AQA

Please write clearly in block capitals.

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AS MATHEMATICS

Paper 1

Wednesday 15 May 2019 Morning Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 State the number of solutions to the equation $\tan 4\theta = 1$ for $0° < \theta < 180°$

Circle your answer.

[1 mark]

1 2 (4) 8

2 Dan believes that

for every positive integer n, at least one of $2^n - 1$ and $2^n + 1$ is prime.

Which value of n shown below is a counter example to Dan's belief?

Circle your answer.

[1 mark]

$n = 3$ $n = 4$ $n = 5$ ($n = 6$)

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3 It is given that $(x + 1)$ and $(x - 3)$ are two factors of $f(x)$, where

$$f(x) = px^3 - 3x^2 - 8x + q$$

3 (a) Find the values of p and q.

[3 marks]

$f(-1) = 0$: $-p - 3 + 8 + q = 0 \Rightarrow -p + q = -5$

$f(3) = 0$: $27p - 27 - 24 + q = 0 \Rightarrow 27p + q = 51$

Subtract: $28p = 56$

$p = 2, \quad q = -5 + 2 = -3$

3 (b) Fully factorise $f(x)$.

[2 marks]

$f(x) = 2x^3 - 3x^2 - 8x - 3$

$(x + 1)(x - 3) = x^2 - 2x - 3$

$2x^3 - 3x^2 - 8x - 3 = (x^2 - 2x - 3)(2x + 1)$

$f(x) = (x + 1)(x - 3)(2x + 1)$

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4 Show that $\dfrac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}$ can be expressed in the form $m\sqrt{n}+n\sqrt{m}$, where m and n are integers.

Fully justify your answer.

[4 marks]

$\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{6}(\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$

$= \frac{\sqrt{18}+\sqrt{12}}{3-2}$

$= \sqrt{18}+\sqrt{12}$

$= \sqrt{(9 \times 2)}+\sqrt{(4 \times 3)}$

$= 3\sqrt{2}+2\sqrt{3}$

This is of the form $m\sqrt{n}+n\sqrt{m}$ with $m = 3$ and $n = 2$,

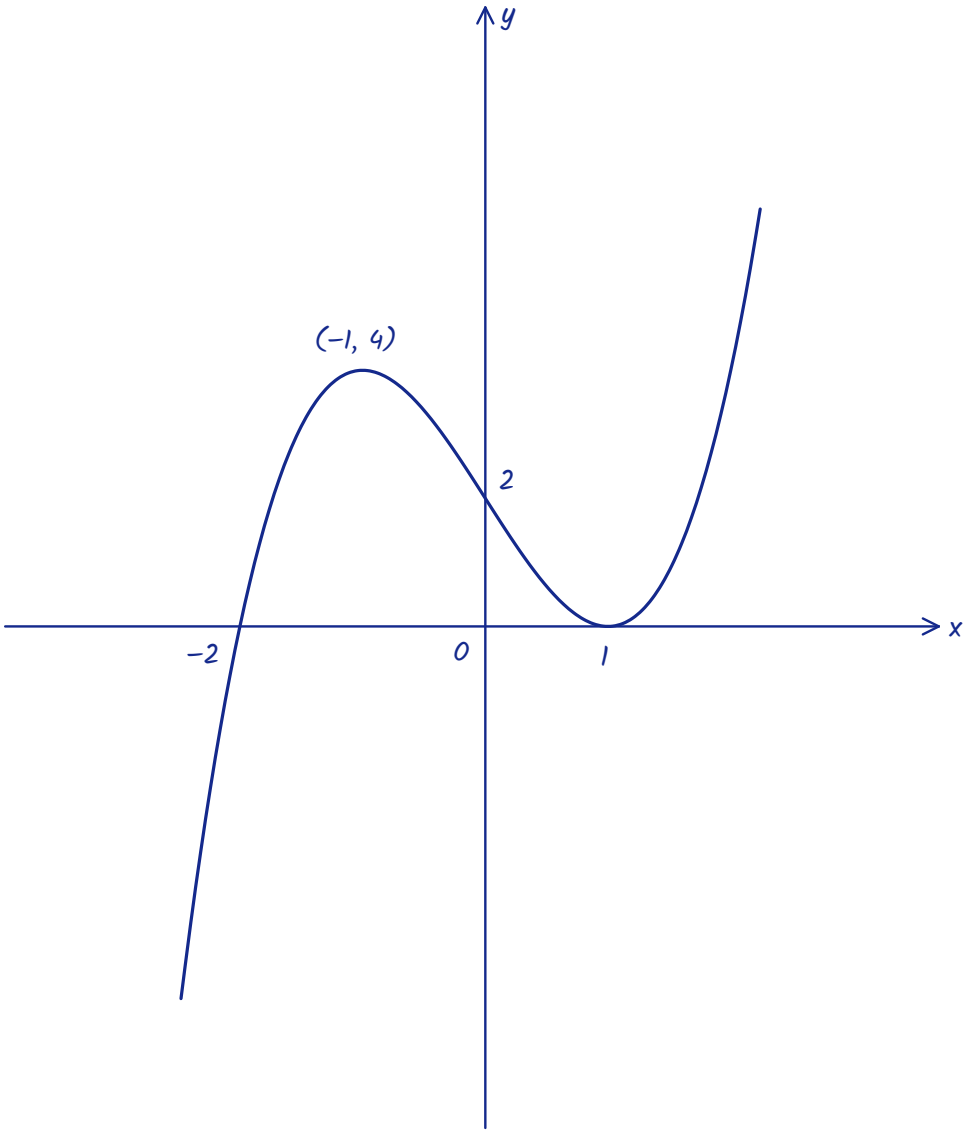
which are integers, as required.

