AQA

Please write clearly in block capitals.

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I declare this is my own work.

AS MATHEMATICS

Paper 2

Thursday 23 May 2024 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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Section A

Answer **all** questions in the spaces provided.

1 Line L has equation

$$5y = 4x + 6$$

Find the gradient of a line parallel to line L

Circle your answer.

[1 mark]

$-\frac{5}{4}$ $-\frac{4}{5}$ $\frac{4}{5}$ $\frac{5}{4}$

2 One of the equations below is true for all values of x

Identify the correct equation.

Tick (✓) **one** box.

[1 mark]

$\cos^2 x = -1 - \sin^2 x$ ☐

$\cos^2 x = -1 + \sin^2 x$ ☐

$\cos^2 x = 1 - \sin^2 x$ ✓

$\cos^2 x = 1 + \sin^2 x$ ☐

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3 It is given that

$$3 \log_a x = \log_a 72 - 2 \log_a 3$$

Solve the equation to find the value of x

Fully justify your answer.

[4 marks]

$3 \log_a x = \log_a x^3$

$2 \log_a 3 = \log_a 3^2 = \log_a 9$

$\log_a x^3 = \log_a 72 - \log_a 9$

$\log_a x^3 = \log_a \frac{72}{9} = \log_a 8$

$x^3 = 8$

$x = 2$ (the only real root, and $x > 0$ as needed for $\log_a x$)

Turn over for the next question

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4 Curve C has equation $y = 8\sin x$

4 (a) Curve C is transformed onto curve C_1 by a translation of vector $\begin{bmatrix} 0 \\ 4 \end{bmatrix}$

Find the equation of C_1

[1 mark]

$y = 8 \sin x + 4$

4 (b) Curve C is transformed onto curve C_2 by a stretch of scale factor 4 in the y direction.

Find the equation of C_2

[1 mark]

$y = 32 \sin x$

4 (c) Curve C is transformed onto curve C_3 by a stretch of scale factor 2 in the x direction.

Find the equation of C_3

[1 mark]

$y = 8 \sin \frac{x}{2}$

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5 A student suggests that for any positive integer n the value of the expression

$$4n^2 + 3$$

is always a prime number.

Prove that the student's statement is false by finding a counter example.

Fully justify your answer.

[3 marks]

$n = 1$: $4(1)^2 + 3 = 7$ (prime)

$n = 2$: $4(2)^2 + 3 = 19$ (prime)

$n = 3$: $4(3)^2 + 3 = 39$

$39 = 3 \times 13$, so 39 is not prime.

So $n = 3$ is a counter example and the student's statement is false.

Turn over for the next question

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6 In the expansion of $(3 + ax)^n$, where a and n are integers, the coefficient of x^2 is 4860

6 (a) Show that

$$3^n a^2 n(n-1) = 87480$$

[3 marks]

x^2 term: ${}^nC_2 \times 3^{n-2} \times (ax)^2$

${}^nC_2 = \frac{n(n-1)}{2}$, $3^{n-2} = \frac{3^n}{9}$

$\frac{n(n-1)}{2} \times \frac{3^n}{9} \times a^2 = 4860$

$3^n a^2 n(n-1) = 2 \times 9 \times 4860$

$3^n a^2 n(n-1) = 87480$ as required

6 (b) The constant term in the expansion is 729

The coefficient of x in the expansion is negative.

6 (b) (i) Verify that $n = 6$

[1 mark]

Constant term $= 3^n = 729$

$3^6 = 729$, so $n = 6$

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6 (b) (ii) Find the value of a

[3 marks]

$n = 6$: $3^6 \times a^2 \times 6 \times 5 = 87480$

$21870a^2 = 87480$

$a^2 = 4$

$a = 2$ or $a = -2$

Coefficient of $x = {}^6C_1 \times 3^5 \times a = 1458a$

Coefficient of x is negative, so $1458a < 0$

$a = -2$

Turn over for the next question

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7 Point A has coordinates (4, 1) and point B has coordinates (−8, 5)

7 (a) Find the equation of the perpendicular bisector of AB

[5 marks]

Midpoint of $AB = \left(\frac{4 + (-8)}{2}, \frac{1 + 5}{2}\right) = (-2, 3)$

Gradient of $AB = \frac{5 - 1}{-8 - 4} = \frac{4}{-12} = -\frac{1}{3}$

Gradient of perpendicular bisector $= 3$

$y - 3 = 3(x + 2)$

$y = 3x + 9$

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7 (b) A circle passes through the points A and B

A diameter of the circle lies along the x-axis.

