AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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I declare this is my own work.

AS MATHEMATICS

Paper 1

Thursday 18 May 2023 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 At a point P on a curve, the gradient of the tangent to the curve is 10

State the gradient of the normal to the curve at P

Circle your answer.

[1 mark]

-10 $\quad$ -0.1 $\quad$ 0.1 $\quad$ 10

2 Identify the expression below which is equivalent to $\left(\frac{2x}{5}\right)^{-3}$

Circle your answer.

[1 mark]

$\frac{8x^3}{125}$ $\quad$ $\frac{125x^3}{8}$ $\quad$ $\frac{125}{8x^3}$ $\quad$ $\frac{8}{125x^3}$

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3 The coefficient of x^2 in the binomial expansion of $(1 + ax)^6$ is $\frac{20}{3}$

Find the two possible values of a

[3 marks]

Term in x^2: ${}^6C_2(ax)^2 = 15a^2x^2$

so the coefficient of x^2 is $15a^2$

$15a^2 = \frac{20}{3}$

$a^2 = \frac{20}{45} = \frac{4}{9}$

$a = \frac{2}{3}$ or $a = -\frac{2}{3}$

Turn over for the next question

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4 It is given that $5\cos^2\theta - 4\sin^2\theta = 0$

4 (a) Find the possible values of $\tan\theta$, giving your answers in exact form.

[3 marks]

$5\cos^2\theta = 4\sin^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} = \frac{5}{4} \Rightarrow \tan^2\theta = \frac{5}{4}$

$\tan\theta = \frac{\sqrt{5}}{2}$ or $\tan\theta = -\frac{\sqrt{5}}{2}$

4 (b) Hence, or otherwise, solve the equation

$$5\cos^2\theta - 4\sin^2\theta = 0$$

giving all solutions of θ to the nearest 0.1° in the interval $0° \leq \theta \leq 360°$

[2 marks]

$\tan\theta = \frac{\sqrt{5}}{2}$: $\theta = \tan^{-1}\frac{\sqrt{5}}{2} = 48.18968...°$, $180° + 48.19° = 228.19...°$

$\tan\theta = -\frac{\sqrt{5}}{2}$: $\theta = 180° - 48.19° = 131.81...°$, $360° - 48.19° = 311.81...°$

$\theta = 48.2°, 131.8°, 228.2°, 311.8°$

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5 (a) Given that $y = x\sqrt{x}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

[2 marks]

$y = x\sqrt{x} = x^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$ $\left(= \frac{3}{2}\sqrt{x}\right)$

5 (b) The line, L, has equation $6x - 2y + 5 = 0$

L is a tangent to the curve with equation $y = x\sqrt{x} + k$

Find the value of k

[5 marks]

$6x - 2y + 5 = 0 \Rightarrow 2y = 6x + 5 \Rightarrow y = 3x + 2.5$, so gradient of $L = 3$

Curve: $\frac{dy}{dx} = \frac{3}{2}x^{1/2}$ (adding k does not change the gradient)

At the point of contact: $\frac{3}{2}x^{1/2} = 3 \Rightarrow x^{1/2} = 2 \Rightarrow x = 4$

On L when $x = 4$: $2y = 6 \times 4 + 5 = 29 \Rightarrow y = 14.5$

On the curve: $14.5 = 4\sqrt{4} + k = 4 \times 2 + k = 8 + k$

$k = 6.5$

Do not write outside the box

6 (a) The curve C_1 has equation $y = 2x^2 - 20x + 42$

Express the equation of C_1 in the form

$$y = a(x - b)^2 + c$$

where a, b and c are integers.

[3 marks]

$y = 2x^2 - 20x + 42 = 2(x^2 - 10x + 21)$

$= 2[(x - 5)^2 - 25 + 21]$

$= 2[(x - 5)^2 - 4]$

$y = 2(x - 5)^2 - 8$

$(a = 2,\ b = 5,\ c = -8)$

6 (b) Write down the coordinates of the minimum point of C_1

[1 mark]

$(5, -8)$

6 (c) The curve C_1 is mapped onto the curve C_2 by a stretch in the y-direction.

