AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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Candidate signature _____

I declare this is my own work.

AS MATHEMATICS

Paper 1

Thursday 16 May 2024 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 It is given that $\tan \theta° = k$, where k is a constant.

Find $\tan (\theta + 180)°$

Circle your answer.

[1 mark]

$-k$ $\quad -\frac{1}{k}$ $\quad \frac{1}{k}$ $\quad (k)$

2 Curve C has equation $y = \frac{1}{(x-1)^2}$

State the equations of the asymptotes to curve C

Tick (✓) **one** box.

[1 mark]

$x = 0$ and $y = 0$ ☐

$x = 0$ and $y = 1$ ☐

$x = 1$ and $y = 0$ ☑

$x = 1$ and $y = 1$ ☐

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3 Express $\dfrac{\sqrt{3}+3\sqrt{5}}{\sqrt{5}-\sqrt{3}}$ in the form $a+\sqrt{b}$, where a and b are integers.

Fully justify your answer.

[4 marks]

$\dfrac{\sqrt{3}+3\sqrt{5}}{\sqrt{5}-\sqrt{3}} \times \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ (multiply by the conjugate)

$= \dfrac{\sqrt{15}+\sqrt{9}+3\sqrt{25}+3\sqrt{15}}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \dfrac{\sqrt{15}+3+15+3\sqrt{15}}{5-3}$

$= \dfrac{18+4\sqrt{15}}{2}$

$= 9+2\sqrt{15}$

$= 9+\sqrt{4}\times\sqrt{15} = 9+\sqrt{60}$

$\therefore\ a=9,\ b=60$

Turn over for the next question

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4 (a) (i) By using a suitable trigonometric identity, show that the equation

$$\sin\theta \tan\theta = 4\cos\theta$$

can be written as

$$\tan^2\theta = 4$$

[1 mark]

Using $\tan\theta = \frac{\sin\theta}{\cos\theta}$, divide both sides by $\cos\theta$:

$\frac{\sin\theta}{\cos\theta} \times \tan\theta = 4 \Rightarrow \tan\theta \times \tan\theta = 4$

$\therefore \tan^2\theta = 4$ as required

4 (a) (ii) Hence solve the equation

$$\sin\theta \tan\theta = 4\cos\theta$$

where $0° < \theta < 360°$

Give your answers to the nearest degree.

[3 marks]

$\tan^2\theta = 4 \Rightarrow \tan\theta = 2$ or $\tan\theta = -2$

$\tan\theta = 2$: $\theta = \tan^{-1} 2 = 63.43...°$

$\theta = 180° + 63.43...° = 243.43...°$

$\tan\theta = -2$: $\theta = 180° - 63.43...° = 116.56...°$

$\theta = 360° - 63.43...° = 296.56...°$

$\theta = 63°, 117°, 243°, 297°$ (nearest degree)

Do not write outside the box

4 (b) Deduce all solutions of the equation

$$\sin 3\alpha \tan 3\alpha = 4 \cos 3\alpha$$

where $0° < \alpha < 180°$

Give your answers to the nearest degree.

[3 marks]

This is the equation in (a) with $\theta = 3\alpha$, so $\tan 3\alpha = \pm 2$

$0° < \alpha < 180° \Rightarrow 0° < 3\alpha < 540°$

$3\alpha = 63.43°, 116.57°, 243.43°, 296.57°, 423.43°, 476.57°$

Divide by 3:

$\alpha = 21.14°, 38.86°, 81.14°, 98.86°, 141.14°, 158.86°$

$\alpha = 21°, 39°, 81°, 99°, 141°, 159°$ (nearest degree)

Turn over for the next question

Do not write outside the box

5 A student is looking for factors of the polynomial $f(x)$

They suggest that $(x - 2)$ is a factor of $f(x)$

The method they use to check this suggestion is to calculate $f(-2)$
They correctly calculate that $f(-2) = 0$

They conclude that their suggestion is correct.

5 (a) Make **one** comment about the student's **method**.

[1 mark]

The student should have calculated f(2), not f(−2).

By the factor theorem, (x − 2) is a factor only if f(2) = 0

5 (b) Make **two** comments about the student's **conclusion**.