Find the equation of the circle.

[4 marks]

The centre lies on the perpendicular bisector of AB and on the x-axis.

$y = 0$: $0 = 3x + 9 \Rightarrow x = -3$, centre $(-3, 0)$

$r^2 = (4 - (-3))^2 + (1 - 0)^2 = 49 + 1 = 50$

$r = \sqrt{50}$

Check B: $(-8 + 3)^2 + 5^2 = 25 + 25 = 50$

$(x + 3)^2 + y^2 = 50$

Turn over for the next question

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8 Prove that the graph of the curve with equation

$$y = x^3 + 15x - \frac{18}{x}$$

has no stationary points.

[5 marks]

$y = x^3 + 15x - 18x^{-1}$

$\frac{dy}{dx} = 3x^2 + 15 + 18x^{-2} = 3x^2 + 15 + \frac{18}{x^2}$

For stationary points $\frac{dy}{dx} = 0$:

$3x^2 + 15 + \frac{18}{x^2} = 0$

$3x^4 + 15x^2 + 18 = 0$

$x^4 + 5x^2 + 6 = 0$

$(x^2 + 2)(x^2 + 3) = 0$

$x^2 = -2$ or $x^2 = -3$

x^2 cannot be negative for real x, so there are no real solutions for x.

$\therefore \frac{dy}{dx} \neq 0$ for any x, so the curve has no stationary points.

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Turn over for the next question

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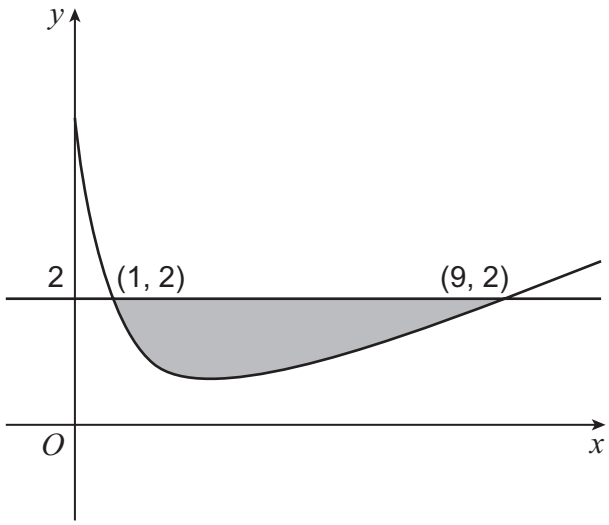
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9 A curve has equation

$$y = x - a\sqrt{x} + b$$

where a and b are constants.

The curve intersects the line $y = 2$ at points with coordinates (1, 2) and (9, 2), as shown in the diagram below.



9 (a) Show that a has the value 4 and find the value of b

[3 marks]

(1, 2): $2 = 1 - a\sqrt{1} + b \Rightarrow 1 = -a + b$ (1)

(9, 2): $2 = 9 - a\sqrt{9} + b \Rightarrow -7 = -3a + b$ (2)

(1) − (2): $8 = 2a$

$a = 4$ as required

$b = 1 + a = 1 + 4$

$b = 5$

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9 (b) On the diagram, the region enclosed between the curve and the line $y = 2$ is shaded.

Show that the area of this shaded region is $\frac{16}{3}$

Fully justify your answer.

[6 marks]

$y = x - 4\sqrt{x} + 5 = x - 4x^{1/2} + 5$

Area between curve and x-axis from $x = 1$ to $x = 9$:

$\int_1^9 (x - 4x^{1/2} + 5)\,dx = \left[\frac{1}{2}x^2 - \frac{8}{3}x^{3/2} + 5x\right]_1^9$

$= \left(\frac{81}{2} - 72 + 45\right) - \left(\frac{1}{2} - \frac{8}{3} + 5\right)$

$= \frac{27}{2} - \frac{17}{6} = \frac{32}{3}$

The curve lies below $y = 2$ and above the x-axis for $1 < x < 9$.

