Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____
I declare this is my own work.

AS
MATHEMATICS

Paper 1

Thursday 18 May 2023 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
TOTAL	

Do not write outside the box

Section A

Answer **all** questions in the spaces provided.

1 At a point P on a curve, the gradient of the tangent to the curve is 10

State the gradient of the normal to the curve at P

Circle your answer.

[1 mark]

-10 $\quad$ -0.1 $\quad$ 0.1 $\quad$ 10

2 Identify the expression below which is equivalent to $\left(\frac{2x}{5}\right)^{-3}$

Circle your answer.

[1 mark]

$\frac{8x^3}{125}$ $\quad$ $\frac{125x^3}{8}$ $\quad$ $\frac{125}{8x^3}$ $\quad$ $\frac{8}{125x^3}$

Do not write outside the box

3 The coefficient of x^2 in the binomial expansion of $(1 + ax)^6$ is $\frac{20}{3}$

Find the two possible values of a

[3 marks]

Turn over for the next question

Do not write outside the box

4 It is given that $5\cos^2\theta - 4\sin^2\theta = 0$

4 (a) Find the possible values of $\tan\theta$, giving your answers in exact form.

[3 marks]

4 (b) Hence, or otherwise, solve the equation

$$5\cos^2\theta - 4\sin^2\theta = 0$$

giving all solutions of θ to the nearest 0.1° in the interval $0° \leq \theta \leq 360°$

[2 marks]

Do not write outside the box

5 (a) Given that $y = x\sqrt{x}$, find $\frac{dy}{dx}$

[2 marks]

5 (b) The line, L, has equation $6x - 2y + 5 = 0$

L is a tangent to the curve with equation $y = x\sqrt{x} + k$

Find the value of k

[5 marks]

Do not write outside the box

6 (a) The curve C_1 has equation $y = 2x^2 - 20x + 42$

Express the equation of C_1 in the form

$$y = a(x - b)^2 + c$$

where a, b and c are integers.

[3 marks]

6 (b) Write down the coordinates of the minimum point of C_1

[1 mark]

6 (c) The curve C_1 is mapped onto the curve C_2 by a stretch in the y-direction.

The minimum point of C_2 is at $(5, -4)$

Find the equation of C_2

[2 marks]

Do not write outside the box

7 Points P and Q lie on the curve with equation $y = x^4$

The x-coordinate of P is x
The x-coordinate of Q is $x + h$

7 (a) Expand $(x + h)^4$

[2 marks]

7 (b) Hence, find an expression, in terms of x and h, for the gradient of the line PQ

[1 mark]

7 (c) Explain how to use the answer from part **(b)** to obtain the gradient function of $y = x^4$

[2 marks]

Do not write outside the box

8 (a) Show that

$$\int_1^a \left(6 - \frac{12}{\sqrt{x}}\right) \mathrm{d}x = 6a - 24\sqrt{a} + 18$$

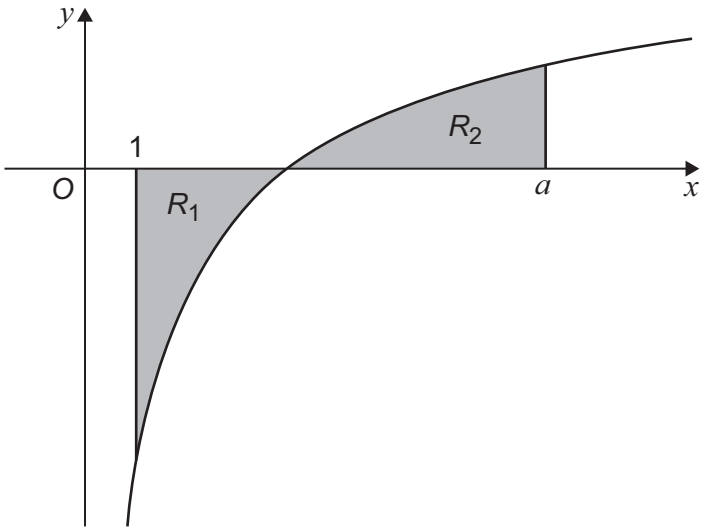
[3 marks]

Do not write outside the box

8 (b) The curve $y = 6 - \frac{12}{\sqrt{x}}$, the line $x = 1$ and the line $x = a$ are shown in the diagram below.

