AQA

Please write clearly in block capitals.

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I declare this is my own work.

AS MATHEMATICS

Paper 2

Thursday 25 May 2023 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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Section A

Answer **all** questions in the spaces provided.

1 Simplify $\log_2 8^a$

Circle your answer.

[1 mark]

a^3 $\quad$ $2a$ $\quad$ (3a) $\quad$ $8a$

2 It is given that $\sin\theta = \frac{4}{5}$ and $90° < \theta < 180°$

Find the value of $\cos\theta$

Circle your answer.

[1 mark]

$-\frac{3}{4}$ $\quad$ ($-\frac{3}{5}$) $\quad$ $\frac{3}{5}$ $\quad$ $\frac{3}{4}$

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3 (a) Find $\int \left(2x^3 + \frac{8}{x^2}\right) \mathrm{d}x$

[3 marks]

$\int (2x^3 + 8x^{-2})\, dx$

$= \frac{2x^4}{4} + \frac{8x^{-1}}{-1} + c$

$= \frac{1}{2}x^4 - \frac{8}{x} + c$

3 (b) A curve has gradient function $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x^3 + \frac{8}{x^2}$

The x-intercept of the curve is at the point (2, 0)

Find the equation of the curve.

[2 marks]

$y = \frac{1}{2}x^4 - \frac{8}{x} + c$

At (2, 0): $0 = \frac{1}{2}(2)^4 - \frac{8}{2} + c$

$0 = 8 - 4 + c \Rightarrow c = -4$

$y = \frac{1}{2}x^4 - \frac{8}{x} - 4$

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4 Find the exact solution of the equation $\ln(x+1) + \ln(x-1) = \ln 15 - 2\ln 7$

Fully justify your answer.

[5 marks]

$\ln(x+1) + \ln(x-1) = \ln[(x+1)(x-1)] = \ln(x^2-1)$

$2\ln 7 = \ln 7^2 = \ln 49$

$\ln 15 - \ln 49 = \ln\frac{15}{49}$

So $\ln(x^2-1) = \ln\frac{15}{49}$

$x^2 - 1 = \frac{15}{49}$

$x^2 = \frac{64}{49}$

$x = \pm\frac{8}{7}$

If $x = -\frac{8}{7}$ then $x+1 = -\frac{1}{7} < 0$ and $x-1 = -\frac{15}{7} < 0$

and ln of a negative number is not defined, so $\ln(x+1)$

and $\ln(x-1)$ would not exist. Reject $x = -\frac{8}{7}$

$\therefore\ x = \frac{8}{7}$

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5 It is given that $\sin 15° = \frac{\sqrt{6} - \sqrt{2}}{4}$ and $\cos 15° = \frac{\sqrt{6} + \sqrt{2}}{4}$

Use these two expressions to show that $\tan 15° = 2 - \sqrt{3}$

Fully justify your answer.

[4 marks]

$\tan 15° = \frac{\sin 15°}{\cos 15°} = \frac{\sqrt{6} - \sqrt{2}}{4} \div \frac{\sqrt{6} + \sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}$

$= \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} \times \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}$

$= \frac{6 - \sqrt{12} - \sqrt{12} + 2}{6 - 2} = \frac{8 - 2\sqrt{12}}{4}$

$\sqrt{12} = 2\sqrt{3}$, so $= \frac{8 - 4\sqrt{3}}{4}$

$= 2 - \sqrt{3}$ as required

Turn over for the next question

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6 A curve has equation

$$y = 2x^2 + px + 1$$

A line has equation

$$y = 5x - 2$$

Find the set of values of p for which the line intersects the curve at two distinct points.

Give your answer in exact form.

