AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS MATHEMATICS

Paper 2

Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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Section A

Answer **all** questions in the spaces provided.

1 Find $\int 12x^3 \, dx$

Circle your answer.

[1 mark]

$36x^2 + c$ $\quad$ $3x^4 + c$ $\quad$ $3x^2 + c$ $\quad$ $36x^4 + c$

2 Given that

$$\cos(\theta - 20°) = \cos 60°$$

which one of the following is a possible value for θ?

Circle your answer.

[1 mark]

40° $\quad$ 140° $\quad$ 280° $\quad$ 320°

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3 A curve has equation $y = k\sqrt{x}$ where k is a constant.

Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at the point (4, 2k) on the curve, giving your answer as an expression in terms of k.

[5 marks]

$y = k\sqrt{x} = kx^{1/2}$

$\frac{dy}{dx} = \frac{k}{2}x^{-1/2}$

$\frac{d^2y}{dx^2} = -\frac{k}{4}x^{-3/2}$

At (4, 2k): $x^{-3/2} = 4^{-3/2} = \frac{1}{8}$

$\frac{d^2y}{dx^2} = -\frac{k}{4} \times \frac{1}{8} = -\frac{k}{32}$

Turn over for the next question

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4 The equation $9x^2 + 4x + p^2 = 0$ has no real solutions for x.

Find the set of possible values of p.

Fully justify your answer.

[4 marks]

No real solutions, so the discriminant $b^2 - 4ac < 0$

$4^2 - 4 \times 9 \times p^2 < 0$

$16 - 36p^2 < 0$

$36p^2 > 16 \Rightarrow p^2 > \frac{4}{9}$

Critical values: $p = \frac{2}{3}$ and $p = -\frac{2}{3}$

$p < -\frac{2}{3}$ or $p > \frac{2}{3}$

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5 Kaya is investigating the function

$$f(x) = 2x^3 - 7x^2 - 12x + 45$$

Kaya makes two statements.

Statement 1: $f(3) = 0$

Statement 2: this shows that $(x + 3)$ must be a factor of $f(x)$.

5 (a) State, with a reason, whether each of Kaya's statements is correct.

[2 marks]

Statement 1: Correct: $f(3) = 54 - 63 - 36 + 45 = 0$

so Kaya is correct that $f(3) = 0$

Statement 2: Incorrect: $f(3) = 0$ shows that $(x - 3)$ is a factor,

not $(x + 3)$. $f(-3) = -54 - 63 + 36 + 45 = -36 \neq 0$

so $(x + 3)$ is not a factor of $f(x)$.

5 (b) Fully factorise $f(x)$.

[3 marks]

$f(3) = 0$ so $(x - 3)$ is a factor

$2x^3 - 7x^2 - 12x + 45 = (x - 3)(2x^2 + bx - 15)$

x^2 terms: $-7 = b - 6 \Rightarrow b = -1$

$f(x) = (x - 3)(2x^2 - x - 15)$

$2x^2 - x - 15 = (2x + 5)(x - 3)$

$f(x) = (x - 3)^2(2x + 5)$

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6 An on-line science website states:

'To find a dog's equivalent human age in years, multiply the natural logarithm of the dog's age in years by 16 then add 31.'

6 (a) Calculate the equivalent age to the nearest human year of a dog aged 5 years.

[1 mark]

Human age = $16 \ln 5 + 31 = 56.751...$

$= 57$ years

6 (b) A dog's equivalent age in human years is 40 years. Find the dog's actual age to the nearest month.

[3 marks]

$16 \ln x + 31 = 40$

$\ln x = \frac{9}{16}$

$x = e^{9/16} = 1.7550...$ years

$0.7550... \times 12 = 9.06$ months

Age = 1 year 9 months (21 months)

6 (c) Explain why the behaviour of the natural logarithm for values close to zero means that the formula given on the website cannot be true for very young dogs.

[2 marks]

As $x \to 0$, $\ln x \to -\infty$ ($\ln x$ is negative for $0 < x < 1$),

so $16 \ln x + 31$ becomes negative for very young dogs,

e.g. $x = 0.1$ gives $16 \ln 0.1 + 31 = -5.8$ years,

but an equivalent human age cannot be negative.

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7 The expression

$$\frac{3-\sqrt{n}}{2+\sqrt{n}}$$

can be written in the form $a+b\sqrt{n}$, where a and b and n are rational but $\sqrt{n}$ is irrational.

Find expressions for a and b in terms of n.