Rectangle under $y = 2$ from $x = 1$ to 9: area $= 2 \times 8 = 16$

Shaded area $= 16 - \frac{32}{3}$

$= \frac{16}{3}$ as required

Turn over for the next question

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10 A singer has a social media account with a number of followers. The singer releases a new song and the number of followers grows exponentially.

The number of followers, F, may be modelled by the formula

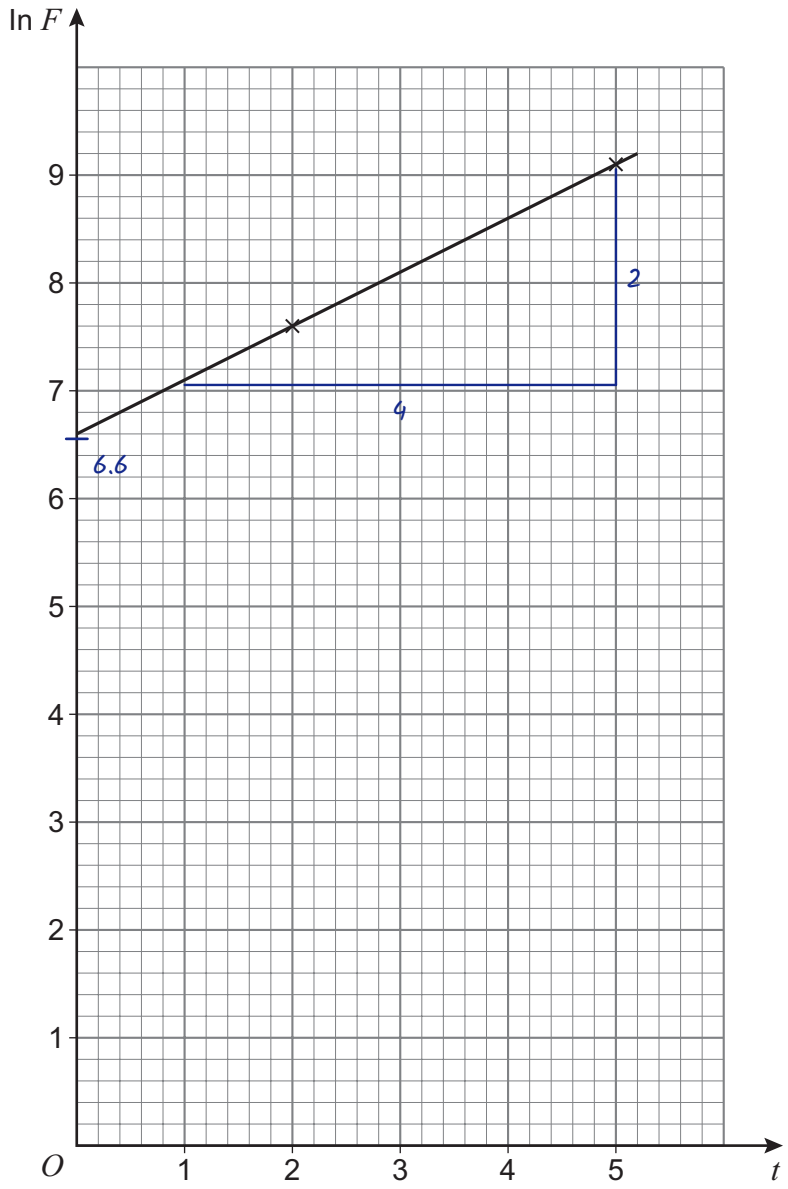
$$F = ae^{kt}$$

where t is the number of days since the song was released and a and k are constants.

- Two days after the song is released the account has 2050 followers.
- Five days after the song is released the account has 9200 followers.

On the graph below ln F has been plotted against t for these two pieces of data.

A line has been drawn passing through these two data points.



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10 (a) (i) Show that $\ln F = \ln a + kt$

[2 marks]

$F = ae^{kt}$

$\ln F = \ln(ae^{kt})$

$\ln F = \ln a + \ln e^{kt}$

$\ln F = \ln a + kt$ as required

10 (a) (ii) Using the graph, estimate the value of the constant a and the value of the constant k

[4 marks]

Intercept on the ln F axis: $\ln a = 6.6$

$a = e^{6.6} = 735.09...$

Gradient $= k = \frac{9.1 - 7.1}{5 - 1} = \frac{2}{4} = 0.5$

$a \approx 735, \quad k \approx 0.5$

Question 10 continues on the next page

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10 (b) (i) Show that $\frac{\mathrm{d}F}{\mathrm{d}t} = kF$

[2 marks]

$F = ae^{kt}$

$\frac{dF}{dt} = a \times ke^{kt} = k(ae^{kt})$

$\frac{dF}{dt} = kF$ as required

10 (b) (ii) Using the model, estimate the **rate** at which the number of followers is increasing 5 days after the song is released.

