AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname StemUp Model Solution

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AS MATHEMATICS

Paper 2

Wednesday 22 May 2019 Morning Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Find the gradient of the curve $y = e^{-3x}$ at the point where it crosses the y-axis.

Circle your answer.

[1 mark]

-3 $\quad -1 \quad$ 1 $\quad$ 3

2 Find the centre of the circle $x^2 + y^2 + 4x - 6y = 12$

Tick (✓) **one** box.

[1 mark]

$(-2, -3)$ ☐

$(-2, 3)$ ☑

$(2, -3)$ ☐

$(2, 3)$ ☐

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3 It is given that $\sin\theta = -0.1$ and $180° < \theta < 270°$

Find the exact value of $\cos\theta$

[2 marks]

$\sin^2\theta + \cos^2\theta = 1$

$(-0.1)^2 + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - 0.01 = 0.99$

$180° < \theta < 270°$ (third quadrant), so $\cos\theta < 0$

$\cos\theta = -\sqrt{0.99} = -\frac{\sqrt{99}}{10}$

$\cos\theta = -\frac{3}{10}\sqrt{11}$

4 Show that, for $x > 0$

$$\log_{10}\frac{x^4}{100} + \log_{10} 9x - \log_{10} x^3 \equiv 2(-1 + \log_{10} 3x)$$

[4 marks]

$LHS = \log_{10}\frac{x^4}{100} + \log_{10} 9x - \log_{10} x^3$

$= \log_{10} x^4 - \log_{10} 100 + \log_{10} 9 + \log_{10} x - \log_{10} x^3$

$= 4\log_{10} x - 2 + \log_{10} 9 + \log_{10} x - 3\log_{10} x$

$= 2\log_{10} x - 2 + \log_{10} 3^2$

$= 2\log_{10} x - 2 + 2\log_{10} 3$

$= 2(-1 + \log_{10} 3 + \log_{10} x)$

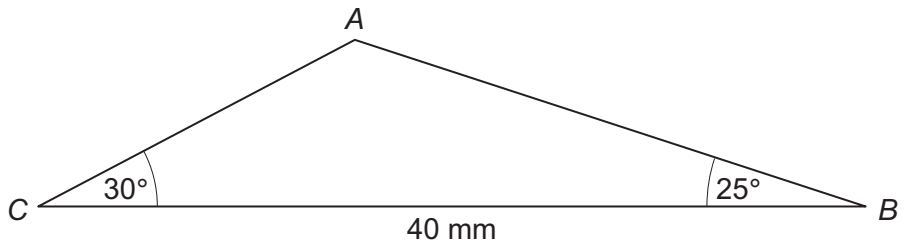
$= 2(-1 + \log_{10} 3x)$

$\therefore \log_{10}\frac{x^4}{100} + \log_{10} 9x - \log_{10} x^3 \equiv 2(-1 + \log_{10} 3x)$

Turn over for the next question

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5 A triangular prism has a cross section ABC as shown in the diagram below.



Angle $ABC = 25°$

Angle $ACB = 30°$

$BC = 40$ millimetres.

The length of the prism is 300 millimetres.

Calculate the volume of the prism, giving your answer to three significant figures.

[4 marks]

Angle BAC = 180° − 30° − 25° = 125°

Sine rule: $\frac{AB}{\sin 30°} = \frac{40}{\sin 125°}$

$AB = \frac{40 \sin 30°}{\sin 125°} = 24.415...$ mm

Area of ABC = $\frac{1}{2} \times BC \times AB \times \sin B$

$= \frac{1}{2} \times 40 \times 24.415... \times \sin 25°$

$= 206.368...$ mm^2

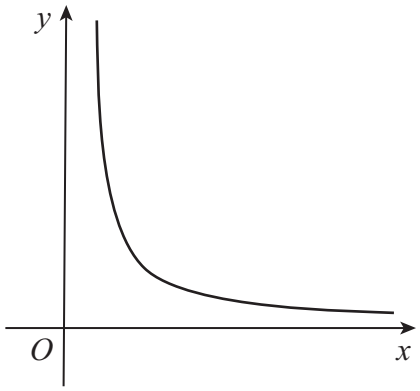
Volume = area × length = 206.368... × 300

$= 61910.5...$ mm^3

Volume = 61 900 mm^3 (3 s.f.)