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5 (a) Sketch the curve $y = g(x)$ where

$$g(x) = (x + 2)(x - 1)^2$$

[3 marks]



5 (b) Hence, solve $g(x) \leq 0$

[2 marks]

$g(x) \leq 0$ where the curve is on or below the x-axis:

$x \leq -2$ or $x = 1$

Turn over ▶

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6 (a) (i) Show that $\cos\theta = \frac{1}{2}$ is one solution of the equation

$$6\sin^2\theta + 5\cos\theta = 7$$

[2 marks]

$\sin^2\theta = 1 - \cos^2\theta$: $6(1 - \cos^2\theta) + 5\cos\theta = 7$

$6 - 6\cos^2\theta + 5\cos\theta = 7$

$6\cos^2\theta - 5\cos\theta + 1 = 0$

$(2\cos\theta - 1)(3\cos\theta - 1) = 0$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = \frac{1}{3}$

so $\cos\theta = \frac{1}{2}$ is one solution of the equation.

6 (a) (ii) Find all the values of θ that solve the equation

$$6\sin^2\theta + 5\cos\theta = 7$$

for $0° \leq \theta \leq 360°$

Give your answers to the nearest degree.

[2 marks]

$\cos\theta = \frac{1}{2}$: $\theta = 60°$, $360° - 60° = 300°$

$\cos\theta = \frac{1}{3}$: $\theta = 70.528...° = 71°$, $360° - 70.528...° = 289°$

$\theta = 60°, 71°, 289°, 300°$

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6 (b) Hence, find all the solutions of the equation

$$6\sin^2 2\theta + 5\cos 2\theta = 7$$

for $0° \leq \theta \leq 360°$

Give your answers to the nearest degree.

[2 marks]

Replace θ with 2θ, where 0° ≤ 2θ ≤ 720°:

2θ = 60°, 70.53°, 289.47°, 300°, 420°, 430.53°, 649.47°, 660°

θ = 30°, 35°, 145°, 150°,

210°, 215°, 325°, 330°

Turn over for the next question

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7 Given that $y \in \mathbb{R}$, prove that

$$(2 + 3y)^4 + (2 - 3y)^4 \geq 32$$

Fully justify your answer.