[5 marks]

$2x^2 + px + 1 = 5x - 2$

$2x^2 + (p - 5)x + 3 = 0$

Two distinct points of intersection $\Rightarrow$ $b^2 - 4ac > 0$

$(p - 5)^2 - 4(2)(3) > 0$

$p^2 - 10p + 25 - 24 > 0$

$p^2 - 10p + 1 > 0$

Critical values: $p = \frac{10 \pm \sqrt{(100 - 4)}}{2} = \frac{10 \pm \sqrt{96}}{2} = \frac{10 \pm 4\sqrt{6}}{2}$

$p = 5 \pm 2\sqrt{6}$

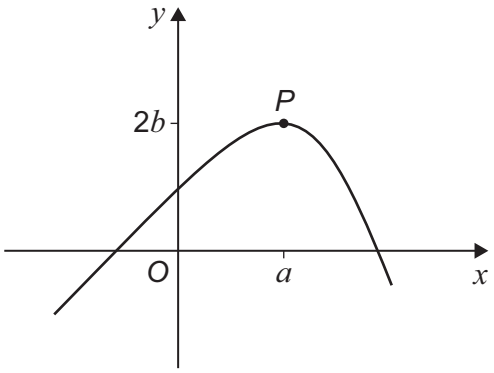
$p^2 - 10p + 1$ is a positive quadratic, so it is > 0 outside the roots:

$p < 5 - 2\sqrt{6}$ or $p > 5 + 2\sqrt{6}$

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7 The curve C has equation $y = f(x)$

C has a maximum point at P with coordinates $(a, 2b)$ as shown in the diagram below.



7 (a) C is mapped by a single transformation onto curve C_1 with equation $y = f(x + 2)$

State the coordinates of the maximum point on curve C_1

[1 mark]

Translation by 2 to the left: (a − 2, 2b)

7 (b) C is mapped by a single transformation onto curve C_2 with equation $y = 4f(x)$

State the coordinates of the maximum point on curve C_2

[1 mark]

Stretch in y-direction, scale factor 4: (a, 8b)

7 (c) C is mapped by a stretch in the x-direction onto curve C_3 with equation $y = f(3x)$

State the scale factor of the stretch.

[1 mark]

Scale factor $\frac{1}{3}$

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8 Prove that the sum of the cubes of two consecutive odd numbers is always a multiple of 4.

[5 marks]

Let the two consecutive odd numbers be $2k - 1$ and $2k + 1$,

where k is an integer.

$(2k + 1)^3 = 8k^3 + 12k^2 + 6k + 1$

$(2k - 1)^3 = 8k^3 - 12k^2 + 6k - 1$

Sum $= (2k + 1)^3 + (2k - 1)^3 = 16k^3 + 12k$

$= 4(4k^3 + 3k)$

k is an integer, so $4k^3 + 3k$ is an integer,

so the sum has a factor of 4.

Hence the sum of the cubes of two consecutive odd numbers

is always a multiple of 4.

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Turn over for the next question

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9 A craft artist is producing items and selling them in a local market.

The selling price, £P, of an item is inversely proportional to the number of items produced, n

9 (a) When $n = 10$, $P = 24$

Show that $P = \dfrac{240}{n}$

[1 mark]

$P = \frac{k}{n}$, $24 = \frac{k}{10}$ $\Rightarrow$ $k = 240$

$\therefore$ $P = \frac{240}{n}$

9 (b) The production cost, £C, of an item is inversely proportional to the square of the number of items produced, n

When $n = 10$, $C = 60$

Find the set of values of n for which $P > C$

[4 marks]

$C = \frac{l}{n^2}$, $60 = \frac{l}{10^2}$ $\Rightarrow$ $l = 6000$, so $C = \frac{6000}{n^2}$

$P > C$: $\frac{240}{n} > \frac{6000}{n^2}$

$n > 0$, so multiply both sides by n^2:

$240n > 6000$

$n > 25$

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9 (c) Explain the significance to the craft artist of the range of values found in part **(b)**.

[1 mark]

If the artist produces (and sells) more than 25 items, the selling price of each item is greater than its production cost, so the artist makes a profit.

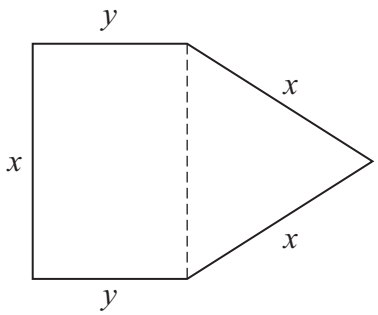
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10 A piece of wire of length 66 cm is bent to form the five sides of a pentagon.

The pentagon consists of three sides of a rectangle and two sides of an equilateral triangle.

The sides of the rectangle measure x cm and y cm and the sides of the triangle measure x cm, as shown in the diagram below.