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6 A curve has equation $y = \dfrac{2}{x\sqrt{x}}$



The region enclosed between the curve, the x-axis and the lines $x = 1$ and $x = a$ has area 3 units.

Given that $a > 1$, find the value of a.

Fully justify your answer.

[5 marks]

$y = \frac{2}{x\sqrt{x}} = 2x^{-3/2}$

For $x > 0$, $y > 0$, so the curve is above the x-axis and the area is given by the integral: $\int_1^a 2x^{-3/2}\,dx = 3$

$\int 2x^{-3/2}\,dx = \frac{2x^{-1/2}}{-1/2} = -4x^{-1/2}$

$[-4x^{-1/2}]_1^a = 3$

$(-4a^{-1/2}) - (-4 \times 1^{-1/2}) = 3$

$-4a^{-1/2} + 4 = 3$

$4a^{-1/2} = 1 \Rightarrow a^{-1/2} = \frac{1}{4}$

$\sqrt{a} = 4$

$a = 16$

Turn over ▶

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7 The points A $(a, 3)$ and B $(10, 6)$ lie on a circle.

AB is a diameter of the circle and passes through the point $(2, 4)$

The circle has equation

$$(x - c)^2 + (y - d)^2 = e$$

where c, d and e are rational numbers.

Find the values of a, c, d and e.

[6 marks]

(2, 4) lies on AB, so A, (2, 4) and B are collinear.

Gradient from (2, 4) to B = $\frac{6-4}{10-2} = \frac{1}{4}$

Gradient from A to B: $\frac{6-3}{10-a} = \frac{1}{4}$

$10 - a = 12 \Rightarrow a = -2$

AB is a diameter, so the centre is the midpoint of AB:

centre = $\left(\frac{-2+10}{2}, \frac{3+6}{2}\right) = (4, 4.5)$

so $c = 4$, $d = 4.5$

$e = r^2 = (10 - 4)^2 + (6 - 4.5)^2$

$= 36 + 2.25 = 38.25$

$(x - 4)^2 + (y - 4.5)^2 = 38.25$

$a = -2$, $c = 4$, $d = 4.5$, $e = 38.25$

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Turn over for the next question

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8 A curve has equation

$$y = x^3 + px^2 + qx - 45$$

The curve passes through point R (2, 3)

The gradient of the curve at R is 8

8 (a) Find the value of p and the value of q.

[5 marks]

R (2, 3) on the curve: $3 = 2^3 + p(2)^2 + q(2) - 45$

$3 = 8 + 4p + 2q - 45 \Rightarrow 4p + 2q = 40 \Rightarrow 2p + q = 20$ (1)

$\frac{dy}{dx} = 3x^2 + 2px + q$

At $x = 2$, $\frac{dy}{dx} = 8$: $8 = 3(2)^2 + 2p(2) + q$

$8 = 12 + 4p + q \Rightarrow 4p + q = -4$ (2)

(2) − (1): $2p = -24 \Rightarrow p = -12$

$q = 20 - 2(-12) = 44$

$p = -12$, $q = 44$

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8 (b) Calculate the area enclosed between the normal to the curve at R and the coordinate axes.

[5 marks]

Gradient of tangent at $R = 8$, so gradient of normal $= -\frac{1}{8}$

Normal at R: $y - 3 = -\frac{1}{8}(x - 2)$

$y = -\frac{1}{8}x + \frac{13}{4}$

$x = 0$: $y = \frac{13}{4}$ so it meets the y-axis at $(0, \frac{13}{4})$

$y = 0$: $\frac{1}{8}x = \frac{13}{4} \Rightarrow x = 26$ so it meets the x-axis at $(26, 0)$

Area $= \frac{1}{2} \times 26 \times \frac{13}{4}$

$= \frac{169}{4} = 42.25$ square units

Turn over ▶

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9 A curve C has equation $y = f(x)$ where

$$f(x) = (x - 2)(x - 3)^2$$

9 (a) Find the exact coordinates of the turning points of C.

Determine the nature of each turning point.

Fully justify your answer.