The shaded region R_1 is bounded by the curve, the line $x = 1$ and the x-axis.

The shaded region R_2 is bounded by the curve, the line $x = a$ and the x-axis.



It is given that the areas of R_1 and R_2 are equal.

Find the value of a

Fully justify your answer.

[4 marks]

Do not write outside the box

9 A continuous curve has equation $y = \mathrm{f}(x)$

The curve passes through the points $A(2, 1)$, $B(4, 5)$ and $C(6, 1)$

It is given that $\mathrm{f}'(4) = 0$

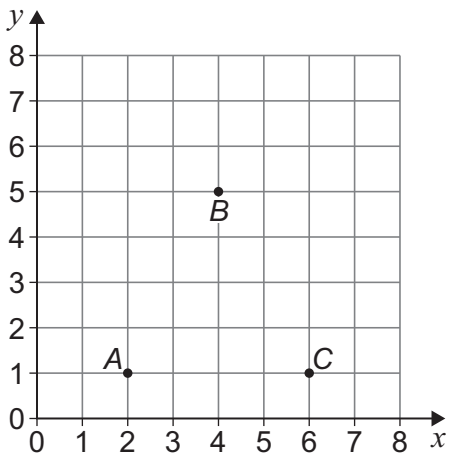
Jasmin made two statements about the nature of the curve $y = \mathrm{f}(x)$ at the point B:

Statement 1: There is a turning point at B

Statement 2: There is a maximum point at B

9 (a) Draw a sketch of the curve $y = \mathrm{f}(x)$ such that Statement 1 is correct and Statement 2 is correct.

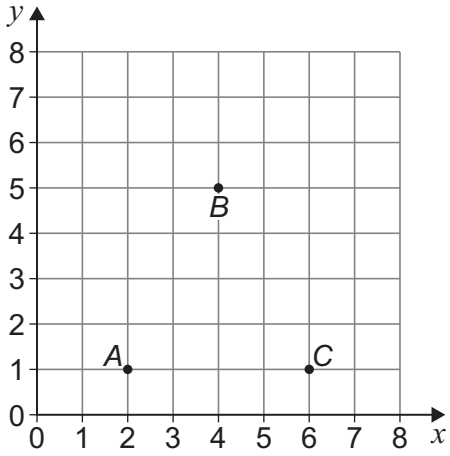
[1 mark]



Do not write outside the box

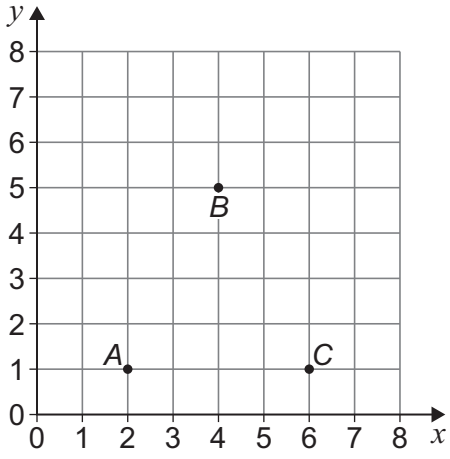
9 (b) Draw a sketch of the curve $y = \mathrm{f}(x)$ such that Statement 1 is correct and Statement 2 is **not** correct.

[1 mark]



9 (c) Draw a sketch of the curve $y = \mathrm{f}(x)$ such that Statement 1 is **not** correct and Statement 2 is **not** correct.

[1 mark]



Do not write outside the box

10 Charlie buys a car for £18 000 on 1 January 2016.

The value of the car decreases exponentially.

The car has a value of £12 000 on 1 January 2018.

10 (a) Charlie says:

- because the car has lost £6000 after two years, after another two years it will be worth £6000.