10 (a) (i) You are given that $\sin 60° = \frac{\sqrt{3}}{2}$

Explain why the area of the triangle is $\frac{\sqrt{3}}{4}x^2$

[1 mark]

Area = $\frac{1}{2}$ ab sin C with a = b = x and C = 60° (equilateral)

Area = $\frac{1}{2}$ × x × x × sin 60° = $\frac{1}{2}x^2 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}x^2$

10 (a) (ii) Show that the area enclosed by the wire, A cm^2, can be expressed by the formula

$$A = 33x - \frac{1}{4}(6 - \sqrt{3})x^2$$

[3 marks]

Length of wire: $x + 2y + 2x = 66 \Rightarrow 3x + 2y = 66$

$y = 33 - \frac{3}{2}x$

Area of rectangle = $xy = x(33 - \frac{3}{2}x) = 33x - \frac{3}{2}x^2$

$A = 33x - \frac{3}{2}x^2 + \frac{\sqrt{3}}{4}x^2$

$A = 33x - \frac{6}{4}x^2 + \frac{\sqrt{3}}{4}x^2$

$A = 33x - \frac{1}{4}(6 - \sqrt{3})x^2$ as required

Do not write outside the box

10 (b) Use calculus to find the value of x for which the wire encloses the maximum area.

Give your answer in the form $p + q\sqrt{3}$, where p and q are integers.

Fully justify your answer.

[7 marks]

$\frac{dA}{dx} = 33 - \frac{1}{2}(6 - \sqrt{3})x$

At a stationary point $\frac{dA}{dx} = 0$

$33 - \frac{1}{2}(6 - \sqrt{3})x = 0$

$(6 - \sqrt{3})x = 66$

$x = \frac{66}{6 - \sqrt{3}} \times \frac{6 + \sqrt{3}}{6 + \sqrt{3}} = \frac{66(6 + \sqrt{3})}{36 - 3} = \frac{66(6 + \sqrt{3})}{33}$

$x = 2(6 + \sqrt{3}) = 12 + 2\sqrt{3}$

$\frac{d^2A}{dx^2} = -\frac{1}{2}(6 - \sqrt{3})$

$6 > \sqrt{3}$, so this is negative for all x

so the stationary point is a maximum.

The wire encloses the maximum area when $x = 12 + 2\sqrt{3}$

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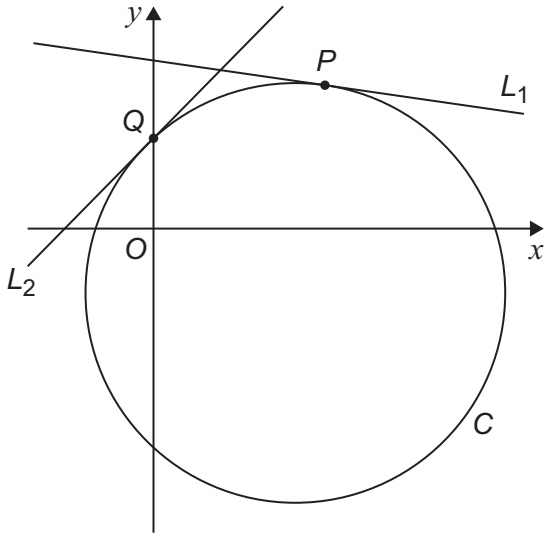
11 The line L_1 has equation $x + 7y - 41 = 0$

L_1 is a tangent to the circle C at the point $P(6, 5)$

The line L_2 has equation $y = x + 3$

L_2 is a tangent to the circle C at the point $Q(0, 3)$

The lines L_1 and L_2 and the circle C are shown in the diagram below.



11 (a) Show that the equation of the radius of the circle through P is $y = 7x - 37$

[3 marks]

L_1: $7y = -x + 41 \Rightarrow y = -\frac{1}{7}x + \frac{41}{7}$

Gradient of $L_1 = -\frac{1}{7}$

The radius through P is perpendicular to the tangent L_1

so gradient of radius = 7 (since $-\frac{1}{7} \times 7 = -1$)

$y - 5 = 7(x - 6)$

$y = 7x - 42 + 5$

$y = 7x - 37$ as required

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11 (b) Find the equation of C

[4 marks]

Gradient of $L_2 = 1$, so gradient of radius through $Q = -1$

Radius through $Q(0, 3)$: $y = -x + 3$

The centre is where the two radii meet:

$7x - 37 = -x + 3$

$8x = 40 \Rightarrow x = 5, \quad y = -5 + 3 = -2$

Centre $(5, -2)$

$r^2 = (5 - 0)^2 + (-2 - 3)^2 = 25 + 25 = 50 \qquad (r = \sqrt{50})$

Equation of C: $(x - 5)^2 + (y + 2)^2 = 50$

END OF SECTION A

TURN OVER FOR SECTION B

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Section B

Answer **all** questions in the spaces provided.

