AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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Candidate signature _____

I declare this is my own work.

AS MATHEMATICS

Paper 1

Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Express as a single logarithm

$$\log_{10} 2 - \log_{10} x$$

Circle your answer.

[1 mark]

$\log_{10}(2 + x)$ $\log_{10}(2 - x)$ $\log_{10}(2x)$ $\log_{10}\left(\frac{2}{x}\right)$

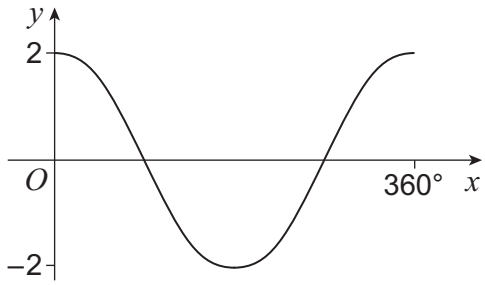
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2 The graph of the function $y = \cos\frac{1}{2}x$ for $0° \leq x \leq 360°$ is one of the graphs shown below.

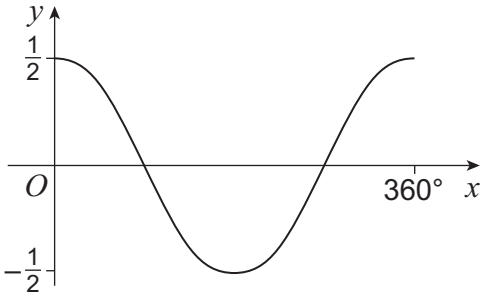
Identify the correct graph.

Tick (✓) **one** box.

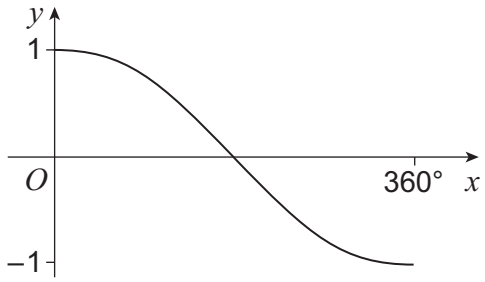
[1 mark]



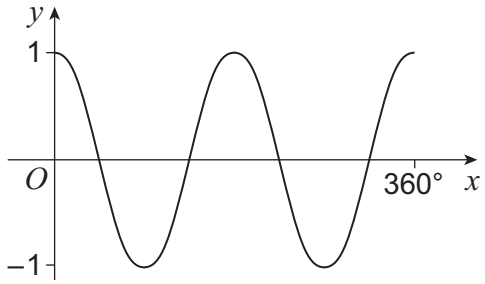
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3 Find the coefficient of the x^3 term in the expansion of $\left(3x+\frac{1}{2}\right)^4$

[3 marks]

Term in x^3: ${}^4C_3\ (3x)^3\ (\frac{1}{2})^1$

$= 4 \times 27x^3 \times \frac{1}{2}$

$= 54x^3$

Coefficient of $x^3 = 54$

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4 Find all the solutions of the equation

$$\cos^2\theta = 10\sin\theta + 4$$

for $0° < \theta < 360°$, giving your answers to the nearest degree.

Fully justify your answer.

[5 marks]

$\cos^2\theta = 1 - \sin^2\theta$

$1 - \sin^2\theta = 10\sin\theta + 4$

$\sin^2\theta + 10\sin\theta + 3 = 0$

$\sin\theta = \frac{-10 \pm \sqrt{(10^2 - 4 \times 1 \times 3)}}{2} = \frac{-10 \pm \sqrt{88}}{2}$

$\sin\theta = -0.3095...$ or $\sin\theta = -9.6904...$

$\sin\theta = -9.69...$ has no solutions since $-1 \leq \sin\theta \leq 1$

$\sin^{-1}(-0.3095...) = -18.03...°$

$\theta = 180° + 18.03...° = 198.03...°$

$\theta = 360° - 18.03...° = 341.96...°$

$\theta = 198°$ or $\theta = 342°$

Turn over for the next question

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5 Express $3x^3 + 5x^2 - 27x + 10$ in the form $(x - 2)(ax^2 + bx + c)$, where a, b and c are integers.