Charlie's friend Kaya says:

- because the car has lost one third of its value after two years, after another two years it will be worth £8000.

Explain whose statement is correct, justifying the value they have stated.

[2 marks]

Do not write outside the box

10 (b) The value of Charlie's car, £V, t years after 1 January 2016 may be modelled by the equation

$$V = A\mathrm{e}^{-kt}$$

where A and k are positive constants.

Find the value of t when the car has a value of £10 000, giving your answer to two significant figures.

[5 marks]

10 (c) Give a reason why the model, in this context, will not be suitable to calculate the value of the car when $t = 30$

[1 mark]

Turn over ▶

Do not write outside the box

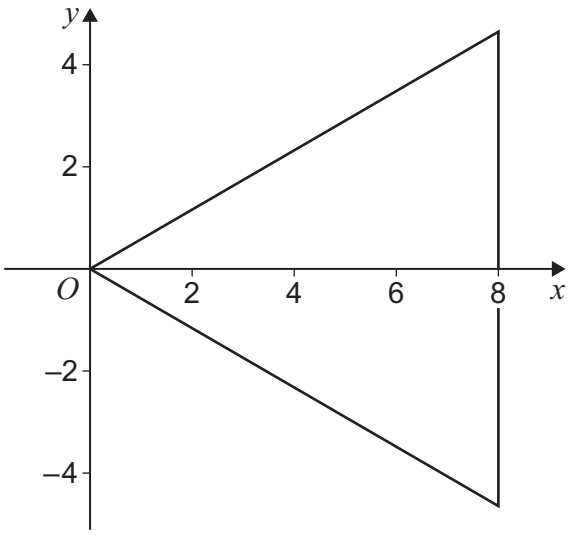
11 (a) A circle has equation

$$x^2 + y^2 - 10x - 6 = 0$$

Find the centre and the radius of the circle.

[2 marks]

11 (b) An equilateral triangle has one vertex at the origin, and one side along the line $x = 8$, as shown in the diagram below.



11 (b) (i) Show that the vertex at the origin lies inside the circle $x^2 + y^2 - 10x - 6 = 0$

[1 mark]

Do not write outside the box

11 (b) (ii) Prove that the triangle lies completely within the circle $x^2 + y^2 - 10x - 6 = 0$

[4 marks]

END OF SECTION A

TURN OVER FOR SECTION B

Do not write outside the box

Section B

Answer **all** questions in the spaces provided.

12 A particle, initially at rest, starts to move forward in a straight line with constant acceleration, $a\,\mathrm{m\,s^{-2}}$

After 6 seconds the particle has a velocity of $3\,\mathrm{m\,s^{-1}}$

Find the value of a

Circle your answer.

[1 mark]

-2 $\quad$ -0.5 $\quad$ 0.5 $\quad$ 2

13 A resultant force of $\begin{bmatrix} -2 \\ 6 \end{bmatrix}$ N acts on a particle.

The acceleration of the particle is $\begin{bmatrix} -6 \\ y \end{bmatrix}\,\mathrm{m\,s^{-2}}$

Find the value of y

Circle your answer.

[1 mark]

2 $\quad$ 3 $\quad$ 10 $\quad$ 18

Do not write outside the box

14 A ball, initially at rest, is dropped from a vertical height of h metres above the Earth's surface.

After 4 seconds the ball's height above the Earth's surface is $0.2h$ metres.

14 (a) Assuming air resistance can be ignored, show that

$$h = 10g$$

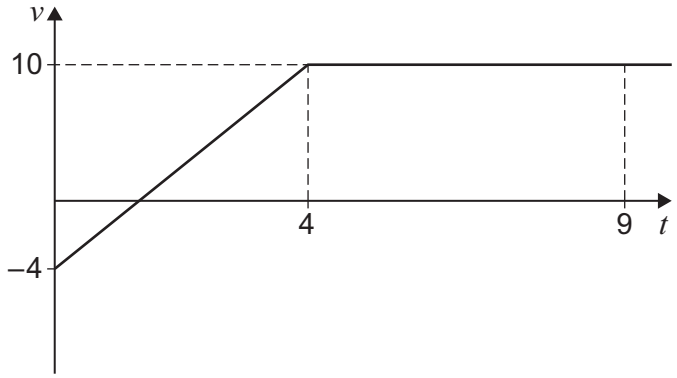
[3 marks]

14 (b) Assuming air resistance cannot be ignored, explain the effect that this would have on the value of h in part **(a)**.