The minimum point of C_2 is at $(5, -4)$

Find the equation of C_2

[2 marks]

Minimum moves from $(5, -8)$ to $(5, -4)$: scale factor $= \frac{-4}{-8} = \frac{1}{2}$

$y = \frac{1}{2}[2(x - 5)^2 - 8]$

$y = (x - 5)^2 - 4 \quad (= x^2 - 10x + 21)$

Do not write outside the box

7 Points P and Q lie on the curve with equation $y = x^4$

The x-coordinate of P is x
The x-coordinate of Q is $x + h$

7 (a) Expand $(x + h)^4$

[2 marks]

$(x + h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

7 (b) Hence, find an expression, in terms of x and h, for the gradient of the line PQ

[1 mark]

gradient $PQ = \dfrac{(x + h)^4 - x^4}{(x + h) - x} = \dfrac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$

7 (c) Explain how to use the answer from part **(b)** to obtain the gradient function of $y = x^4$

[2 marks]

Cancel h: gradient of $PQ = 4x^3 + 6x^2h + 4xh^2 + h^3$

Then let $h \to 0$, so Q moves towards P and the gradient of the chord

PQ tends to the gradient of the tangent at P.

The terms $6x^2h$, $4xh^2$ and h^3 all $\to 0$, so the limit is

the gradient function: $dy/dx = 4x^3$

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8 (a) Show that

$$\int_1^a \left(6 - \frac{12}{\sqrt{x}}\right) \mathrm{d}x = 6a - 24\sqrt{a} + 18$$

[3 marks]

$\int_1^a (6 - \frac{12}{\sqrt{x}})\ dx = \int_1^a (6 - 12x^{-1/2})\ dx$

$= [6x - \frac{12x^{1/2}}{1/2}]_1^a = [6x - 24\sqrt{x}]_1^a$

$= (6a - 24\sqrt{a}) - (6 \times 1 - 24\sqrt{1})$

$= 6a - 24\sqrt{a} - 6 + 24$

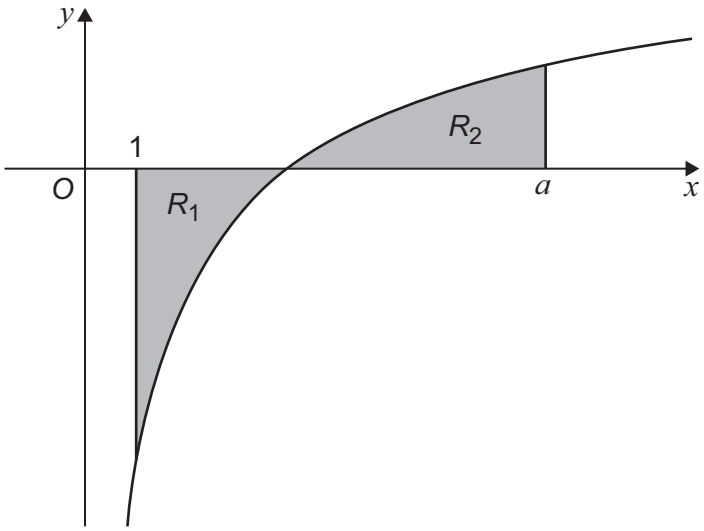
$= 6a - 24\sqrt{a} + 18$ as required

Do not write outside the box

8 (b) The curve $y = 6 - \frac{12}{\sqrt{x}}$, the line $x = 1$ and the line $x = a$ are shown in the diagram below.

The shaded region R_1 is bounded by the curve, the line $x = 1$ and the x-axis.

The shaded region R_2 is bounded by the curve, the line $x = a$ and the x-axis.



It is given that the areas of R_1 and R_2 are equal.

Find the value of a

Fully justify your answer.

[4 marks]

R_1 is below the x-axis (integral < 0); R_2 is above (integral > 0).

Areas equal ⇒ the integral from 1 to a is zero:

$6a - 24\sqrt{a} + 18 = 0 \quad \Rightarrow \quad a - 4\sqrt{a} + 3 = 0$

$(\sqrt{a} - 1)(\sqrt{a} - 3) = 0 \quad \Rightarrow \quad \sqrt{a} = 1$ or $\sqrt{a} = 3$

$\sqrt{a} = 1$ gives $a = 1$, which is the line $x = 1$ itself (zero area), so reject.

Curve crosses the x-axis where $6 = 12/\sqrt{x}$, i.e. $x = 4$, so $a > 4$.