[6 marks]

$(2 + 3y)^4 = 2^4 + 4(2^3)(3y) + 6(2^2)(3y)^2 + 4(2)(3y)^3 + (3y)^4$

$= 16 + 96y + 216y^2 + 216y^3 + 81y^4$

$(2 - 3y)^4 = 2^4 + 4(2^3)(-3y) + 6(2^2)(-3y)^2 + 4(2)(-3y)^3 + (-3y)^4$

$= 16 - 96y + 216y^2 - 216y^3 + 81y^4$

Adding, the odd powers cancel:

$(2 + 3y)^4 + (2 - 3y)^4 = 32 + 432y^2 + 162y^4$

For $y \in \mathbb{R}$, $y^2 \geq 0$ and $y^4 = (y^2)^2 \geq 0$

so $432y^2 \geq 0$ and $162y^4 \geq 0$

$\therefore\ 32 + 432y^2 + 162y^4 \geq 32$

Hence $(2 + 3y)^4 + (2 - 3y)^4 \geq 32$ for all real y.

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8 Prove that the curve with equation

$$y = 2x^5 + 5x^4 + 10x^3 - 8$$

has **only one** stationary point, stating its coordinates.

[6 marks]

$\frac{dy}{dx} = 10x^4 + 20x^3 + 30x^2$

Stationary points where $\frac{dy}{dx} = 0$:

$10x^4 + 20x^3 + 30x^2 = 0$

$10x^2(x^2 + 2x + 3) = 0$

$x = 0$ or $x^2 + 2x + 3 = 0$

For $x^2 + 2x + 3$: discriminant $b^2 - 4ac = 2^2 - 4(1)(3) = 4 - 12 = -8$

$-8 < 0$, so $x^2 + 2x + 3 = 0$ has no real roots.

So $x = 0$ is the only solution of $\frac{dy}{dx} = 0$

When $x = 0$, $y = 0 + 0 + 0 - 8 = -8$

Hence the curve has only one stationary point, at $(0, -8)$.

Turn over for the next question

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9 A curve cuts the x-axis at (2, 0) and has gradient function

$$\frac{dy}{dx} = \frac{24}{x^3}$$

9 (a) Find the equation of the curve.

[4 marks]

$y = \int \frac{24}{x^3}\,dx = \int 24x^{-3}\,dx$

$y = \frac{24x^{-2}}{-2} + c = -\frac{12}{x^2} + c$

At (2, 0): $0 = -\frac{12}{2^2} + c$

$0 = -3 + c \Rightarrow c = 3$

$y = 3 - \frac{12}{x^2}$

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9 (b) Show that the perpendicular bisector of the line joining $A(-2, 8)$ to $B(-6, -4)$ is the normal to the curve at (2, 0)

[6 marks]

Midpoint of AB = $\left(\frac{-2 + (-6)}{2}, \frac{8 + (-4)}{2}\right) = (-4, 2)$

Gradient of AB = $\frac{8 - (-4)}{-2 - (-6)} = \frac{12}{4} = 3$

Gradient of perpendicular bisector = $-\frac{1}{3}$

Perpendicular bisector: $y - 2 = -\frac{1}{3}(x + 4)$

$y = -\frac{1}{3}x + \frac{2}{3}$

Gradient of curve at (2, 0): $\frac{dy}{dx} = \frac{24}{2^3} = \frac{24}{8} = 3$

Gradient of normal = $-\frac{1}{3}$

Normal at (2, 0): $y - 0 = -\frac{1}{3}(x - 2)$

$y = -\frac{1}{3}x + \frac{2}{3}$

The two equations are identical (same gradient −1/3, and (2, 0) lies on the bisector: −2/3 + 2/3 = 0), so the perpendicular bisector of AB is the normal to the curve at (2, 0).

Turn over ▶

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10 On 18 March 2019 there were 12 hours of daylight in Inverness.

On 16 June 2019, 90 days later, there will be 18 hours of daylight in Inverness.

Jude decides to model the number of hours of daylight in Inverness, N, by the formula

$$N = A + B\sin t°$$

where t is the number of days after 18 March 2019.

10 (a) (i) State the value that Jude should use for A.

[1 mark]

$A = 12$

10 (a) (ii) State the value that Jude should use for B.

[1 mark]

$B = 6$

10 (a) (iii) Using Jude's model, calculate the number of hours of daylight in Inverness on 15 May 2019, 58 days after 18 March 2019.