[1 mark]

Do not write outside the box

15 A particle is moving in a straight line such that its velocity, $v\,\mathrm{m\,s^{-1}}$, changes with respect to time, t seconds, as shown in the graph below.



15 (a) Show that the acceleration of the particle over the first 4 seconds is $3.5\,\mathrm{m\,s^{-2}}$

[1 mark]

15 (b) The particle is initially at a fixed point P

Show that the displacement of the particle from P, when $t = 9$, is 62 metres.

[3 marks]

Do not write outside the box

16 A toy remote control speed boat is launched from one edge of a small pond and moves in a straight line across the pond's surface.

The boat's velocity, $v\,\mathrm{m\,s^{-1}}$, is modelled in terms of time, t seconds after the boat is launched, by the expression

$$v = 0.9 + 0.16t - 0.06t^2$$

16 (a) Find the acceleration of the boat when $t = 2$

[3 marks]

16 (b) Find the displacement of the boat, from the point where it was launched, when $t = 2$

[4 marks]

Do not write outside the box

17 A particle, P, is initially at rest on a smooth horizontal surface.

A resultant force of $\begin{bmatrix} 12 \\ 9 \end{bmatrix}$ N is then applied to P, so that it moves in a straight line.

17 (a) Find the magnitude of the resultant force.

[1 mark]

17 (b) Two fixed points A and B have position vectors

$$\overrightarrow{OA} = \begin{bmatrix} 3 \\ 7 \end{bmatrix} \text{ metres} \qquad \text{and} \qquad \overrightarrow{OB} = \begin{bmatrix} k \\ k-1 \end{bmatrix} \text{ metres}$$

with respect to a fixed origin, O

P moves in a straight line parallel to $\overrightarrow{AB}$

17 (b) (i) Find $\overrightarrow{AB}$ in terms of k

[1 mark]

Do not write outside the box

17 (b) (ii) Find the value of k

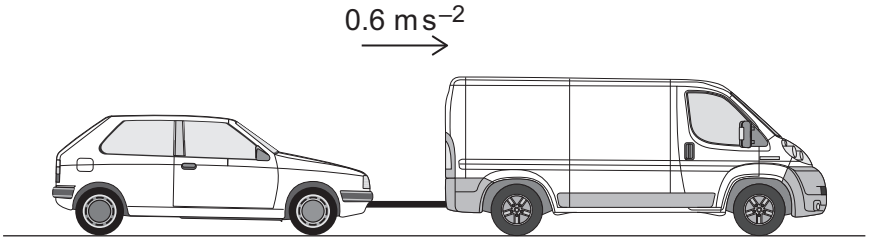
[2 marks]

Turn over for the next question

Do not write outside the box

18 A rescue van is towing a broken-down car by using a tow bar.

The van and the car are moving with a constant acceleration of $0.6\,\text{m}\,\text{s}^{-2}$ along a straight horizontal road as shown in the diagram below.



The van has a total mass of 2780 kg

The car has a total mass of 1620 kg

The van experiences a driving force of D newtons.

The van experiences a total resistance force of R newtons.

The car experiences a total resistance force of $0.6R$ newtons.

18 (a) The tension in the tow bar, T newtons, may be modelled by

$$T = kD - 18$$

where k is a constant.

Find k

[5 marks]

Do not write outside the box

18 (b) State one assumption that must be made in answering part **(a)**.

[1 mark]

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Copyright information

For confidentiality purposes, all acknowledgements of third-party copyright material are published in a separate booklet. This booklet is published after each live examination series and is available for free download from www.aqa.org.uk.

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team.

Copyright © 2023 AQA and its licensors. All rights reserved.