[2 marks]

At $t = 5$, $F = 9200$

$\frac{dF}{dt} = kF = 0.5 \times 9200 = 4600$

About 4600 followers per day

10 (c) The singer claims that 30 days after the song is released, the account will have more than a billion followers.

Comment on the singer's claim.

[1 mark]

Unlikely: in reality the number of followers will not keep growing exponentially (the model is extrapolated far beyond days 2 to 5), so the claim is unrealistic.

END OF SECTION A

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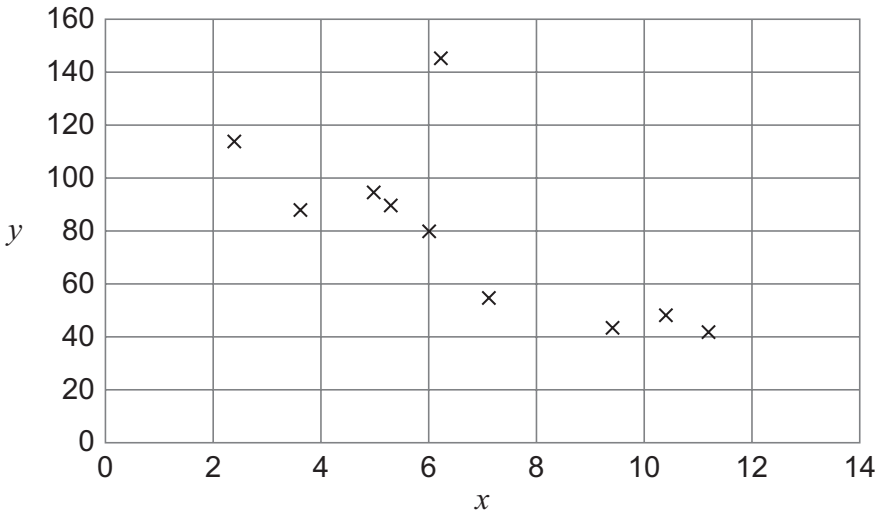
Section B

Answer **all** questions in the spaces provided.

11 The table below shows the daily salt intake, x grams, and the daily Vitamin C intake, y milligrams, for a group of 10 adults.

Adult	A	B	C	D	E	F	G	H	I	J
x	5.3	6.2	3.6	10.4	2.4	9.4	6	5	7.1	11.2
y	90	145	88	48	114	44	80	95	55	41

A scatter diagram of the data is shown below.



One of the adults is an outlier. Identify the letter of the adult that is the outlier.

Circle your answer below.

[1 mark]

A **B** **E** **J**

12 Which **one** of the following is **not** a measure of spread?

Circle your answer.

[1 mark]

median range standard deviation variance

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13 The headteacher of a school wishes to collect the opinions of the students on a new timetable structure.

To do this, a random sample of size 50, stratified by year group, will be selected.

The school has a total of 720 students.

The number of students in each of the year groups at this school is shown below.

Year group	10	11	12	13
Number of students	200	240	150	130

13 (a) Find the number of students from each year group that should be selected in the stratified random sample.

[3 marks]

Year 10: $\frac{200}{720} \times 50 = 13.88... \approx 14$

Year 11: $\frac{240}{720} \times 50 = 16.66... \approx 17$

Year 12: $\frac{150}{720} \times 50 = 10.41... \approx 10$

Year 13: $\frac{130}{720} \times 50 = 9.02... \approx 9$

Check: $14 + 17 + 10 + 9 = 50$

Year 10: 14, Year 11: 17, Year 12: 10, Year 13: 9

13 (b) State **one** advantage of using a stratified random sample.

[1 mark]

It ensures that every year group is represented in the sample, in proportion to its size.