[8 marks]

$f(x) = (x - 2)(x^2 - 6x + 9) = x^3 - 8x^2 + 21x - 18$

$f'(x) = 3x^2 - 16x + 21$

At a turning point $f'(x) = 0$:

$3x^2 - 16x + 21 = 0$

$(3x - 7)(x - 3) = 0 \quad \Rightarrow \quad x = \frac{7}{3}$ or $x = 3$

$f(\frac{7}{3}) = (\frac{7}{3} - 2)(\frac{7}{3} - 3)^2 = \frac{1}{3} \times \frac{4}{9} = \frac{4}{27}$

$f(3) = (3 - 2)(3 - 3)^2 = 0$

$f''(x) = 6x - 16$

$f''(\frac{7}{3}) = 14 - 16 = -2 < 0 \quad \Rightarrow$ maximum

$f''(3) = 18 - 16 = 2 > 0 \quad \Rightarrow$ minimum

Maximum point at $(\frac{7}{3}, \frac{4}{27})$

Minimum point at $(3, 0)$

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9 (b) State the coordinates of the turning points of the curve

$$y = \mathrm{f}(x + 1) - 4$$

[2 marks]

Translation by 1 unit left and 4 units down: $(x, y) \rightarrow (x - 1, y - 4)$

$\left(\frac{7}{3} - 1, \frac{4}{27} - 4\right) = \left(\frac{4}{3}, -\frac{104}{27}\right)$

$(3 - 1, 0 - 4) = (2, -4)$

Turning points: $\left(\frac{4}{3}, -\frac{104}{27}\right)$ and $(2, -4)$

Turn over for the next question

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10 As part of an experiment, Zena puts a bucket of hot water outside on a day when the outside temperature is 0°C.

She measures the temperature of the water after 10 minutes and after 20 minutes. Her results are shown below.

Time (minutes)	10	20
Temperature (degrees Celsius)	30	12

Zena models the relationship between θ, the temperature of the water in °C, and t, the time in minutes, by

$$\theta = A \times 10^{-kt}$$

where A and k are constants.

10 (a) Using $t = 0$, explain how the value of A relates to the experiment.

[1 mark]

When $t = 0$, $\theta = A \times 10^0 = A$, so A is the initial temperature (°C) of the water when the bucket was put outside.

10 (b) Show that

$$\log_{10} \theta = \log_{10} A - kt$$

[1 mark]

$\theta = A \times 10^{-kt}$

$\log_{10} \theta = \log_{10} (A \times 10^{-kt})$

$= \log_{10} A + \log_{10} 10^{-kt}$

$= \log_{10} A - kt$

10 (c) Using Zena's results, calculate the values of A and k.

[4 marks]

$t = 10, \theta = 30$: $\log_{10} 30 = \log_{10} A - 10k$ (1)

$t = 20, \theta = 12$: $\log_{10} 12 = \log_{10} A - 20k$ (2)

(1) − (2): $\log_{10} 30 - \log_{10} 12 = 10k$

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$10k = \log_{10} \frac{30}{12} = \log_{10} 2.5$

$k = \frac{1}{10} \log_{10} 2.5 = 0.039794... = 0.0398$ (3 s.f.)

$\log_{10} A = \log_{10} 30 + 10k = \log_{10} 30 + \log_{10} 2.5 = \log_{10} 75$

$A = 75$

$A = 75$, $k = 0.0398$

10 (d) Zena states that the temperature of the water will be less than 1°C after 45 minutes.

Determine whether the model supports this statement.

[3 marks]

$t = 45$: $\theta = 75 \times 10^{-0.039794... \times 45}$

$\theta = 1.2143... = 1.21$ °C

$1.21 > 1$, so after 45 minutes the water is still above 1 °C.

The model does not support Zena's statement.

10 (e) Explain why Zena's model is unlikely to accurately give the value of θ after 45 minutes.

[1 mark]

The outside temperature may change during the 45 minutes, and the model implies the water never cools all the way down to 0 °C.

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Section B

Answer **all** questions in the spaces provided.

11 A survey is undertaken to find out the most popular political party in London.

The first 1100 available people from London are surveyed.

Identify the name of this type of sampling.

Circle your answer.

[1 mark]

simple random | opportunity | stratified | quota

12 Manny is studying the price and number of pages of a random sample of books.

He calculates the value of the product moment correlation coefficient between the price and number of pages in each book as 1.05

Which of the following best describes the value 1.05?

Tick (✓) **one** box.