[4 marks]

$\frac{3-\sqrt{n}}{2+\sqrt{n}} \times \frac{2-\sqrt{n}}{2-\sqrt{n}}$

$= \frac{6-3\sqrt{n}-2\sqrt{n}+n}{4-2\sqrt{n}+2\sqrt{n}-n}$

$= \frac{6+n-5\sqrt{n}}{4-n}$

$= \frac{6+n}{4-n} + \frac{-5}{4-n}\sqrt{n}$

$a = \frac{6+n}{4-n}$, $b = \frac{-5}{4-n}$

Turn over for the next question

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8 Triangle *ABC* has sides of length $(m-n)$, m and $(m+n)$ where $0 < 2n < m$

Angle *A* is the largest angle in the triangle.

8 (a) (i) Explain why angle *A* must be opposite the side of length $(m+n)$.

[1 mark]

Since $n > 0$, $(m+n)$ is the longest side, and the largest angle is always opposite the longest side, so angle A must be opposite the side of length $(m+n)$.

8 (a) (ii) Using the cosine rule, show that $\cos A = \dfrac{m-4n}{2(m-n)}$

[3 marks]

$a = m+n, \quad b = m, \quad c = m-n$

$(m+n)^2 = m^2 + (m-n)^2 - 2m(m-n)\cos A$

$\cos A = \dfrac{m^2 + (m-n)^2 - (m+n)^2}{2m(m-n)}$

$= \dfrac{m^2 + m^2 - 2mn + n^2 - m^2 - 2mn - n^2}{2m(m-n)}$

$= \dfrac{m^2 - 4mn}{2m(m-n)}$

$= \dfrac{m(m-4n)}{2m(m-n)}$

$\cos A = \dfrac{m-4n}{2(m-n)}$ as required

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8 (b) You are given that BC is the diameter of a circle, and A lies on the circumference of the circle. The value of m is 8

Calculate the value of n.

[3 marks]

BC is a diameter, so A = 90° (angle in a semicircle)

cos A = cos 90° = 0

$\frac{m - 4n}{2(m - n)} = 0 \Rightarrow m - 4n = 0$

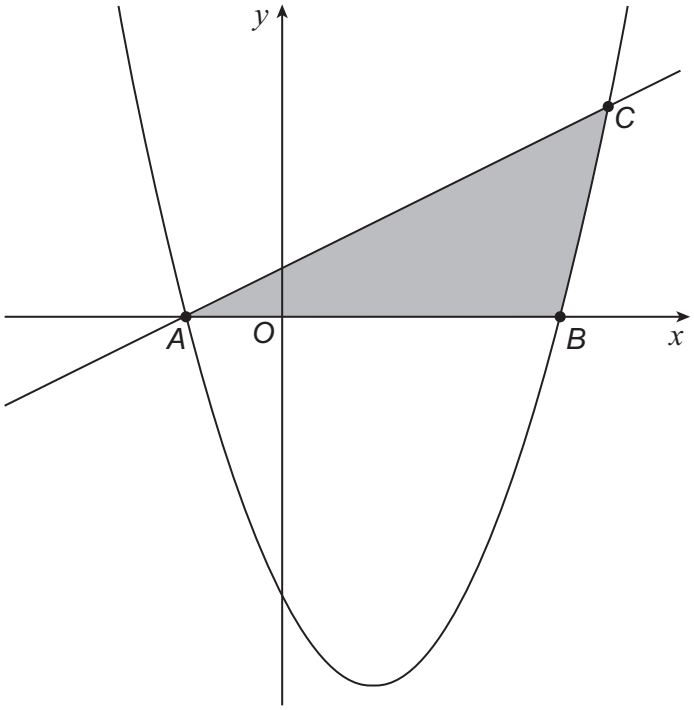
$8 - 4n = 0$

$n = 2$

Turn over for the next question

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9 The diagram below shows the graphs of $y = x^2 - 4x - 12$ and $y = x + 2$



9 (a) Write down three inequalities which together describe the shaded region.

[2 marks]

$y \leq x + 2$

$y \geq x^2 - 4x - 12$

$y \geq 0$

9 (b) Find the coordinates of the points *A*, *B* and *C*.

[4 marks]

A: $x + 2 = 0 \Rightarrow x = -2$, so $A(-2, 0)$

B: $x^2 - 4x - 12 = 0 \Rightarrow (x - 6)(x + 2) = 0 \Rightarrow x = 6$

so $B(6, 0)$

C: $x^2 - 4x - 12 = x + 2$

$x^2 - 5x - 14 = 0 \Rightarrow (x - 7)(x + 2) = 0$

$x = 7 \Rightarrow y = 7 + 2 = 9$, so $C(7, 9)$

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$A(-2, 0)$, $B(6, 0)$, $C(7, 9)$

9 (c) Find the exact area of the shaded region.