[3 marks]

$(x - 2)(ax^2 + bx + c) = 3x^3 + 5x^2 - 27x + 10$

x^3 terms: $a = 3$

constant terms: $-2c = 10 \Rightarrow c = -5$

x^2 terms: $b - 2a = 5 \Rightarrow b = 5 + 6 = 11$

check x terms: $c - 2b = -5 - 22 = -27$ ✓

$3x^3 + 5x^2 - 27x + 10 = (x - 2)(3x^2 + 11x - 5)$

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Turn over for the next question

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6 *AB* is a diameter of a circle where *A* is (1, 4) and *B* is (7, −2)

6 (a) Find the coordinates of the midpoint of *AB*.

[1 mark]

Midpoint = $\left(\frac{1+7}{2}, \frac{4+(-2)}{2}\right) = (4, 1)$

6 (b) Show that the equation of the circle may be written as

$$x^2 + y^2 - 8x - 2y = 1$$

[4 marks]

Centre C = midpoint of AB = (4, 1)

$r = AC = \sqrt{((4-1)^2 + (1-4)^2)} = \sqrt{(9+9)} = \sqrt{18}$

$r^2 = 18$

$(x-4)^2 + (y-1)^2 = 18$

$x^2 - 8x + 16 + y^2 - 2y + 1 = 18$

$x^2 + y^2 - 8x - 2y = 1$ as required

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6 (c) The circle has centre C and crosses the x-axis at points D and E.

Find the exact area of triangle DEC.

[4 marks]

$C = (4, 1)$. On the x-axis $y = 0$:

$x^2 - 8x = 1 \Rightarrow x^2 - 8x - 1 = 0$

$x = \frac{8 \pm \sqrt{(64 + 4)}}{2} = \frac{8 \pm \sqrt{68}}{2} = 4 \pm \sqrt{17}$

$D = (4 - \sqrt{17}, 0)$, $E = (4 + \sqrt{17}, 0)$

$DE = (4 + \sqrt{17}) - (4 - \sqrt{17}) = 2\sqrt{17}$

Height of triangle = distance of C from x-axis = 1

Area $= \frac{1}{2} \times 2\sqrt{17} \times 1$

Area of triangle $DEC = \sqrt{17}$

Turn over ▶

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7 A curve has equation $y = a^2 - x^2$, where $a > 0$

The area enclosed between the curve and the x-axis is 36 units.

Find the value of a.

Fully justify your answer.

[6 marks]

Curve meets x-axis when $a^2 - x^2 = 0$

$(a + x)(a - x) = 0 \quad \Rightarrow \quad x = -a \text{ or } x = a$

For $-a \leq x \leq a$, $y \geq 0$ so the area is given by

$\int_{-a}^{a} (a^2 - x^2)\, dx = 36$

$\left[a^2x - \frac{x^3}{3} \right]_{-a}^{a} = 36$

$\left(a^3 - \frac{a^3}{3}\right) - \left(-a^3 + \frac{a^3}{3}\right) = 36$

$\frac{2a^3}{3} + \frac{2a^3}{3} = 36$

$\frac{4a^3}{3} = 36$

$a^3 = 27$

$a = 3 \quad (a > 0)$

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Turn over for the next question

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8 A curve has equation

$$y = x^3 - 6x + \frac{9}{x}$$

8 (a) Show that the x coordinates of the stationary points of the curve satisfy the equation

$$x^4 - 2x^2 - 3 = 0$$

[3 marks]

$y = x^3 - 6x + 9x^{-1}$

$\frac{dy}{dx} = 3x^2 - 6 - 9x^{-2} = 3x^2 - 6 - \frac{9}{x^2}$

At stationary points $\frac{dy}{dx} = 0$: $3x^2 - 6 - \frac{9}{x^2} = 0$

$\times x^2$: $3x^4 - 6x^2 - 9 = 0$

$\div 3$: $x^4 - 2x^2 - 3 = 0$ as required

8 (b) Deduce that the curve has exactly two stationary points.

[3 marks]

$x^4 - 2x^2 - 3 = 0$

$(x^2 - 3)(x^2 + 1) = 0$

$x^2 = 3 \Rightarrow x = \sqrt{3}$ or $x = -\sqrt{3}$ (two solutions)

$x^2 = -1$ has no real solutions

So $\frac{dy}{dx} = 0$ has exactly two real solutions,

therefore the curve has exactly two stationary points

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8 (c) Find the coordinates and nature of the two stationary points.