[2 marks]

1 f(−2) = 0 shows that (x + 2) is a factor of f(x)

2 f(−2) = 0 does not tell us whether (x − 2) is a factor or not:

it may or may not be a factor (they need to check f(2) = 0)

Do not write outside the box

6 Determine the set of values of x which satisfy the inequality

$$3x^2 + 3x > x + 6$$

Give your answer in exact form using set notation.

[4 marks]

$3x^2 + 3x - x - 6 > 0$

$3x^2 + 2x - 6 > 0$

Critical values: $3x^2 + 2x - 6 = 0$

$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-6)}}{2 \times 3} = \frac{-2 \pm \sqrt{76}}{6} = \frac{-2 \pm 2\sqrt{19}}{6} = \frac{-1 \pm \sqrt{19}}{3}$

Coefficient of x^2 is positive, so the quadratic is > 0 outside the roots:

$x < \frac{-1 - \sqrt{19}}{3}$ or $x > \frac{-1 + \sqrt{19}}{3}$

$\{ x : x < \frac{-1 - \sqrt{19}}{3} \} \cup \{ x : x > \frac{-1 + \sqrt{19}}{3} \}$

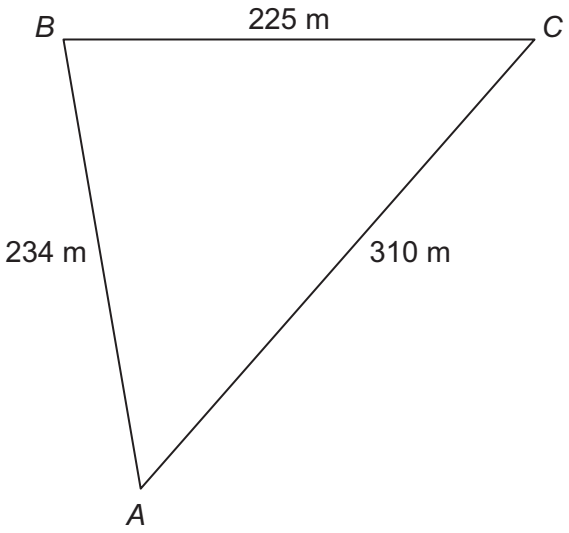
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7 A triangular field of grass, ABC, has boundaries with lengths as follows:

$AB = 234$ m $\quad BC = 225$ m $\quad AC = 310$ m

The field is shown in the diagram below.



7 (a) Find angle A

[2 marks]

$a^2 = b^2 + c^2 - 2bc \cos A$: $\quad 225^2 = 234^2 + 310^2 - 2 \times 234 \times 310 \cos A$

$\cos A = \dfrac{234^2 + 310^2 - 225^2}{2 \times 234 \times 310} = \dfrac{100231}{145080}$

$\cos A = 0.690867...$

$A = \cos^{-1}(0.690867...) = 46.301...°$

$A = 46.3°$ (1 d.p.)

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7 (b) Farmers calculate the number of sheep they can keep in a field, by allowing one sheep for every 1200 m^2 of grass.

Find the maximum number of sheep which can be kept in the field *ABC*

[3 marks]

Area = $\frac{1}{2}$ bc sin A = $\frac{1}{2}$ × 234 × 310 × sin 46.301...°

= 26222.5... m^2

Number of sheep = 26222.5... ÷ 1200

= 21.85...

Must round down: 22 sheep would need 22 × 1200 = 26400 m^2

which is more than the area of the field.

Maximum number of sheep = 21

Turn over for the next question

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8 It is given that

$$\ln x - \ln y = 3$$

8 (a) Express x in terms of y in a form not involving logarithms.