$\sqrt{a} = 3 \quad \Rightarrow \quad a = 9$

Check: area $R_1 = |24 - 48 + 18| = 6$, area $R_2 = (54 - 72) - (24 - 48) = 6$ ✓

$\therefore \quad a = 9$

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9 A continuous curve has equation $y = \mathrm{f}(x)$

The curve passes through the points $A(2, 1)$, $B(4, 5)$ and $C(6, 1)$

It is given that $\mathrm{f}'(4) = 0$

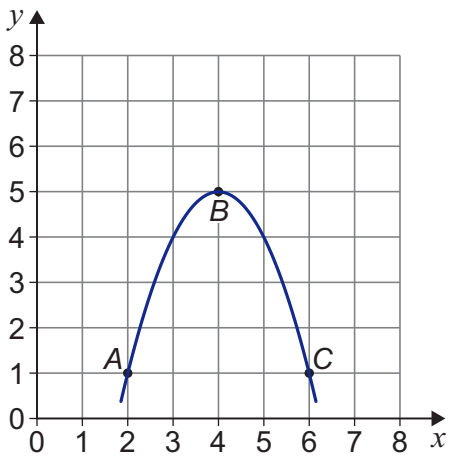
Jasmin made two statements about the nature of the curve $y = \mathrm{f}(x)$ at the point B:

Statement 1: There is a turning point at B

Statement 2: There is a maximum point at B

9 (a) Draw a sketch of the curve $y = \mathrm{f}(x)$ such that Statement 1 is correct and Statement 2 is correct.

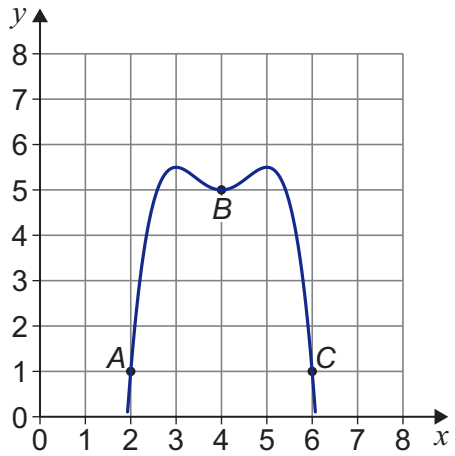
[1 mark]



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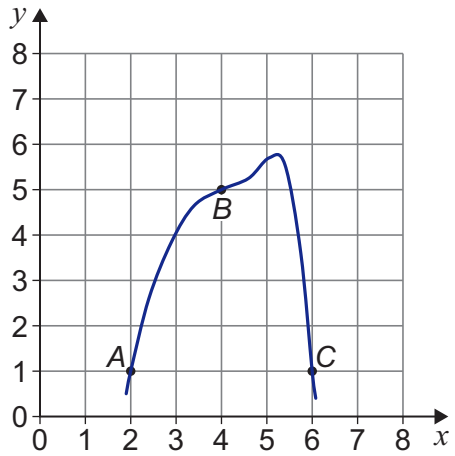
9 (b) Draw a sketch of the curve $y = f(x)$ such that Statement 1 is correct and Statement 2 is **not** correct.

[1 mark]



9 (c) Draw a sketch of the curve $y = f(x)$ such that Statement 1 is **not** correct and Statement 2 is **not** correct.

[1 mark]



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10 Charlie buys a car for £18 000 on 1 January 2016.

The value of the car decreases exponentially.

The car has a value of £12 000 on 1 January 2018.

10 (a) Charlie says:

- because the car has lost £6000 after two years, after another two years it will be worth £6000.

Charlie's friend Kaya says:

- because the car has lost one third of its value after two years, after another two years it will be worth £8000.

Explain whose statement is correct, justifying the value they have stated.

[2 marks]

Kaya is correct (Charlie is wrong).

In exponential decay the value falls by the same proportion in equal time intervals, not by the same amount.

In 2 years the value went from £18000 to £12000, i.e. it was multiplied by $\frac{2}{3}$

so after another 2 years it is $\frac{2}{3} \times £12000 = £8000$

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10 (b) The value of Charlie's car, £V, t years after 1 January 2016 may be modelled by the equation

$$V = A\mathrm{e}^{-kt}$$

where A and k are positive constants.

Find the value of t when the car has a value of £10 000, giving your answer to two significant figures.