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14 The discrete random variables X and Y can be modelled by the distributions

$$X \sim \text{B}(40 , p)$$

$$Y \sim \text{B}(25 , 0.6)$$

It is given that the mean of X is equal to the variance of Y

14 (a) Find the value of p

[3 marks]

Mean of $X = 40p$

$\text{Var}(Y) = 25 \times 0.6 \times 0.4 = 6$

$40p = 6$

$p = \frac{6}{40} = \frac{3}{20}$

14 (b) Find $\text{P}(Y = 17)$

[1 mark]

$\text{P}(Y = 17) = {}^{25}C_{17} \times 0.6^{17} \times 0.4^{8} = 0.11997... = 0.120$

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15 The number of flowers which grow on a certain type of plant can be modelled by the discrete random variable X

The probability distribution of X is given in the table below.

x	0	1	2	3	4	5 or more
$\mathbf{P}(X = x)$	0.03	0.15	0.22	0.31	0.09	p

15 (a) Find the value of p

[2 marks]

$0.03 + 0.15 + 0.22 + 0.31 + 0.09 + p = 1$

$0.8 + p = 1$

$p = 0.2$

15 (b) Two plants of this type are randomly selected from a large batch received from a local garden centre.

Find the probability that the two plants will produce a **total** of three flowers.

[3 marks]

Total of 3: $(0, 3), (3, 0), (1, 2), (2, 1)$

$P = 2 \times P(X = 0)P(X = 3) + 2 \times P(X = 1)P(X = 2)$

$= 2 \times 0.03 \times 0.31 + 2 \times 0.15 \times 0.22$

$= 0.0186 + 0.066$

$= 0.0846$

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15 (c) (i) State **one** assumption necessary for the calculation in part **(b)** to be valid.

[1 mark]

The number of flowers on one plant is independent of the number of flowers on the other plant.

15 (c) (ii) Comment on whether, in reality, this assumption is likely to be valid.

[1 mark]

Probably not valid: plants from the same batch are likely to be grown in the same soil and conditions, so their numbers of flowers may not be independent.

Turn over for the next question

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16 An investigation into the hydrocarbon emissions, X g/km, from cars in the Large Data Set was carried out.

The results are summarised below.

$$\sum x = 128.657 \qquad \sum x^2 = 8.701\,707 \qquad n = 2405$$

where n is the total number of cars which had a measured hydrocarbon emission in the Large Data Set.

16 (a) (i) Find the mean of X

[1 mark]

Mean $= \frac{128.657}{2405} = 0.053495\ldots = 0.0535$ g/km

16 (a) (ii) Find the standard deviation of X

[2 marks]

Variance $= \frac{8.701707}{2405} - \left(\frac{128.657}{2405}\right)^2$

$= 0.00075639\ldots$

sd $= \sqrt{0.00075639\ldots} = 0.027502\ldots = 0.0275$ g/km

16 (b) (i) The Large Data Set is a sample taken from the entire UK Department for Transport Stock Vehicle Database.

It is claimed that the values in part **(a)(i)** and part **(a)(ii)** obtained from the Large Data Set should be reliable estimates for the mean and standard deviation of the hydrocarbon emissions for the entire UK Department for Transport Stock Vehicle Database.

State, with a reason, whether this claim is likely to be correct.

[1 mark]

Likely to be incorrect: the Large Data Set only includes cars from certain regions and two years, so it may not be representative of the whole database.

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16 (b) (ii) State **one** type of emission where **more than** 80% of the data is known for cars in the entire UK Department for Transport Stock Vehicle Database.

[1 mark]

CO_2 (carbon dioxide emissions)

Turn over for the next question

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17 The proportion of vegans in a city is thought to be 8%

The owner of an organic food café in this city believes that the proportion of their customers who are vegan is greater than 8%

To test this belief, a random sample of 50 customers at the café were interviewed and it was found that 7 of them were vegan.

Investigate, at the 5% level, whether this sample supports the owner's belief.

[5 marks]

p = proportion of the café's customers who are vegan

H_0: $p = 0.08$

H_1: $p > 0.08$

X = number of vegans in the sample; under H_0, $X \sim B(50, 0.08)$

$P(X \geq 7) = 1 - P(X \leq 6)$

$= 1 - 0.89812...$

$= 0.10187... = 0.102$

$0.102 > 0.05$, so do not reject H_0

There is insufficient evidence at the 5% level to suggest that the proportion of the café's customers who are vegan is greater than 8%.

END OF QUESTIONS

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