[3 marks]

$\ln x - \ln y = 3$

$\ln \frac{x}{y} = 3$

$\frac{x}{y} = e^3$

$x = e^3 y$

8 (b) Given also that

$$x + y = 10$$

find the exact value of y and the exact value of x

[3 marks]

Substitute $x = e^3 y$ into $x + y = 10$:

$e^3 y + y = 10$

$y(e^3 + 1) = 10$

$y = \frac{10}{1 + e^3}$

$x = e^3 y = e^3 \times \frac{10}{1 + e^3}$

$y = \frac{10}{1 + e^3}$ $\quad x = \frac{10e^3}{1 + e^3}$

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9 A curve has equation $y = f(x)$ where

$$f(x) = x\,(6 - x)$$

9 (a) Find $f'(x)$

[2 marks]

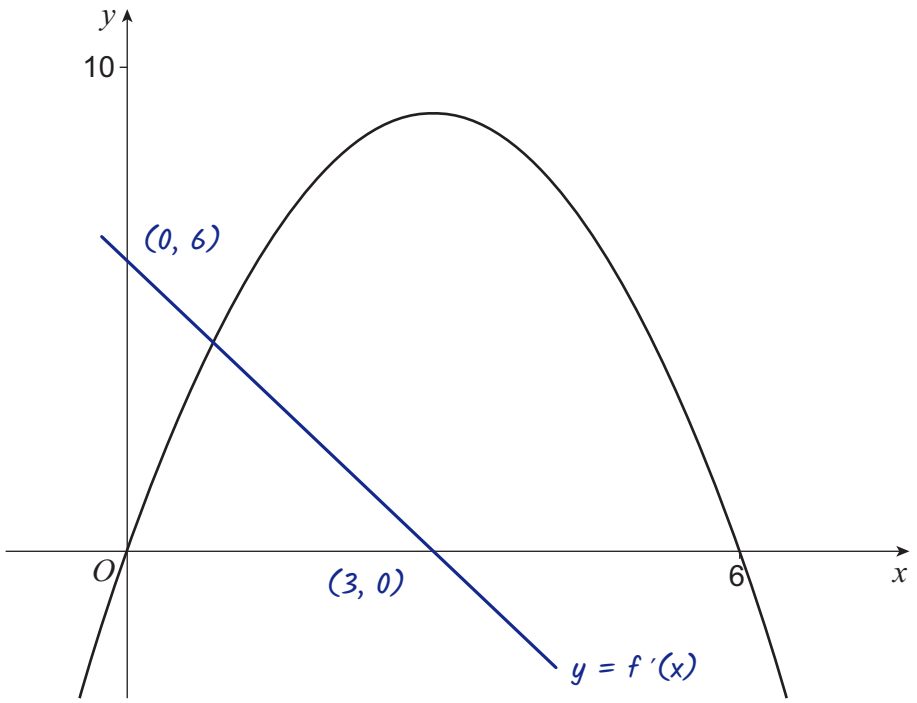
$f(x) = x(6 - x) = 6x - x^2$

$f'(x) = 6 - 2x$

9 (b) The diagram below shows the graph of $y = f(x)$

On the same diagram sketch the gradient function for this curve, stating the coordinates of any points where the gradient function cuts the axes.

[3 marks]



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10 It is given that

$$\frac{dy}{dx} = (x + 2)(2x - 1)^2$$

and when $x = 6$, $y = 900$

Find y in terms of x

[6 marks]

$\frac{dy}{dx} = (x + 2)(2x - 1)^2 = (x + 2)(4x^2 - 4x + 1)$

$= 4x^3 - 4x^2 + x + 8x^2 - 8x + 2$

$= 4x^3 + 4x^2 - 7x + 2$

$y = \int (4x^3 + 4x^2 - 7x + 2)\, dx$

$y = x^4 + \frac{4}{3}x^3 - \frac{7}{2}x^2 + 2x + c$

When $x = 6$, $y = 900$:

$900 = 6^4 + \frac{4}{3}(6^3) - \frac{7}{2}(6^2) + 2(6) + c$

$900 = 1296 + 288 - 126 + 12 + c$

$900 = 1470 + c \quad \Rightarrow \quad c = -570$

$y = x^4 + \frac{4}{3}x^3 - \frac{7}{2}x^2 + 2x - 570$

Do not write outside the box

11 It is given that for the continuous function g

- $g'(1) = 0$
- $g'(4) = 0$
- $g''(x) = 2x - 5$

11 (a) Determine the nature of each of the turning points of g

Fully justify your answer.