[1 mark]

- definitely correct ☐
- probably correct ☐
- probably incorrect ☐
- definitely incorrect ☑

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Turn over for the next question

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13 Denzel wants to buy a car with a propulsion type **other than** petrol or diesel.

He takes a sample, from the Large Data Set, of the CO_2 emissions, in g/km, of cars with one particular propulsion type.

The sample is as follows

82 13 96 49 96 92 70 81

13 (a) Using your knowledge of the Large Data Set, state which propulsion type this sample is for, giving a reason for your answer.

[2 marks]

Petrol/electric hybrid (electric/petrol)

Electric cars have zero CO_2 emissions, and hybrid is the only other propulsion type in the Large Data Set with this many values.

13 (b) Calculate the mean of the sample.

[1 mark]

Mean = $\frac{579}{8}$ = 72.375 = 72.4 g/km

13 (c) Calculate the standard deviation of the sample.

[1 mark]

s = 28.650... = 28.7 g/km

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13 (d) Denzel claims that the value 13 is an outlier.

13 (d) (i) Any value more than 2 standard deviations from the mean can be regarded as an outlier.

Verify that Denzel's claim is correct.

[1 mark]

mean − 2 × s.d. = 72.375 − 2 × 28.650...

= 15.07...

13 < 15.07..., so 13 is more than 2 standard deviations below

the mean: 13 is an outlier and Denzel's claim is correct.

13 (d) (ii) State what effect, if any, removing the value 13 from the sample would have on the standard deviation.

[1 mark]

Removing 13 would decrease the standard deviation.

Turn over for the next question

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14 A probability distribution is given by

$$P(X = x) = c(4 - x), \text{ for } x = 0, 1, 2, 3$$

where c is a constant.

14 (a) Show that $c = \frac{1}{10}$

[2 marks]

$\Sigma P(X = x) = 1$

$c(4 - 0) + c(4 - 1) + c(4 - 2) + c(4 - 3) = 1$

$4c + 3c + 2c + c = 1 \quad \Rightarrow \quad 10c = 1$

$c = \frac{1}{10}$

14 (b) Calculate $P(X \geq 1)$

[2 marks]

$P(X \geq 1) = P(X = 1) + P(X = 2) + P(X = 3)$

$= 3c + 2c + c = 6c$

$= 6 \times \frac{1}{10} = 0.6$

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15 Two independent events, A and B, are such that

$$P(A) = 0.2$$

$$P(A \cup B) = 0.8$$

15 (a) (i) Find $P(B)$

[4 marks]

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

A and B are independent, so $P(A \cap B) = P(A) \times P(B) = 0.2\,P(B)$

$0.8 = 0.2 + P(B) - 0.2\,P(B)$

$0.6 = 0.8\,P(B)$

$P(B) = \frac{0.6}{0.8} = 0.75$

15 (a) (ii) Find $P(A \cap B)$

[1 mark]

$P(A \cap B) = 0.2 \times 0.75 = 0.15$

15 (b) State, with a reason, whether or not the events A and B are mutually exclusive.

[1 mark]

Not mutually exclusive, since $P(A \cap B) = 0.15 \neq 0$

(independent events with non-zero probabilities cannot be).

Turn over ▶

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16 Andrea is the manager of a company which makes mobile phone chargers.

In the past, she had found that 12% of all chargers are faulty.

16 (a) Andrea decides to move the manufacture of chargers to a different factory.

Andrea tests 60 of the new chargers and finds that 4 chargers are faulty.

Investigate, at the 10% level of significance, whether the proportion of faulty chargers has reduced.

[7 marks]

Let p = proportion of faulty chargers from the new factory.

$H_0 : p = 0.12$

$H_1 : p < 0.12$

Under H_0, $X \sim B(60, 0.12)$, X = number of faulty chargers in 60

$P(X \leq 4) = 0.13880... = 0.139$

$0.139 > 0.10$, so the result is not significant at the 10% level.

Accept H_0

There is insufficient evidence, at the 10% level, to suggest that the proportion of faulty chargers has reduced.

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16 (b) State, **in context**, two assumptions that are necessary for the distribution that you have used in part **(a)** to be valid.

[2 marks]

1. The probability that a charger is faulty is fixed at 0.12 for every charger.

2. Whether a charger is faulty is independent of whether any other charger is faulty.

END OF QUESTIONS

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