Fully justify your answer.

[4 marks]

$\frac{d^2y}{dx^2} = 6x + 18x^{-3} = 6x + \frac{18}{x^3}$

$x = \sqrt{3}$: $y = (\sqrt{3})^3 - 6\sqrt{3} + 9/\sqrt{3} = 3\sqrt{3} - 6\sqrt{3} + 3\sqrt{3} = 0$

$\frac{d^2y}{dx^2} = 6\sqrt{3} + \frac{18}{3\sqrt{3}} = 8\sqrt{3} > 0 \Rightarrow$ minimum

$x = -\sqrt{3}$: $y = -3\sqrt{3} + 6\sqrt{3} - 3\sqrt{3} = 0$

$\frac{d^2y}{dx^2} = -6\sqrt{3} - \frac{18}{3\sqrt{3}} = -8\sqrt{3} < 0 \Rightarrow$ maximum

$(\sqrt{3}, 0)$ is a minimum point, $(-\sqrt{3}, 0)$ is a maximum point

8 (d) Write down the equation of a line which is a tangent to the curve in two places.

[1 mark]

$y = 0$

Turn over for the next question

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9 Integers m and n are both odd.

Prove that $m^2 + n^2$ is a multiple of 2 but **not** a multiple of 4

[5 marks]

Let $m = 2p + 1$ and $n = 2q + 1$, where p and q are integers

$m^2 + n^2 = (2p + 1)^2 + (2q + 1)^2$

$= 4p^2 + 4p + 1 + 4q^2 + 4q + 1$

$= 2(2p^2 + 2q^2 + 2p + 2q + 1)$

$2p^2 + 2q^2 + 2p + 2q + 1$ is an integer, so $m^2 + n^2$ is a multiple of 2

Also $m^2 + n^2 = 4(p^2 + q^2 + p + q) + 2$

$p^2 + q^2 + p + q$ is an integer, so $m^2 + n^2$ is 2 more than

a multiple of 4: it leaves remainder 2 when divided by 4

(equivalently $2p^2 + 2q^2 + 2p + 2q + 1$ is odd)

$\therefore m^2 + n^2$ is a multiple of 2 but not a multiple of 4

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Turn over for the next question

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10 Curve C has equation $y = \frac{\sqrt{2}}{x^2}$

10 (a) Find an equation of the tangent to C at the point $\left(2, \frac{\sqrt{2}}{4}\right)$

[4 marks]

$y = \sqrt{2}\,x^{-2}$

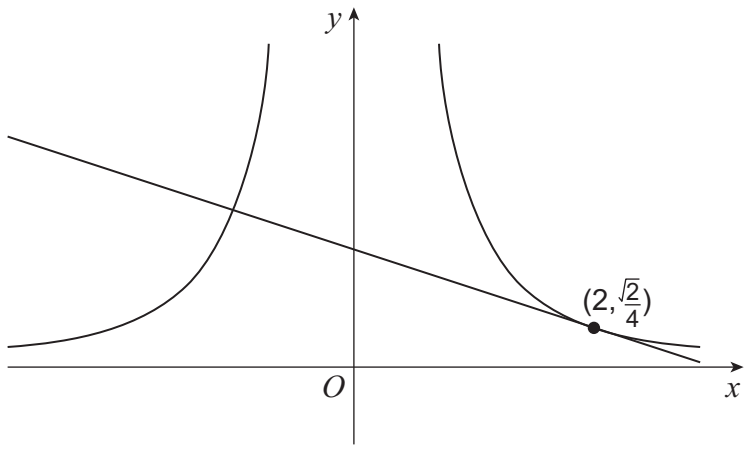
$\frac{dy}{dx} = -2\sqrt{2}\,x^{-3} = -\frac{2\sqrt{2}}{x^3}$

At $x = 2$: $\frac{dy}{dx} = -\frac{2\sqrt{2}}{8} = -\frac{\sqrt{2}}{4}$

Tangent: $y - \frac{\sqrt{2}}{4} = -\frac{\sqrt{2}}{4}(x - 2)$

$y = \frac{3\sqrt{2}}{4} - \frac{\sqrt{2}}{4}x$

10 (b) Show that the tangent to C at the point $\left(2, \frac{\sqrt{2}}{4}\right)$ is also a normal to the curve at a different point.