12 The mass of a bag of nuts produced by a company is known to have a mean of 40 grams and a standard deviation of 3 grams.

The company produces five different flavours of nuts.

The bags of nuts are packed in large boxes.

Given the information above, identify the continuous variable from the options below.

Tick (✓) **one** box.

[1 mark]

The flavours of the bags of nuts	☐
The known standard deviation of the mass of a bag of nuts	☐
The mass of an individual bag of nuts	✓
The number of bags of nuts in a large box	☐

Do not write outside the box

13 The table below shows the frequencies for a set of data from a continuous variable X

x	Frequency
$11 < x \leq 21$	7
$21 < x \leq 24$	9
$24 < x \leq 42$	36
$42 < x \leq 50$	18

A histogram is drawn to represent this data.

Find the frequency density of the bar in the histogram representing the class $24 < x \leq 42$

Circle your answer.

[1 mark]

(2) 18 36 70

Turn over for the next question

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14 The manager of a factory wants to introduce a bonus scheme.

The factory has 65 employees who work in production and 28 employees who work in the office.

The manager decides to collect the opinions of a sample of these 93 employees.

14 (a) Explain how the manager could collect a simple random sample of 20 employees.

[3 marks]

Number the 93 employees from 1 to 93. Use the random number generator on a calculator to generate random whole numbers between 1 and 93. Select the employees with these numbers, ignoring any repeats and any numbers out of range, and continue until 20 different employees have been selected.

14 (b) The manager collected a simple random sample of 20 employees.

The manager noticed that all 20 of the employees in the sample worked in production and therefore the sample was not representative.

State a different method of sampling that would give a representative sample.

[1 mark]

Stratified sampling (by production and office staff)

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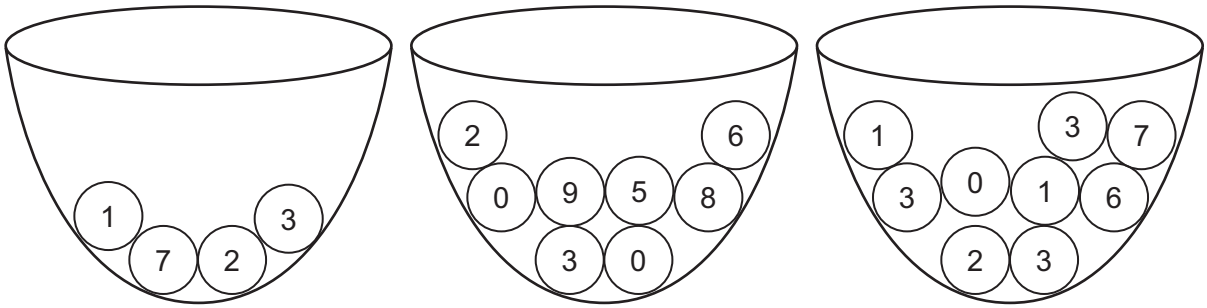
15 Numbered balls are placed in bowls A, B and C

In bowl A there are four balls numbered 1, 2, 3 and 7

In bowl B there are eight balls numbered 0, 0, 2, 3, 5, 6, 8 and 9

In bowl C there are nine balls numbered 0, 1, 1, 2, 3, 3, 3, 6 and 7

This information is shown in the diagram below.



A B C

A three-digit number is generated using the following method:

- a ball is selected at random from each bowl
- the first digit of the number is the ball drawn from bowl A
- the second digit of the number is the ball drawn from bowl B
- the third digit of the number is the ball drawn from bowl C

15 (a) Find the probability that the number generated is even.