[3 marks]

$g''(1) = 2(1) - 5 = -3$

$-3 < 0$, so the turning point at $x = 1$ is a maximum

$g''(4) = 2(4) - 5 = 3$

$3 > 0$, so the turning point at $x = 4$ is a minimum

11 (b) Find the set of values of x for which g is an increasing function.

[2 marks]

$g'(x) = \int (2x - 5)\,dx = x^2 - 5x + c, \quad g'(1) = 0 \Rightarrow 1 - 5 + c = 0 \Rightarrow c = 4$

$g'(x) = x^2 - 5x + 4 = (x - 1)(x - 4)$

g is increasing when $g'(x) > 0$: $x < 1$ or $x > 4$

$\{ x : x < 1 \} \cup \{ x : x > 4 \}$

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12 The monthly mean temperature of a city, T degrees Celsius, may be modelled by the equation

$$T = 15 + 8\sin(30m - 120)^\circ$$

where m is the month number, counting January = 1, February = 2, through to December = 12

12 (a) Using this model, calculate the monthly mean temperature of the city for May, the fifth month.

[2 marks]

$m = 5$: $T = 15 + 8 \sin (30 \times 5 - 120)^\circ$

$= 15 + 8 \sin 30^\circ$

$= 15 + 8 \times 0.5$

$T = 19\ ^\circ C$

12 (b) Using this model, find the month with the highest mean temperature.

[2 marks]

T is greatest when $\sin (30m - 120)^\circ = 1$ (its maximum value)

$30m - 120 = 90$

$30m = 210 \Rightarrow m = 7$

Highest mean temperature is in July ($m = 7$)

12 (c) Climate change may affect the parameters, 8, 30, 120 and 15, used in this model.

12 (c) (i) State, with a reason, which parameter would be increased because of an overall rise in temperatures.

[1 mark]

15: increasing 15 translates the graph upwards, so the mean temperature of every month is raised by the same amount.

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12 (c) (ii) State, with a reason, which parameter would be increased because of the occurrence of more extreme temperatures.

[1 mark]

B: it is the amplitude; increasing B makes the high temperatures higher and the low temperatures lower (more extreme).

END OF SECTION A

Turn over for Section B

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Section B

Answer **all** questions in the spaces provided.

13 A particle is moving in a straight line with constant acceleration a m s^{-2}

The particle's velocity, v m s^{-1}, varies with time, t seconds, so that

$$v = 3 - 4t$$

Deduce the value of a

Circle your answer.

[1 mark]

(−4) −1 3 4

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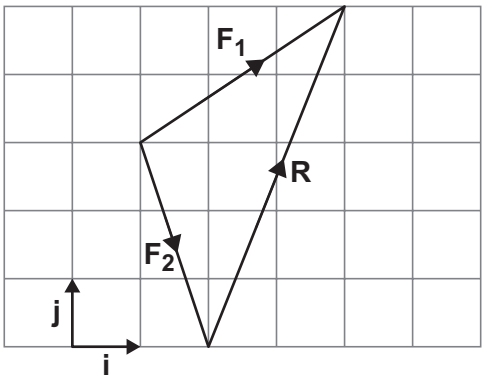
14 Two forces, $\mathbf{F_1} = 3\mathbf{i} + 2\mathbf{j}$ newtons and $\mathbf{F_2} = \mathbf{i} - 3\mathbf{j}$ newtons, are added together to find a resultant force, **R** newtons.

This vector addition can be represented using a diagram.

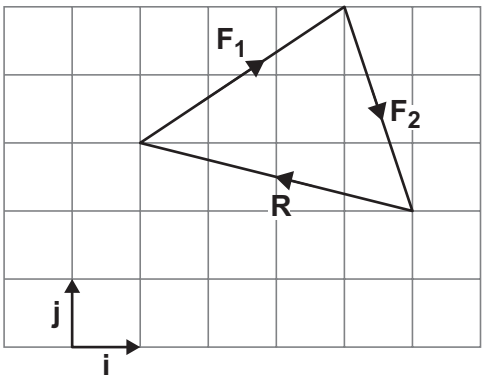
Identify the diagram below which correctly represents this vector addition.

Tick (✓) **one** box.