[5 marks]

$t = 0$: $V = 18000 \Rightarrow A = 18000$

$t = 2$: $12000 = 18000e^{-2k}$

$e^{-2k} = \frac{12000}{18000} = \frac{2}{3} \Rightarrow -2k = \ln\frac{2}{3}$

$k = \frac{1}{2}\ln 1.5 = 0.20273...$

$10000 = 18000e^{-kt}$

$e^{-kt} = \frac{10000}{18000} = \frac{5}{9} \Rightarrow kt = \ln 1.8$

$t = \frac{\ln 1.8}{0.20273...} = 2.8993...$

$t = 2.9$ (2 s.f.)

10 (c) Give a reason why the model, in this context, will not be suitable to calculate the value of the car when $t = 30$

[1 mark]

At $t = 30$ the model gives $V = 18000e^{-30k} \approx$ £41, which is unrealistic:

the car would probably have been scrapped by then (or its scrap value

would be worth more than the model suggests).

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11 (a) A circle has equation

$$x^2 + y^2 - 10x - 6 = 0$$

Find the centre and the radius of the circle.

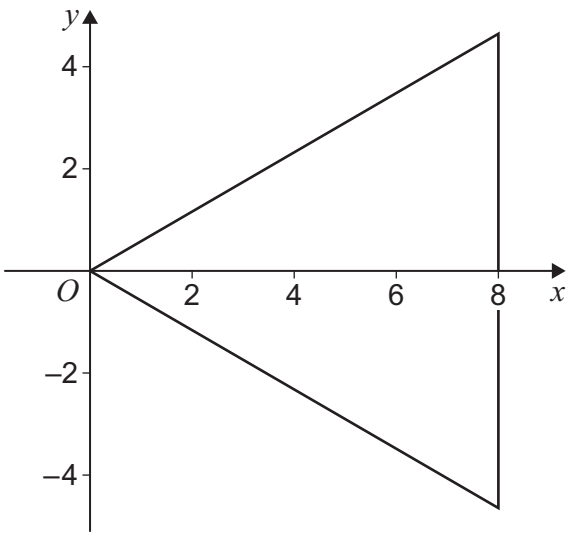
[2 marks]

$x^2 - 10x + y^2 = 6$

$(x - 5)^2 - 25 + y^2 = 6 \quad \Rightarrow \quad (x - 5)^2 + y^2 = 31$

Centre (5, 0), radius $\sqrt{31}$ (= 5.57)

11 (b) An equilateral triangle has one vertex at the origin, and one side along the line $x = 8$, as shown in the diagram below.



11 (b) (i) Show that the vertex at the origin lies inside the circle $x^2 + y^2 - 10x - 6 = 0$

[1 mark]

Distance from centre (5, 0) to O = $\sqrt{(5^2 + 0^2)} = 5$

$5 < \sqrt{31}$ (= 5.57), so the vertex at the origin is inside the circle.

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11 (b) (ii) Prove that the triangle lies completely within the circle $x^2 + y^2 - 10x - 6 = 0$

[4 marks]

Angles of an equilateral triangle are 60°, so each side from O makes 30° with Ox.

$\tan 30° = \frac{y}{8} \Rightarrow y = 8 \tan 30° = \frac{8}{\sqrt{3}}$ $(= 4.62)$

Other vertices: $(8, \frac{8}{\sqrt{3}})$ and $(8, -\frac{8}{\sqrt{3}})$

Distance from (5, 0) to $(8, \frac{8}{\sqrt{3}}) = \sqrt{(3^2 + \frac{64}{3})} = \sqrt{\frac{91}{3}} = 5.507...$

$\sqrt{\frac{91}{3}} = \sqrt{30.33...} < \sqrt{31} = 5.567...$, so $(8, \frac{8}{\sqrt{3}})$ is inside the circle.

By symmetry in the x-axis, $(8, -8/\sqrt{3})$ is also inside; O is inside from (b)(i).

All three vertices lie inside the circle and the inside of a circle is convex,

so every point of the triangle is inside: it lies completely within the circle.

END OF SECTION A

TURN OVER FOR SECTION B

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Section B

Answer **all** questions in the spaces provided.

12 A particle, initially at rest, starts to move forward in a straight line with constant acceleration, $a\,\text{m s}^{-2}$

After 6 seconds the particle has a velocity of $3\,\text{m s}^{-1}$

Find the value of a

Circle your answer.