[1 mark]

Third digit (bowl C) must be even: 0, 2, 6. $P(\text{even}) = \frac{3}{9} = \frac{1}{3}$

15 (b) Find the probability that the number generated is 703

[2 marks]

$P(703) = P(A = 7) \times P(B = 0) \times P(C = 3)$

$= \frac{1}{4} \times \frac{2}{8} \times \frac{3}{9}$

$= \frac{1}{48}$

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15 (c) Find the probability that the number generated is divisible by 111

[2 marks]

Divisible by 111 means all three digits are the same.

Bowl B has no 1 or 7, so the number must be 222 or 333

$P(222) = \frac{1}{4} \times \frac{1}{8} \times \frac{1}{9} = \frac{1}{288}$ $P(333) = \frac{1}{4} \times \frac{1}{8} \times \frac{3}{9} = \frac{1}{96}$

$P(\text{divisible by 111}) = \frac{1}{288} + \frac{1}{96} = \frac{4}{288} = \frac{1}{72}$

Turn over for the next question

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16 The discrete random variable X has probability distribution

x	1	2	3	4	5	6
$\mathbf{P}(X = x)$	0.3	0.1	0.2	0.1	0.1	0.2

The discrete random variable Y has probability distribution

y	2	3	4	5	6	7
$\mathbf{P}(Y = y)$	0.3	p	0.2	0.1	p	3p + 0.05

It is claimed that $P(X \geq 3)$ is greater than $P(Y \leq 4)$

Determine if this claim is correct.

Fully justify your answer.

[4 marks]

Probabilities for Y sum to 1:

$0.3 + p + 0.2 + 0.1 + p + 3p + 0.05 = 1$

$0.65 + 5p = 1 \Rightarrow 5p = 0.35 \Rightarrow p = 0.07$

$P(X \geq 3) = 0.2 + 0.1 + 0.1 + 0.2 = 0.6$

$P(Y \leq 4) = 0.3 + 0.07 + 0.2 = 0.57$

$0.6 > 0.57$, so $P(X \geq 3)$ is greater than $P(Y \leq 4)$

The claim is correct.

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17 An archer is training for the Olympics.

Each of the archer's training sessions consists of 30 attempts to hit the centre of a target.

The archer consistently hits the centre of the target with 79% of their attempts.

It can be assumed that the number of times the centre of the target is hit in any training session can be modelled by a binomial distribution.

17 (a) Find the mean of the number of times that the archer hits the centre of the target during a training session.

[1 mark]

$X \sim B(30, 0.79)$ Mean $= np = 30 \times 0.79 = 23.7$

17 (b) Find the probability that the archer hits the centre of the target exactly 22 times during a particular training session.

[1 mark]

$P(X = 22) = 0.123857... = 0.124$

17 (c) Find the probability that the archer hits the centre of the target 18 times or less during a particular training session.

[1 mark]

$P(X \leq 18) = 0.0139937... = 0.0140$

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17 (d) Find the probability that the archer hits the centre of the target more than 26 times in a training session.

[2 marks]

$P(X > 26) = 1 - P(X \leq 26)$

$= 1 - 0.901559...$

$= 0.098440... = 0.0984$

Turn over for the next question

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18 It is believed that 25% of the customers at a bakery buy a loaf of bread.

In an attempt to increase this proportion, the manager of the bakery provided free samples for the customers to taste.

To decide whether providing free samples had been effective, a large random sample of customers leaving the bakery were asked whether they had purchased a loaf of bread.

A hypothesis test at the 5% significance level was carried out on the data collected.

The test statistic calculated was found to be in the critical region.

18 (a) State the Null and Alternative hypotheses for this test.

[1 mark]

H_0: $p = 0.25$ H_1: $p > 0.25$

where p is the proportion of customers who buy a loaf of bread

18 (b) State, in context, the conclusion to this test.

[2 marks]

The test statistic is in the critical region, so reject H_0. There is sufficient evidence at the 5% level to suggest that the proportion of customers buying a loaf of bread has increased.