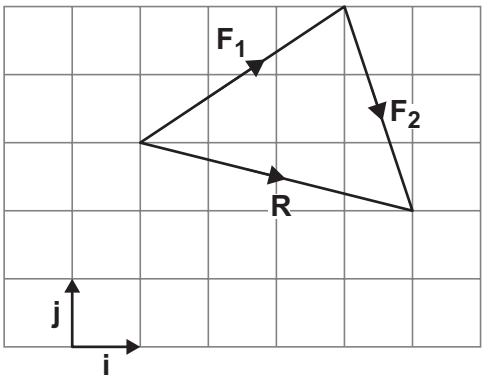
[1 mark]



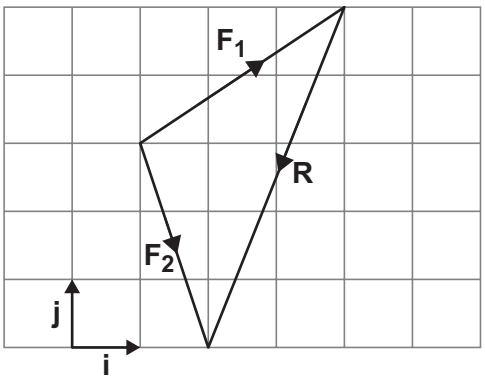
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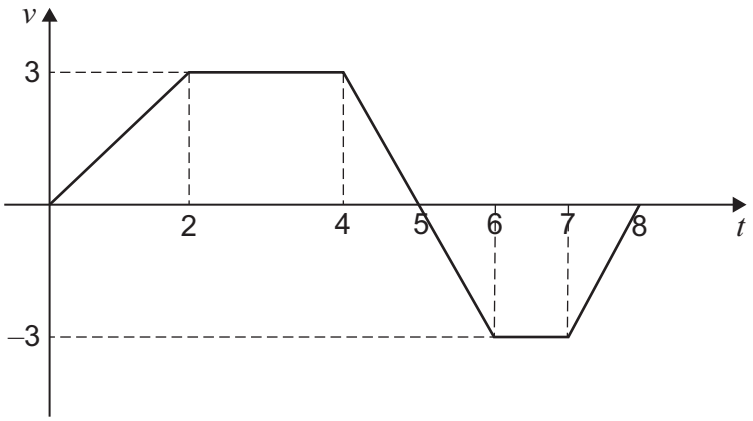


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Turn over for the next question

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15 A graph indicating how the velocity, v m s^{-1}, of a particle changes with respect to time, t seconds, is shown below.



15 (a) Find the total distance travelled by the particle over the 8 second period shown.

[3 marks]

Distance = total area between the graph and the t-axis

Area above axis ($0 \leq t \leq 5$): $\frac{1}{2} \times (5 + 2) \times 3 = 10.5$

Area below axis ($5 \leq t \leq 8$): $\frac{1}{2} \times (3 + 1) \times 3 = 6$

Total distance = 10.5 + 6

Total distance travelled = 16.5 m

Do not write outside the box

15 (b) A student claims that

"The displacement of the particle is less than the distance travelled."

State the range of values of t for which this claim is true.

[1 mark]

After $t = 5$ the velocity is negative, so the particle moves back towards its start: displacement < distance for $t > 5$

$t > 5$ (i.e. $5 < t \leq 8$)

Turn over for the next question

Do not write outside the box

16 **In this question use $g = 9.8$ m s^{-2}**

A ball is launched vertically upwards from the Earth's surface with velocity u m s^{-1}

The ball reaches a maximum height of 15 metres.

You may assume that air resistance can be ignored.

Find the value of u

[3 marks]

Taking upwards as positive: $u = u$, $v = 0$, $a = -9.8$, $s = 15$

$v^2 = u^2 + 2as$

$0 = u^2 + 2(-9.8)(15)$

$u^2 = 294$

$u = \sqrt{294} = 17.146...$

$u = 17.1$ (3 s.f.)

Do not write outside the box

17 A particle moves in a straight line with acceleration a m s^{-2}, at time t seconds, where

$$a = 10 - 6t$$

The particle's velocity, v m s^{-1}, and displacement, r metres, are both initially zero.

Show that

$$r = t^2(5 - t)$$

Fully justify your answer.