[1 mark]

$N = 12 + 6 \sin 58° = 17.088... = 17.1$ hours

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10 (a) (iv) Using Jude's model, find how many days during 2019 will have at least 17.4 hours of daylight in Inverness.

[4 marks]

$17.4 = 12 + 6 \sin t°$

$\sin t° = \frac{5.4}{6} = 0.9$

$t = 64.158...$ (≈ 64)

second value: $t = 180 - 64.158... = 115.84...$ (≈ 116)

Number of days $\approx 115.84 - 64.16 = 51.68...$

≈ 52 days

10 (a) (v) Explain why Jude's model will become inaccurate for 2020 and future years.

[1 mark]

The model repeats every 360 days, but a year has 365 days, so it gets more out of step with the real seasons each year.

10 (b) Anisa decides to model the number of hours of daylight in Inverness with the formula

$$N = A + B\sin\left(\frac{360}{365}t\right)°$$

Explain why Anisa's model is better than Jude's model.

[1 mark]

360/365 × t makes the model repeat every 365 days, which matches the number of days in a year.

Turn over ▶

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Section B

Answer **all** questions in the spaces provided.

11 A ball moves in a straight line and passes through two fixed points, *A* and *B*, which are 0.5 m apart.

The ball is moving with a constant acceleration of $0.39\,\text{m s}^{-2}$ in the direction *AB*.

The speed of the ball at *A* is $1.9\,\text{m s}^{-1}$

Find the speed of the ball at *B*.

Circle your answer.

[1 mark]

$2\,\text{m s}^{-1}$ $3.2\,\text{m s}^{-1}$ $3.8\,\text{m s}^{-1}$ $4\,\text{m s}^{-1}$

$v^2 = u^2 + 2as = 1.9^2 + 2(0.39)(0.5) = 3.61 + 0.39 = 4, \quad v = 2$

12 A particle *P*, of mass m kilograms, is attached to one end of a light inextensible string.

The other end of this string is held at a fixed position, *O*.

P hangs freely, in equilibrium, vertically below *O*.

Identify the statement below that correctly describes the tension, T newtons, in the string as m varies.

Tick (✓) **one** box.

[1 mark]

- T varies along the string, with its greatest value at *O* ☐
- T varies along the string, with its greatest value at *P* ☐
- $T = 0$ because the system is in equilibrium ☐
- T is directly proportional to m ☑

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Turn over for the next question

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13 A car, starting from rest, is driven along a horizontal track.

The velocity of the car, $v\,\text{m s}^{-1}$, at time t seconds, is modelled by the equation

$$v = 0.48t^2 - 0.024t^3 \text{ for } 0 \leq t \leq 15$$

13 (a) Find the distance the car travels during the first 10 seconds of its journey.

[3 marks]

$s = \int_0^{10} (0.48t^2 - 0.024t^3)\,dt$

$= [0.16t^3 - 0.006t^4]_0^{10}$

$= 0.16(1000) - 0.006(10000) - 0$

$= 160 - 60$

$= 100$ m

13 (b) Find the maximum speed of the car.

Give your answer to three significant figures.

[4 marks]

$\frac{dv}{dt} = 0.96t - 0.072t^2$

Maximum speed when $\frac{dv}{dt} = 0$:

$0.96t - 0.072t^2 = 0 \Rightarrow t(0.96 - 0.072t) = 0$

$t = 0$ or $t = \frac{0.96}{0.072} = \frac{40}{3} = 13.33...$

$t = 0$ gives $v = 0$ (start from rest), so maximum at $t = 40/3$

$v = 0.48(40/3)^2 - 0.024(40/3)^3$

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$v = 85.333... - 56.888... = 28.444...$

Maximum speed = 28.4 m s^{-1} (3 s.f.)

13 (c) Deduce the range of values of t for which the car is modelled as decelerating.

[2 marks]

Decelerating when $\frac{dv}{dt} < 0$: $t(0.96 - 0.072t) < 0$

for $t > 0$ this needs $t > \frac{40}{3}$, and the model is valid for $t \leq 15$

$\frac{40}{3} < t \leq 15$

Turn over for the next question

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14 Two particles, A and B, lie at rest on a smooth horizontal plane.