[5 marks]

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Tangent meets curve when $\frac{\sqrt{2}}{x^2} = \frac{3\sqrt{2}}{4} - \frac{\sqrt{2}x}{4}$

$\div \sqrt{2}$: $\frac{1}{x^2} = \frac{3}{4} - \frac{x}{4}$

$\times 4x^2$: $4 = 3x^2 - x^3$

$x^3 - 3x^2 + 4 = 0$

$x = 2$ is a repeated root (point of contact):

$(x - 2)^2(x + 1) = 0$

Other intersection: $x = -1$, $y = \frac{\sqrt{2}}{(-1)^2} = \sqrt{2}$

Gradient of curve at $x = -1$: $\frac{dy}{dx} = -\frac{2\sqrt{2}}{(-1)^3} = 2\sqrt{2}$

Gradient of tangent × gradient of curve

$= (-\frac{\sqrt{2}}{4}) \times 2\sqrt{2} = -\frac{4}{4} = -1$

So the line is perpendicular to the curve at $(-1, \sqrt{2})$

$\therefore$ the tangent at $(2, \frac{\sqrt{2}}{4})$ is also the normal to the curve

at the different point $(-1, \sqrt{2})$

Turn over for Section B

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Section B

Answer **all** questions in the spaces provided.

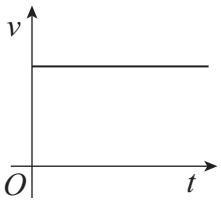
11 A car, initially at rest, moves with constant acceleration along a straight horizontal road.

One of the graphs below shows how the car's velocity, $v\,\text{m}\,\text{s}^{-1}$, changes over time, t seconds.

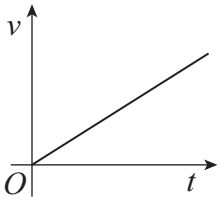
Identify the correct graph.

Tick (✓) **one** box.

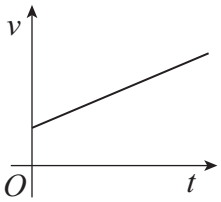
[1 mark]



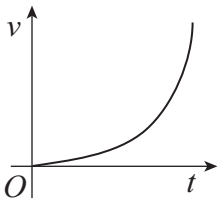
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12 A horizontal force of 30 N causes a crate to travel with an acceleration of $2\,\text{m s}^{-2}$, in a straight line, on a smooth horizontal surface.

Find the **weight** of the crate.

Circle your answer.

[1 mark]

15 kg | (15g N) | 15 N | 15g kg

13 Two points A and B lie in a horizontal plane and have coordinates $(-2, 7)$ and $(3, 19)$ respectively.

A particle moves in a straight line from A to B under the action of a constant resultant force of magnitude 6.5 N

Express the resultant force in vector form.

[3 marks]

$\overrightarrow{AB} = (3 - (-2))i + (19 - 7)j = 5i + 12j$

$|\overrightarrow{AB}| = \sqrt{(5^2 + 12^2)} = \sqrt{169} = 13$

Force acts along AB: $F = \frac{6.5}{13}(5i + 12j)$

$F = 0.5(5i + 12j) = (2.5i + 6j)$ N

Turn over ▶

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14 A ball is released from rest from a height h metres above horizontal ground and falls freely downwards.

When the ball reaches the ground, its speed is $v\,\text{m s}^{-1}$, where $v \leq 10$

Show that

$$h \leq \frac{50}{g}$$

[3 marks]

Taking downwards as positive: $u = 0$, $a = g$, $s = h$, $v = v$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2gh = 2gh$

$v \leq 10 \Rightarrow v^2 \leq 100 \quad (v \geq 0)$

$2gh \leq 100$

$h \leq \frac{100}{2g}$

$h \leq \frac{50}{g}$ as required

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Turn over for the next question

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15 Two particles, P and Q, are initially at rest at the same point on a horizontal plane.

A force of $\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ N is applied to P.

A force of $\begin{bmatrix} 8 \\ 15 \end{bmatrix}$ N is applied to Q.

15 (a) Calculate, to the nearest degree, the acute angle between the two forces.

[2 marks]

Force on P acts along i (angle 0° with i)

Angle of force on Q with i: $\tan^{-1}\left(\frac{15}{8}\right) = 61.927...°$

Acute angle between the forces = 61.927...° − 0°

= 62° (nearest degree)

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15 (b) The particles begin to move under the action of the respective forces.