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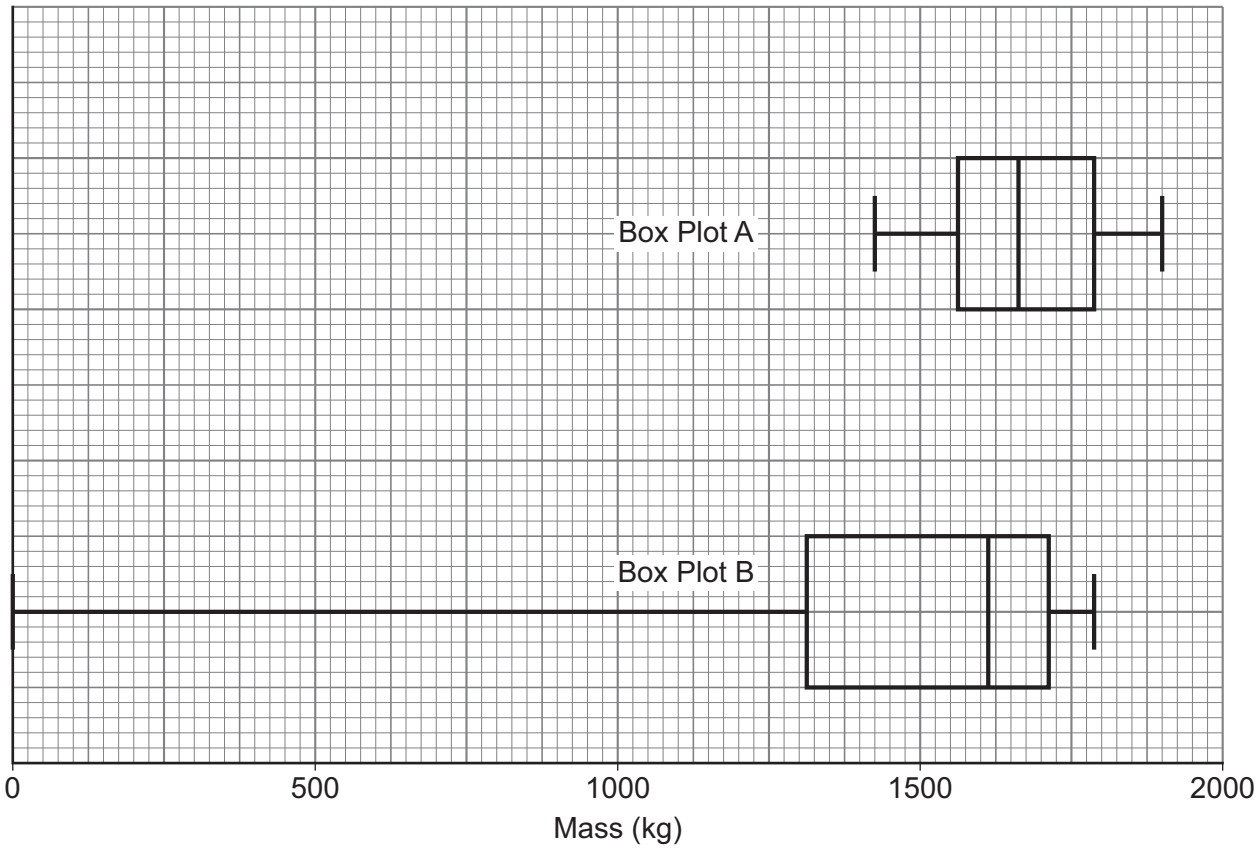
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19 A comparison of the masses (in kg) of convertible cars was made using the Large Data Set.

A sample of 20 masses was chosen from both the 2002 data and the 2016 data.

The masses of the 20 cars in each sample were used to create a box plot for each year.

The box plots were labelled Box Plot A and Box Plot B as shown in the diagram below.



19 (a) Estimate the median of the masses from Box Plot A

[1 mark]

Median ≈ 1662.5 kg

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19 (b) It is claimed that Box Plot B **must be** incorrectly drawn.

19 (b) (i) Give a reason why this claim was made.

[1 mark]

The lower whisker of Box Plot B goes down to 0 kg, which implies at least one car has a mass of zero, which is not possible.

19 (b) (ii) Comment on the validity of this claim.

[1 mark]

The claim is not valid: the Large Data Set contains some car masses recorded as zero, so Box Plot B can be correctly drawn.

19 (c) It is claimed that Box Plot B **must be** from the 2002 data. Give a reason why this claim is correct.

[1 mark]

The 2002 data is the only one of the two years in the Large Data Set to contain masses of zero, and Box Plot B has a minimum of 0 kg.

END OF QUESTIONS

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