A has position vector $\mathbf{r}_A = (13\mathbf{i} - 22\mathbf{j})$ metres
B has position vector $\mathbf{r}_B = (3\mathbf{i} + 2\mathbf{j})$ metres

14 (a) Calculate the distance between A and B.

[2 marks]

$AB = r_A - r_B = (13 - 3)i + (-22 - 2)j = 10i - 24j$

Distance $= \sqrt{(10^2 + (-24)^2)} = \sqrt{676} = 26$ m

14 (b) Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are applied to particle A, where

$$\mathbf{F}_1 = (-2\mathbf{i} + 4\mathbf{j}) \text{ newtons}$$

$$\mathbf{F}_2 = (6\mathbf{i} - 10\mathbf{j}) \text{ newtons}$$

Given that A remains at rest, explain why $\mathbf{F}_3 = (-4\mathbf{i} + 6\mathbf{j})$ newtons

[1 mark]

A is at rest so the resultant force is zero: $F_1 + F_2 + F_3 = 0$

$F_1 + F_2 = (-2i + 4j) + (6i - 10j) = 4i - 6j$

$F_3 = -(4i - 6j) = (-4i + 6j)$ N

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14 (c) A force of $(5\mathbf{i} - 12\mathbf{j})$ newtons, is applied to B, so that B moves, from rest, in a straight line towards A.

B has a mass of 0.8 kg

14 (c) (i) Show that the acceleration of B towards A is $16.25\,\text{m}\,\text{s}^{-2}$

[2 marks]

$|F| = \sqrt{(5^2 + (-12)^2)} = \sqrt{169} = 13$ N

$F = ma$: $13 = 0.8a$

$a = \frac{13}{0.8} = 16.25$ m s^{-2} as required

14 (c) (ii) Hence, find the time taken for B to reach A.

Give your answer to two significant figures.

[2 marks]

$s = ut + \frac{1}{2}at^2$, $u = 0$, $s = 26$, $a = 16.25$

$26 = \frac{1}{2} \times 16.25 \times t^2$

$t^2 = \frac{52}{16.25} = 3.2$

$t = \sqrt{3.2} = 1.7888... = 1.8$ s (2 s.f.)

Turn over ▶

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15 A tractor and its driver have a combined mass of m kilograms.

The tractor is towing a trailer of mass $4m$ kilograms in a straight line along a horizontal road.

The tractor and trailer are connected by a horizontal tow bar, modelled as a light rigid rod.

A driving force of 11 080 N and a total resistance force of 160 N act on the tractor.

A total resistance force of 600 N acts on the trailer.

The tractor and the trailer have an acceleration of $0.8\,\text{m}\,\text{s}^{-2}$

15 (a) Find m.

[3 marks]

Whole system (mass $m + 4m = 5m$): $F = ma$

$11080 - 160 - 600 = 5m \times 0.8$

$10320 = 4m$

$m = 2580$

15 (b) Find the tension in the tow bar.

[2 marks]

Trailer (mass $4 \times 2580 = 10320$ kg):

$T - 600 = 10320 \times 0.8 = 8256$

$T = 8856$ N

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15 (c) At the instant the speed of the tractor reaches $18\,\text{km}\,\text{h}^{-1}$ the tow bar breaks.

The total resistance force acting on the trailer remains constant.

Starting from the instant the tow bar breaks, calculate the time taken until the speed of the trailer reduces to $9\,\text{km}\,\text{h}^{-1}$

[4 marks]

After the bar breaks, only the resistance acts on the trailer:

$-600 = 10320a \quad \Rightarrow \quad a = -\frac{600}{10320} = -\frac{5}{86} = -0.05814\ \text{m s}^{-2}$

$u = 18\ \text{km h}^{-1} = \frac{18000}{3600} = 5\ \text{m s}^{-1}$

$v = 9\ \text{km h}^{-1} = 2.5\ \text{m s}^{-1}$

$v = u + at$: $2.5 = 5 - \frac{5}{86}t$

$\frac{5}{86}t = 2.5 \quad \Rightarrow \quad t = 2.5 \times \frac{86}{5} = 43$

Time taken = 43 seconds

END OF QUESTIONS

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