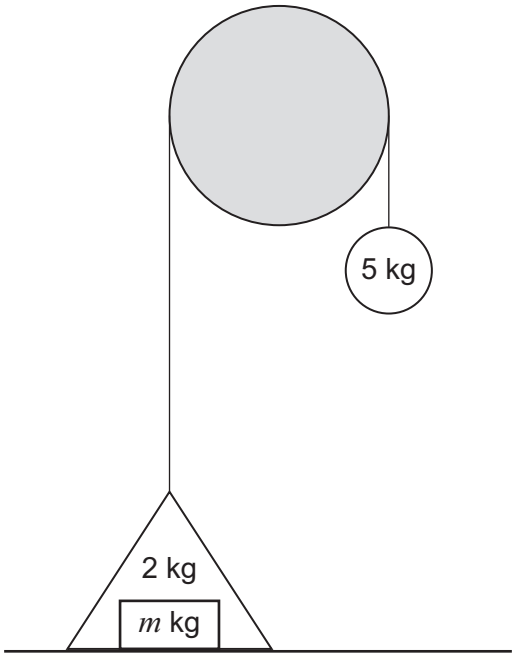
16 A simple lifting mechanism comprises a light inextensible wire which is passed over a smooth fixed pulley.

One end of the wire is attached to a rigid triangular container of mass 2 kg, which rests on horizontal ground.

A load of m kg is placed in the container.

The other end of the wire is attached to a particle of mass 5 kg, which hangs vertically downwards.

The mechanism is initially held at rest as shown in the diagram below.



The mechanism is released from rest, and the container begins to move upwards with acceleration a m s^{-2}

The wire remains taut throughout the motion.

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16 (a) Show that

$$a = \left(\frac{3-m}{m+7}\right) g$$

[4 marks]

Container and load (mass m + 2), upwards:

$T - (m + 2)g = (m + 2)a$

Particle (mass 5), downwards:

$5g - T = 5a$

Adding: $5g - (m + 2)g = (m + 2)a + 5a$

$(3 - m)g = (m + 7)a$

$\therefore\ a = \frac{3 - m}{m + 7} \times g$

16 (b) State the range of possible values of m.

[1 mark]

The container moves upwards, so $a > 0$, so $3 - m > 0$. Also $m > 0$.

$0 < m < 3$

Question 16 continues on the next page

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16 (c) **In this question use $g = 9.8\,\text{m}\,\text{s}^{-2}$**

The load reaches a height of 2 metres above the ground 1 second after it is released.

Find the mass of the load.

[4 marks]

$s = ut + \frac{1}{2}at^2$: $2 = 0 + \frac{1}{2} \times a \times 1^2$

$a = 4\ m\ s^{-2}$

$4 = \frac{3 - m}{m + 7} \times 9.8$

$4(m + 7) = 9.8(3 - m)$

$4m + 28 = 29.4 - 9.8m$

$13.8m = 1.4$

$m = \frac{1.4}{13.8} = 0.10144...$

Mass of the load = 0.10 kg

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16 (d) Ignoring air resistance, describe **one** assumption you have made in your model.

[1 mark]

The container does not reach the pulley, and the 5 kg particle does not hit the ground, during the motion.

END OF QUESTIONS

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