Fully justify your answer.

[6 marks]

Triangle A, C, (7, 0): base $7 - (-2) = 9$, height 9

Area $= \frac{1}{2} \times 9 \times 9 = 40.5$

Subtract area between curve BC and x-axis, $x = 6$ to 7:

$$\int_6^7 (x^2 - 4x - 12)\,dx = \left[\frac{x^3}{3} - 2x^2 - 12x\right]_6^7$$

$$= \left(\frac{343}{3} - 98 - 84\right) - (72 - 72 - 72)$$

$$= -\frac{203}{3} + 72 = \frac{13}{3}$$

Shaded area $= 40.5 - \frac{13}{3} = \frac{243 - 26}{6} = \frac{217}{6} = 36\frac{1}{6}$

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10 A bottle of water has a temperature of 6 °C when it is removed from a refrigerator.

It is placed in a room where the temperature is 20 °C

10 minutes later, the temperature of the water is 12 °C

The temperature of the water, T °C, at time t minutes after it is removed from the refrigerator, may be modelled by the equation

$$T = 20 - a\mathrm{e}^{-kt}$$

10 (a) Find the value of a.

[1 mark]

$t = 0, T = 6$: $6 = 20 - ae^{0}$

$a = 14$

10 (b) Calculate the value of k, giving your answer to two significant figures.

[3 marks]

$t = 10, T = 12$: $12 = 20 - 14e^{-10k}$

$e^{-10k} = \frac{8}{14} = \frac{4}{7}$

$-10k = \ln\frac{4}{7}$

$k = 0.055961...$

$k = 0.056$ (2 s.f.)

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10 (c) Using this model, estimate how long it takes the water to reach a temperature of 18 °C after it is taken out of the refrigerator.

[3 marks]

$18 = 20 - 14e^{-0.056t}$

$e^{-0.056t} = \frac{2}{14} = \frac{1}{7}$

$-0.056t = \ln\frac{1}{7}$

$t = \frac{\ln 7}{0.056} = 34.748...$

$t = 35$ minutes

10 (d) Explain why the model may not be appropriate to predict the temperature of the water three hours after it is taken out of the refrigerator.

[1 mark]

The room temperature is unlikely to stay constant at 20 °C for as long as three hours, so the model may not apply.

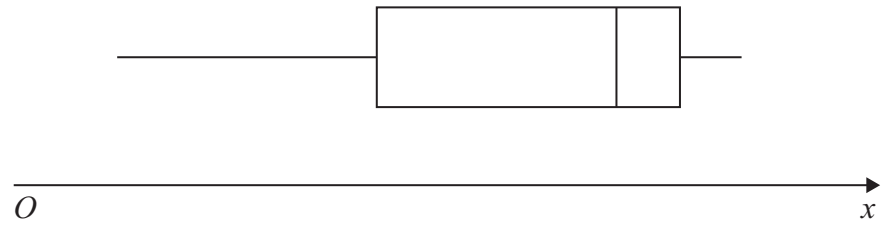
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Section B

Answer **all** questions in the spaces provided.

11 Which of the terms below best describes the distribution represented by the boxplot shown in **Figure 1**?

Figure 1



Circle your answer.

[1 mark]

even | negatively skewed | positively skewed | symmetric

12 Shelly organised an activity weekend for 15 groups of 10 people.

She decided to collect a sample to obtain feedback about the weekend.

To collect the sample Shelly selected two groups at random and then interviewed each member of these two groups.

State the name of this sampling method.

Circle your answer.

[1 mark]

Cluster | Opportunity | Stratified | Systematic

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13 Two random samples of 12 NOX emissions (in g/km) were taken from the Large Data Set.

One sample was taken from the 2002 data and the other sample from the 2016 data.

The sample data are shown below:

2002	0.031	0.019	0.091	0.025	0.030	0.061
	0.047	0.029	0.059	0.363	0.330	0.376

2016	0.005	0.047	0.053	0.063	0.026	0.013
	0.058	0.012	0.010	0.010	0.008	0.008

The mean and standard deviation of the **2002** sample data are 0.122 and 0.137 respectively.

13 (a) Find the mean and standard deviation of the **2016** sample data giving your answers correct to three decimal places.

[2 marks]

$\Sigma x = 0.313, \quad n = 12$

Mean $= 0.313 \div 12 = 0.026083...$

Mean $= 0.026$

Standard deviation $\sigma = 0.021464...$ (calculator)

Standard deviation $= 0.021$

13 (b) Siti claims these samples show that, on average, the NOX emissions across all makes of car in all areas of the UK have fallen by over 75% between 2002 and 2016.