[1 mark]

-2 $\quad$ -0.5 $\quad$ (0.5) $\quad$ 2

13 A resultant force of $\begin{bmatrix} -2 \\ 6 \end{bmatrix}$ N acts on a particle.

The acceleration of the particle is $\begin{bmatrix} -6 \\ y \end{bmatrix}\,\text{m s}^{-2}$

Find the value of y

Circle your answer.

[1 mark]

2 $\quad$ 3 $\quad$ 10 $\quad$ (18)

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14 A ball, initially at rest, is dropped from a vertical height of h metres above the Earth's surface.

After 4 seconds the ball's height above the Earth's surface is $0.2h$ metres.

14 (a) Assuming air resistance can be ignored, show that

$$h = 10g$$

[3 marks]

Taking down as positive: $u = 0$, $t = 4$, $a = g$

distance fallen $s = h - 0.2h = 0.8h$

$s = ut + \frac{1}{2}at^2$

$0.8h = 0 \times 4 + \frac{1}{2} \times g \times 4^2$

$0.8h = 8g$

$h = 10g$ as required

14 (b) Assuming air resistance cannot be ignored, explain the effect that this would have on the value of h in part **(a)**.

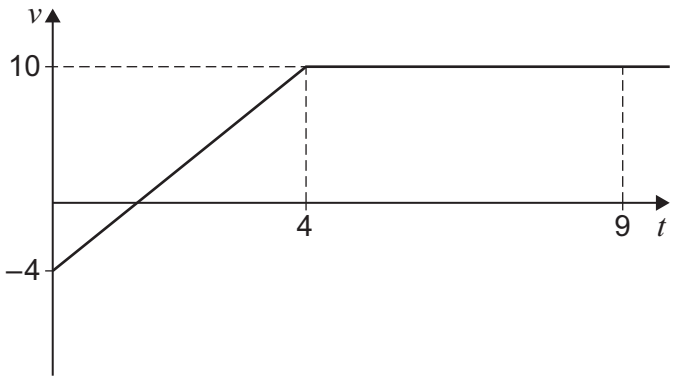
[1 mark]

Air resistance reduces the acceleration below g, so the ball falls less far in 4 seconds: the value of h would be lower ($h < 10g$).

Turn over ▶

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15 A particle is moving in a straight line such that its velocity, $v\,\text{m s}^{-1}$, changes with respect to time, t seconds, as shown in the graph below.



15 (a) Show that the acceleration of the particle over the first 4 seconds is $3.5\,\text{m s}^{-2}$

[1 mark]

$a = \frac{\text{change in v}}{\text{time}} = \frac{10 - (-4)}{4} = \frac{14}{4} = 3.5\ \text{m s}^{-2}$

15 (b) The particle is initially at a fixed point P

Show that the displacement of the particle from P, when $t = 9$, is 62 metres.

[3 marks]

For $0 \leq t \leq 4$: $u = -4$, $a = 3.5$, $t = 4$

$s = ut + \frac{1}{2}at^2 = (-4)(4) + \frac{1}{2}(3.5)(4^2) = -16 + 28 = 12\ \text{m}$

(check by area: $v = 0$ at $t = \frac{8}{7}$; $\frac{1}{2} \times \frac{20}{7} \times 10 - \frac{1}{2} \times \frac{8}{7} \times 4 = \frac{100}{7} - \frac{16}{7} = 12$)

For $4 \leq t \leq 9$: constant velocity $10\ \text{m s}^{-1}$, displacement $= 10 \times 5 = 50\ \text{m}$

Displacement at $t = 9$: $s = 12 + (9 - 4) \times 10 = 12 + 50 = 62\ \text{m}$ as required

Do not write outside the box

16 A toy remote control speed boat is launched from one edge of a small pond and moves in a straight line across the pond's surface.