P and *Q* have the same mass.

P has an acceleration of magnitude $5\,\text{m}\,\text{s}^{-2}$

Find the magnitude of the acceleration of *Q*.

[3 marks]

Magnitude of force on P = 4 N

F = ma: $4 = m \times 5 \Rightarrow m = 0.8$ kg

Magnitude of force on Q = $\sqrt{(8^2 + 15^2)} = \sqrt{289} = 17$ N

F = ma: $17 = 0.8a$

$a = \frac{17}{0.8} = 21.25$

Magnitude of acceleration of Q = 21.25 m s^{-2}

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16 Jermaine and his friend Meena are walking in the same direction along a straight path.

Meena is walking at a constant speed of $u\,\text{m s}^{-1}$

Jermaine is walking $0.2\,\text{m s}^{-1}$ more slowly than Meena.

When Jermaine is d metres behind Meena he starts to run with a constant acceleration of $2\,\text{m s}^{-2}$, for a time of t seconds, until he reaches her.

16 (a) Show that

$$d = t^2 - 0.2t$$

[4 marks]

Jermaine: initial speed $(u - 0.2)$ m s^{-1}, $a = 2$ m s^{-2}

$s = ut + \frac{1}{2}at^2$: $s_J = (u - 0.2)t + \frac{1}{2} \times 2t^2$

$s_J = (u - 0.2)t + t^2$

Meena (constant speed): $s_M = ut$

Jermaine reaches Meena when $s_J = s_M + d$

$(u - 0.2)t + t^2 = ut + d$

$ut - 0.2t + t^2 = ut + d$

$d = t^2 - 0.2t$ as required

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16 (b) When Jermaine's speed is $7.8\,\text{m}\,\text{s}^{-1}$, he reaches Meena.

Given that $u = 1.4$ find the value of d.

[2 marks]

Jermaine: $v = u + at$

$7.8 = (1.4 - 0.2) + 2t$

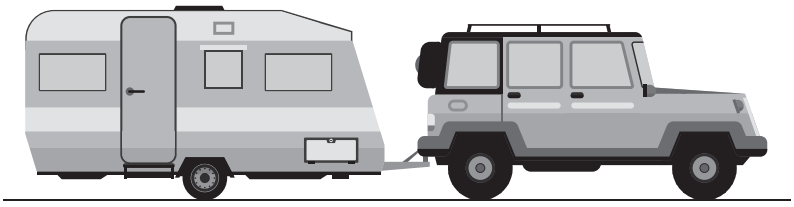
$7.8 = 1.2 + 2t \Rightarrow 2t = 6.6 \Rightarrow t = 3.3$

$d = 3.3^2 - 0.2 \times 3.3 = 10.89 - 0.66$

$d = 10.23$

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17



A car and caravan, connected by a tow bar, move forward together along a horizontal road.

Their velocity $v\,\text{m}\,\text{s}^{-1}$ at time t seconds, for $0 \leq t < 20$, is given by

$$v = 0.5t + 0.01t^2$$

17 (a) Show that when $t = 15$ their acceleration is $0.8\,\text{m}\,\text{s}^{-2}$

[2 marks]

$a = \frac{dv}{dt} = 0.5 + 0.02t$

When $t = 15$: $a = 0.5 + 0.02 \times 15 = 0.5 + 0.3$

$a = 0.8\ \text{m}\ \text{s}^{-2}$ as required

17 (b) The car has a mass of 1500 kg

The caravan has a mass of 850 kg

When $t = 15$ the tension in the tow bar is 800 N and the car experiences a resistance force of 100 N

17 (b) (i) Find the total resistance force experienced by the caravan when $t = 15$

[2 marks]

Caravan: $F = ma$ (R = resistance on caravan)

$800 - R = 850 \times 0.8 = 680$

$R = 120$ N

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17 (b) (ii) Find the driving force being applied by the car when $t = 15$

[3 marks]

Car: $F = ma$ (D = driving force)

$D - 800 - 100 = 1500 \times 0.8$

$D - 900 = 1200$

$D = 2100$ N

17 (c) State one assumption you have made about the tow bar.

[1 mark]

The tow bar is light and inextensible.

END OF QUESTIONS

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