13 (b) (i) Show how Siti's claim of 'over 75%' has been obtained.

[2 marks]

$$\%\text{ decrease in mean} = \frac{0.122 - 0.026}{0.122} \times 100$$

$$= 78.7\%$$

$78.7\% > 75\%$, so the mean has fallen by over 75%

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13 (b) (ii) Using your knowledge of the Large Data Set, make two comments on the validity of Siti's claim.

[2 marks]

Comment 1

The LDS only includes a limited number of makes of car,

so the claim cannot be made about all makes of car.

Comment 2

The LDS does not include cars from all areas of the UK,

so the claim cannot be made about all areas of the UK.

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14 Yingtai visits her local gym regularly.

After each visit she chooses one item to eat from the gym's cafe.

This could be an apple, a banana or a piece of cake.

She chooses the item independently each time.

The probability that Yingtai chooses each of these items on any visit is given by:

$$P(\text{Apple}) = 0.2$$

$$P(\text{Banana}) = 0.35$$

$$P(\text{Cake}) = 0.45$$

For any **four** randomly selected visits to the gym, find the probability that Yingtai chose:

14 (a) at least one banana.

[2 marks]

P(no banana in 4 visits) = 0.65^4 = 0.178506...

P(at least one banana) = $1 - 0.65^4$

= 0.82149... = 0.821

14 (b) the same item each time.

[2 marks]

P(same item) = $0.2^4 + 0.35^4 + 0.45^4$

= 0.0016 + 0.01500625 + 0.04100625

= 0.0576125 = 0.0576

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14 (c) apple twice and cake twice.

[3 marks]

Number of orders of A, A, C, C = $\frac{4!}{2!\,2!} = 6$

$P = 6 \times 0.2^2 \times 0.45^2$

$= 6 \times 0.04 \times 0.2025$

$= 0.0486$

Turn over for the next question

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15 The discrete random variable X is modelled by the probability distribution defined by:

$$\mathrm{P}(X = x) = \begin{cases} cx & x = 1, 2 \\ kx^2 & x = 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

where k and c are constants.

15 (a) State, in terms of k, the probability that $X = 3$

[1 mark]

$P(X = 3) = k \times 3^2 = 9k$

15 (b) Given that $\mathrm{P}(X \geq 3) = 3 \times \mathrm{P}(X \leq 2)$

Find the exact value of k and the exact value of c.

[4 marks]

$P(X = 1) = c, \quad P(X = 2) = 2c, \quad P(X = 3) = 9k, \quad P(X = 4) = 16k$

$\Sigma P = 1: \quad c + 2c + 9k + 16k = 1 \quad \Rightarrow \quad 3c + 25k = 1$

$P(X \geq 3) = 3P(X \leq 2): \quad 9k + 16k = 3(c + 2c)$

$25k = 9c$

Substitute: $\quad 3c + 9c = 1 \quad \Rightarrow \quad 12c = 1$

$c = \frac{1}{12}$

$25k = \frac{9}{12} = \frac{3}{4} \quad \Rightarrow \quad k = \frac{3}{100}$

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16 It is believed that a coin is biased so that the probability of obtaining a head when the coin is tossed is 0.7

16 (a) Assume that the probability of obtaining a head when the coin is tossed is indeed 0.7

16 (a) (i) Find the probability of obtaining exactly 6 heads from 7 tosses of the coin.

[1 mark]

$X \sim B(7, 0.7)$

$P(X = 6) = 7 \times 0.7^6 \times 0.3 = 0.24706... = 0.247$

16 (a) (ii) Find the mean number of heads obtained from 7 tosses of the coin.

[1 mark]

$\text{Mean} = np = 7 \times 0.7 = 4.9$

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16 (b) Harry believes that the probability of obtaining a head for this coin is actually greater than 0.7

To test this belief he tosses the coin 35 times and obtains 28 heads.

Carry out a hypothesis test at the 10% significance level to investigate Harry's belief.

[5 marks]

Let Y = number of heads in 35 tosses, p = P(head)

H_0: $p = 0.7$

H_1: $p > 0.7$

Under H_0, $Y \sim B(35, 0.7)$

$P(Y \geqslant 28) = 1 - P(Y \leqslant 27)$

$= 1 - 0.86735...$

$= 0.13265... = 0.133$

$0.133 > 0.10$, so the result is not significant: accept H_0

There is insufficient evidence at the 10% level to suggest

that the probability of obtaining a head is greater than 0.7

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