The boat's velocity, $v\,\text{m}\,\text{s}^{-1}$, is modelled in terms of time, t seconds after the boat is launched, by the expression

$$v = 0.9 + 0.16t - 0.06t^2$$

16 (a) Find the acceleration of the boat when $t = 2$

[3 marks]

$a = \frac{dv}{dt} = 0.16 - 0.12t$

When $t = 2$: $a = 0.16 - 0.12 \times 2 = 0.16 - 0.24$

$a = -0.08\ \text{m}\ \text{s}^{-2}$

16 (b) Find the displacement of the boat, from the point where it was launched, when $t = 2$

[4 marks]

$s = \int v\,dt = \int (0.9 + 0.16t - 0.06t^2)\,dt$

$s = 0.9t + 0.08t^2 - 0.02t^3 + c$

$s = 0$ when $t = 0 \Rightarrow c = 0$

$t = 2$: $s = 0.9(2) + 0.08(2^2) - 0.02(2^3)$

$= 1.8 + 0.32 - 0.16$

Displacement = 1.96 m

Do not write outside the box

17 A particle, P, is initially at rest on a smooth horizontal surface.

A resultant force of $\begin{bmatrix} 12 \\ 9 \end{bmatrix}$ N is then applied to P, so that it moves in a straight line.

17 (a) Find the magnitude of the resultant force.

[1 mark]

$|F| = \sqrt{(12^2 + 9^2)} = \sqrt{225} = 15$ N

17 (b) Two fixed points A and B have position vectors

$$\overrightarrow{OA} = \begin{bmatrix} 3 \\ 7 \end{bmatrix} \text{ metres} \quad \text{and} \quad \overrightarrow{OB} = \begin{bmatrix} k \\ k-1 \end{bmatrix} \text{ metres}$$

with respect to a fixed origin, O

P moves in a straight line parallel to $\overrightarrow{AB}$

17 (b) (i) Find $\overrightarrow{AB}$ in terms of k

[1 mark]

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{bmatrix} k \\ k-1 \end{bmatrix} - \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} k-3 \\ k-8 \end{bmatrix}$ metres

Do not write outside the box

17 (b) (ii) Find the value of k

[2 marks]

P starts from rest, so it moves in the direction of the force $\begin{bmatrix} 12 \\ 9 \end{bmatrix}$,

so $\overrightarrow{AB}$ is a scalar multiple of $\begin{bmatrix} 12 \\ 9 \end{bmatrix}$: $\begin{bmatrix} k-3 \\ k-8 \end{bmatrix} = \lambda \begin{bmatrix} 12 \\ 9 \end{bmatrix}$

$9(k-3) = 12(k-8)$

$9k - 27 = 12k - 96$

$3k = 69$

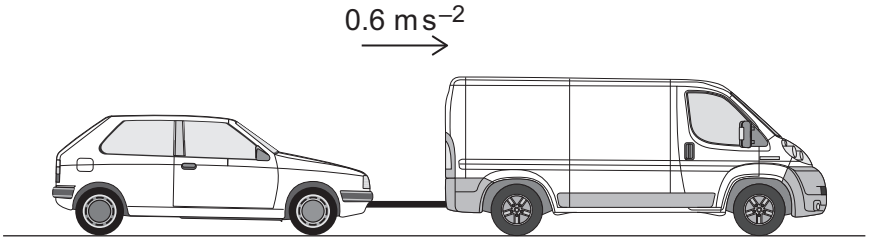
$k = 23$

Turn over for the next question

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18 A rescue van is towing a broken-down car by using a tow bar.

The van and the car are moving with a constant acceleration of $0.6\,\text{m}\,\text{s}^{-2}$ along a straight horizontal road as shown in the diagram below.



The van has a total mass of 2780 kg

The car has a total mass of 1620 kg

The van experiences a driving force of D newtons.

The van experiences a total resistance force of R newtons.

The car experiences a total resistance force of $0.6R$ newtons.

18 (a) The tension in the tow bar, T newtons, may be modelled by

$$T = kD - 18$$

where k is a constant.

Find k

[5 marks]

Car: $T - 0.6R = 1620 \times 0.6 = 972$

Van: $D - R - T = 2780 \times 0.6 = 1668$

From car: $R = \frac{T - 972}{0.6}$

Van: $D - \frac{T - 972}{0.6} - T = 1668$

$0.6D - T + 972 - 0.6T = 1000.8$

$0.6D - 1.6T = 28.8$

$T = \frac{0.6D - 28.8}{1.6} = \frac{3}{8}D - 18$

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Comparing with $T = kD - 18$: $k = \frac{3}{8}$

18 (b) State one assumption that must be made in answering part **(a)**.

[1 mark]

The tow bar is horizontal (and light and inextensible).

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