[4 marks]

$v = \int a\,dt = \int (10 - 6t)\,dt$

$v = 10t - 3t^2 + c$

When $t = 0$, $v = 0$: $0 = 0 - 0 + c \Rightarrow c = 0$

$v = 10t - 3t^2$

$r = \int v\,dt = \int (10t - 3t^2)\,dt$

$r = 5t^2 - t^3 + k$

When $t = 0$, $r = 0$: $0 = 0 - 0 + k \Rightarrow k = 0$

$r = 5t^2 - t^3$

$\therefore \quad r = t^2(5 - t)$ as required

Turn over for the next question

Do not write outside the box

18 It is given that two points A and B have position vectors

$$\overrightarrow{OA} = \begin{bmatrix} 5 \\ -1 \end{bmatrix} \text{ metres} \qquad \text{and} \qquad \overrightarrow{OB} = \begin{bmatrix} 13 \\ 5 \end{bmatrix} \text{ metres.}$$

18 (a) Show that the distance from A to B is 10 metres.

[3 marks]

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{bmatrix} 13 \\ 5 \end{bmatrix} - \begin{bmatrix} 5 \\ -1 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$

$|AB| = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100}$

$|AB| = 10$

So the distance from A to B is 10 metres, as required

Do not write outside the box

18 (b) A constant resultant force, of magnitude R newtons, acts on a particle so that it moves in a straight line passing through the same two points A and B

At A, the speed of the particle is 3 m s^{-1} in the direction from A to B

The particle takes 2 seconds to travel from A to B

The mass of the particle is 150 grams.

Find the value of R

[3 marks]

$s = 10,\ u = 3,\ t = 2,\ a = ?$

$s = ut + \frac{1}{2}at^2$

$10 = 3(2) + \frac{1}{2}a(2^2)$

$10 = 6 + 2a \quad \Rightarrow \quad a = 2\ m\ s^{-2}$

$m = 150\ g = 0.15\ kg$

$R = ma = 0.15 \times 2$

$R = 0.3$

Turn over for the next question

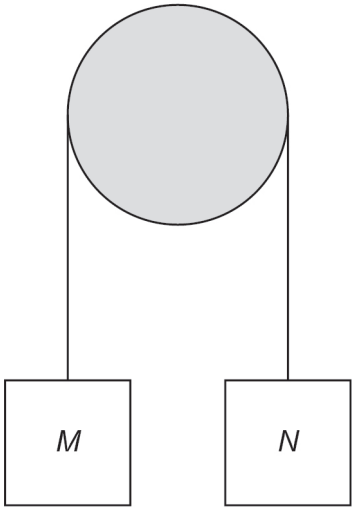
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19 Two objects, *M* and *N*, are connected by a light inextensible string that passes over a smooth peg.

M has a mass of 0.6 kilograms.

N has a mass of 0.5 kilograms.

M and *N* are initially held at rest, with the string taut, as shown in the diagram below.



M and *N* are released at the same instant and begin to move vertically.

You may assume that air resistance can be ignored.

19 (a) It is given that *M* and *N* move with acceleration a m s^{-2}

By forming two equations of motion show that

$$a = \frac{1}{11}g$$

[5 marks]

M (heavier) moves down, N moves up; tension in string = T

M: $0.6g - T = 0.6a$ (1)

N: $T - 0.5g = 0.5a$ (2)

(1) + (2): $0.6g - 0.5g = 0.6a + 0.5a$

$0.1g = 1.1a$

Do not write outside the box

$a = \frac{0.1g}{1.1} = \frac{g}{11}$

$\therefore \quad a = \frac{1}{11}g$ as required

19 (b) The speed of N, 0.5 seconds after its release, is $\frac{g}{k}$ m s^{-1} where k is a constant.

Find the value of k

[2 marks]

$v = u + at$ with $u = 0$, $a = \frac{1}{11}g$, $t = 0.5$

$v = 0 + \frac{1}{11}g \times 0.5 = \frac{g}{22}$

$k = 22$

19 (c) State **one** assumption that must be made for the answer in part **(b)** to be valid.

[1 mark]

N does not reach the peg (and M does not hit the ground)

within the first 0.5 seconds.

END OF QUESTIONS

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